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Base-isolation: Reliability for different design criteria

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ABSTRACT: The degree of protection offered by different design criteria for the seismic isolation of buildings is analyzed. The method used accounts for the non linear response of both the isolator and the structure, and considers the randomness inherent in the seismic motion and in the mechanical parameters of the system. A sensitivity analysis is preliminarly made to assess the separate influence of the various sources of variability on the peak response, this latter expressed in terms of exceedance fractiles of the displacement of the isolator and of the ductility of the structure. The ensuing full probabilistic analysis yields indications on the magnitude of the safety elements that should be introduced in the design procedure in order to ensure the desired reliability against given levels of damage in the isolators and in the structure.

1 INTRODUCTION

Base isolation has now definitely won recognition as a feasible tecnique to reduce the effects of strong seismic motions on the superstructure, to an extent which can be varied from a somewhat reduced vulnerability to a total protection from structural damages. For ordinary types of buildings, the design of a passive isolation system is not a complicated task and suitable procedures are available in the literature (Kelly, 1990; Augenti, Serino, 1991) and are presently included or in the course of being included in the last generation of building codes (SEAOC, 1990; Eurocode 8, 1988). Yet, departing from the universal trend towards risk-based design, comparatively less attention has been paid to the probabilistic calibration of the design procedures aimed at achieving selected levels of protection. This fact stands strangely in contrast with the considerable activity still being spent to assess the design seismic load for non isolated structures in relation to well specified limit-states of damage and collapse. Furthermore, the argument is often heard that the extra cost of isolation is offset by the very fact that the isolated structures are "better" ones: one would expect this attribute be adequately qualified in terms of performances. The study presented in this paper deals with the probabilistic assessment of the levels of protection achievable through isolation depending on the criteria adopted in the design, in particular on the strength conferred to the superstructure. The non linear behaviour of both the isolator and the superstructure are accurately modelled, and the parameters having the major influences on the response quantities of interest are investigated and their random character introduced in the analysis. Using stochastic linearization techniques, the marginal probabilities of exceedance of response displacement thresholds are evaluated and compared with the design requirements. Though the application is made with reference to a specific, if representative case, some of the indications drawn can have a rather general significance.

2 STRUCTURAL MODEL AND SEISMIC INPUT

The isolated structure is modelled as a two degrees-of-freedom system as shown in fig.1).

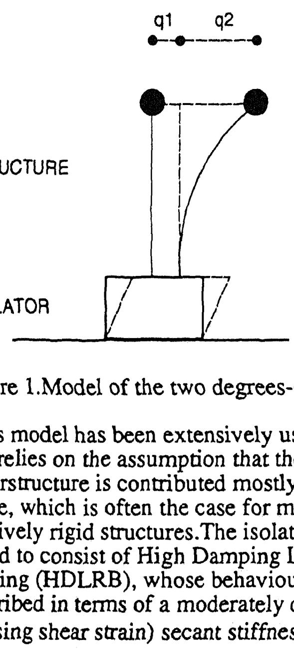

Figure 1. Model of the two degrees-of-freedom system

This model has been extensively used in the literature and relies on the assumption that the response of the superstructure is contributed mostly by its fundamental mode, which is often the case for moderately tall and relatively rigid structures. The isolation system is supposed to consist of High Damping Laminated Rubber Bearing (HDLRB), whose behaviour can be essentially described in terms of a moderately degrading (with increasing shear strain) secant stiffness :k_1 and of an almost stable "equivalent" viscous damping :ξ_1 which can be measured from the hysteretic energy loss per cicle. The supestructure is assigned as a bilinear, hardening, elastoplastic behaviour, characterized by an elastic stiffness k_2, a hardening ratio α_2,and the plastic threshold F_{Y2}.Both force-displacement relationships have been introduced in the analysis in the form:

$$f_i = c\dot{q}_i + \alpha_i k_i q_i + (1-\alpha_i)k_i z_i \qquad i = 1,2 \qquad 1)$$

where q_i is the displacement component associated with f_i, and the auxiliary variable z_i is defined through the following differential equation:

$$\dot{z}_i = A\dot{q}_i - \nu_i |z_i|^n \dot{q}_i - \gamma_i |z_i|^{n-1} z |\dot{q}| \qquad 2)$$

Expression 2)is the well known expression proposed initially by Bouc (1967) and later extended by Wen (1980) which, through a suitable calibration of the four constants : A,ν_i,γ_i,n can be made to represent a wide variety of hysteretic cycles.The seismic motion is modelled as a zero-mean stationary gaussian random process, hence completely defined by its Power Density Spectrum (PDS) for which the classical form of Kanai-Tajimi has been retained.The two parameters of the spectrum, namely the central frequency ω_g and the damping factor ξ_g have been calibrated so as to match as closely as possible the PDS corresponding to the elastic response spectrum of Eurocode 8, for intermediate soil conditions and 5% damping.The response spectrum of EC8 (for a peak ground acceleration of 0.35 g) is shown in fig.2a), while in fig.2b) the associated PDS and the matching Kanai-Tajimi are plotted together for comparison.

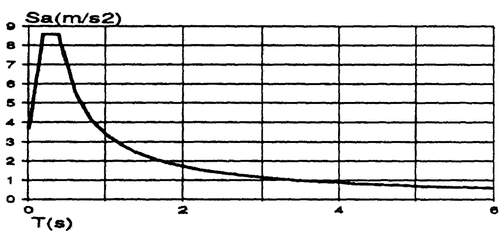

Figure 2a:response spectrum of EC8

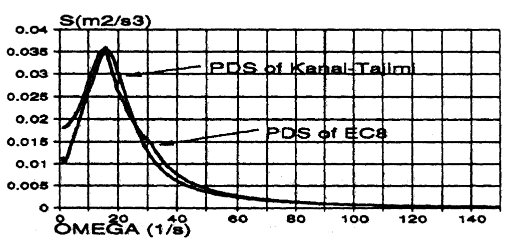

Figure 2b:PDS of EC8 and of the matching Kanai-Tajimi($\omega_g = 18.5;\xi_g = 0.55$)

The reason for having a close approximation is in order to have consistency between the design stage, in which the response spectrum is used, and the subsequent reliability analysis, based on a random processes characterization.

3 METHOD OF ANALYSIS

Among the various alternatives available to obtain the response characteristics of the non linear system in fig.1), statistical linearization has been chosen for its simplicity.Gaussianity of the response has also been assumed, without attempting to remove this approximation using for ex.higher-order expansions of the probability density of the response (Roberts-Spanos, 1990).This approach is justified by the fact that the interest of the study lies not so much in the precision of, say, the exceedance probabilities in absolute terms, but rather on their sensitivity to the design parameters.A summary of the procedure used is given in the following.For the system in fig.1), four equations are written, the first couple expressing the dynamic equilibrium of the masses, the second one giving the hysteretic components of the restoring forces:

$$m_1\ddot{q}_1 + c_1\dot{q}_1 + \alpha_1 k_1 q_1 + (1-\alpha_1)k_1 z_1 - c_2\dot{q}_2 - \alpha_2 k_2 q_2 - (1-\alpha_2)k_2 z_2 = -m_1\ddot{y}_0$$

$$m_2(\ddot{q}_1 + \ddot{q}_2) + c_2\dot{q}_2 + \alpha_2 k_2 q_2 + (1-\alpha_2)k_2 z_2 = -m_2\ddot{y}_0$$

$$\dot{z}_1 = G_1(\dot{q}_1 z_1) = A_1\dot{q}_1 - \nu_1\dot{q}_1|z_1|^n - \gamma_1|\dot{q}_1|\,|z_1|^{n-1}z_1 \qquad 3)$$

$$\dot{z}_2 = G_2(\dot{q}_2, z_2) = A_2\dot{q}_2 - \nu_2\dot{q}_2|z_2|^n - \gamma_2|\dot{q}_2|\,|z_2|^{n-1}z_2$$

where $\ddot{y}_0$represents the excitation process at the base.Upon introducing the response vector:

$q^T = \{q_1, q_2, z_1, z_2\}$the set of eq.3) can be cast in the form:

$$M\ddot{q} + C\dot{q} + Kq + \Phi(\dot{q}) = Q \qquad 4)$$

with the matrices M,C,K and vectors $\Phi(\dot{q}), Q$ being easily identified by comparing eq.3) with eq.4).Linearization implies replacement of eq.4) with the equivalent set :

$$(M + M_e)\ddot{q} + (C + C_e)\dot{q} + (K + K_e)q = Q \qquad 5)$$

the criterion for equivalence being the minimization of the expected value of the norm of the vector difference ε between the original and the replacement sets.If q is assumed to be jointly gaussian, for the chain-type system under study the minimization equations yielding the terms of the matrices M_e, C_e, K_e are uncoupled, and for the particular set of eq.3) one gets (i=1,2) :

$$m_i = 0 \; ; \; c_i = E\left[\frac{dG_i(\dot{q}_i, z_i)}{d\dot{q}_i}\right] ; \; k_i = E\left[\frac{dG_i(\dot{q}_i, z_i)}{dz_i}\right] \qquad 6)$$

Since the derivatives of the functions $G_i(.)$ as defined in eq.3) exist and are continuos, the expectations in eq.6) are easily evaluated, either in a closed form or numerically, once the joint density of the response quantities $\dot{q}_i, z_i$ is known.This is accomplished by means of an iterative procedure on the linearized set 5).For the treatment of the input-output relationships in the linear system 5) the frequency domain approach has been adopted in this study.That is, by denoting by $A(\omega)$ the matrix:

$$A(\omega) = [-\omega^2(M + M_e) + i\omega(C + C_e) + (K + K_e)]^{-1} \qquad 7)$$

one has for the PDS of q:

$$S_q(\omega) = A(\omega)S_Q(\omega)A^{T*}(\omega) \qquad 8)$$

where $S_Q(\omega)$ is the 4x4 cross spectral density matrix of Q and $A^{T*}(\omega)$ is the complex conjugate of the transposed of $A(\omega)$.The matrix $S_q(\omega)$, in particular its two diagonal terms $S_{q_i,q_i}(\omega)$ i=1,2, are used to calculate the first three moments :

$$\lambda_i = \int_0^{\omega_{max}} \omega^r S_{q_{1,2}}(\omega) d\omega \qquad r = 0, 1, 2 \qquad 9)$$

which, following Vanmarke(1976), are the sole quantities necessary for evaluating the exceedance probabilities of the peak responses.In presenting the results of the analysis, selected fractiles of the peak responses q_1 and q_2 are more useful than the full distributions.The expression adopted for the fractile q^* of generic component q reads:

$$q^* = r^* \sigma_q \qquad 10)$$

where σ_q is the standard deviation of q and r^* is the so-called peak-factor, depending on λ_i and on the selected value of the exceedance probability.

4 DESIGN CRITERIA FOR THE ISOLATED SYSTEM

The present application refers to a building having a rigid-base period T_0=0.5 sec.This value is deemed to fall about in the middle of the range of periods for which isolation is normally adopted.The criterion adopted for the isolation consists in lengthening the period of the system to a value : $T_1 = 3T_0 = 1.5$ sec, which is again a common choice and leads in this case to a period well inside the low energy portion of the excitation.For the modal mass of the superstructure : m_2, imagined to correspond to a five storey building, a value of $m_2 = 800000\ Kg_m$ is assumed while for m_1 a fifth of the former is retained, i.e.$m_1 = 160000\ Kg_m$.The elastic response spectrum of EC8 shown in fig.2a), for a peak ground acceleration of 0.35 g and assuming for the HDLRB system an equivalent damping of 10% yields the values of 1.62 $\frac{m}{s^2}$ for the acceleration and of 0.09 m for the displacement.With the elements above the design of both the isolation and the superstructure becomes straightforward.The effective stiffness and the damping of the isolator must in fact satisfy the relations:

$$K_1 = (m_1 + m_2)\left(\frac{2\pi}{T_1}\right)^2 = 16830\frac{KN}{m} \qquad 11)$$

$$\xi_1 = \frac{1}{4\pi}\frac{\Delta W}{W} = 0.10$$

where ΔW is the total energy loss in one cycle with displacement amplitude δ=0.09m, and W is the corresponding elastic energy defined as : $W = \frac{1}{2}k_1\delta^2$.The force-displacement relationship thus obtained is shown in fig.3a), together with the value of the parameters of the hysteretic model of eq.1) and eq.2).The stiffness of the superstructure is obtained through the relation:

$$k_2 = m_2\left(\frac{2\pi}{T_0}\right)^2 = 126300\frac{KN}{m} \qquad 12)$$

while the maximum inertia force to which it may be subjected is :

$$F_2 = m_2 S_a = 800*1.62 = 1300KN \qquad 13)$$

Three design criteria are used to derive the yield strength : F_{Y2} of the superstructure, namely :

a)$F_{Y2} = F_2 = 1300KN$

b)$F_{Y2} = \frac{F_2}{1.5} = 867KN$

c)$F_{Y2} = \frac{F_2}{2.5} = 520KN$

Alternative a) corresponds to the "full isolation " concept, as it is normally implemented, while alternatives b) and c) are along the line of the SEAOC(1990) recommendations, which indicate a factor equal to $R_w/2$ where R_w is the force reduction factor (behaviour factor) that would be used for the building if it were not isolated (for r.c.structures R_w might typically be in the order of 6 to 8).

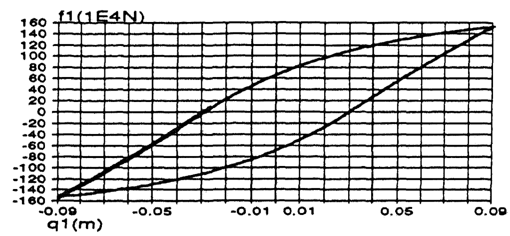

Figure 3a:force-displacement relationship of the isolator with:
$c_1 = 0 \quad \alpha_1 = 0 \quad k_1 = 1515 \quad A = 19 \quad \nu = 10$
$\gamma = 6 \quad n = 1$

5 RANDOM PARAMETERS

Almost all of the parameters entering the design procedure are characterized by a more or less pronounced degree of uncertainty and/or variability.The major source of uncertainty is associated with the seismic input, which in the present case is defined in terms of a spectral shape and by the peak ground acceleration : a_g as a scaling factor .This latter is not treated as r.v. here, the aim of the study being that of assessing the level of protection provided by the design procedure, given a particular value of a_g.Performing the convolution between the conditional probabilities of exceedance of the response as functions of a_g, and the probability density of a_g would actually subtract interest to the study, since the operation would mix together the effects of the design procedure, which is of general application, with those of the seismic hazard, which is site-specific.The frequency content of the motion, on the other hand, is unavoidably specifiable but with considerable uncertainty for any given site, and this source of variability (incomplete knowledge) should be introduced in the study since it could potentially impair the effectiveness of the isolation procedure.This has been implemented by assuming the central frequency ω_g of the Kanai-Tajimi spectrum to be a random variable.Specifically ω_g is taken as normally distributed, with a mean value giving the best fit with EC8 spectrum and a c.o.v.of 0.20.Thus, ω_g=N(18.5,3.70).The attention has then been focussed on the effective stiffness of the isolator, in consideration of its importance in keeping the period of the isolated structure at the desired distance from the predominant period of the seismic motion as well as that of the superstructure, and on the stiffness and strength of the superstructure owing to the well recognized uncertainty which affects their determination on one side, and to the obvious relevance of these two quantities on the response.The variability of k_1 stems from a double source :the random scatter within samples due to the production process, and the (unsystematic) inaccuracy of the analytical model adopted.The effects of the two sources have been lumped together, yielding for k_1 a normally distributed variable with mean value equal to the design one and a c.o.v.of 0.10.Thus : k_1=N(16830,1683).k_2 is also gaussian, with mean value equal to the design value and a c.o.v.of 0.20.Thus k_2=N(126300,25300), while F_{Y2} has been modelled as a lognormal, with mean value equal to the design one and a c.o.v.of 0.20 .Thus :F_{Y2}=LN(1300,260).

6 SENSITIVITY ANALYSIS

The response of the system is described by the two variables q_1, q_2, the former giving the horizontal deformation of the isolator, the latter the deformation of the structure with respect to the base.A sensitivity analysis for the fractile peak values of these two variables as function of the parameters discussed in the previous paragraphs has been carried out preliminarly to the full probabilistic analysis to ascertain their relative importance and to possibly discard those of them proving to be as scarcely influent on the response.The parameters $\omega_g, k_1, k_2, F_{Y2}$ have been made to vary by 2.5 times their standard deviations from either side of the mean.The results are illustrated in the figures 4) and 5) which give the 50% and 90% fractiles of the peak response q_1 and $\overline{q}_2$ as functions of some of the parameters above.The response $\overline{q}_2$ has been normalized with respect to the yield displacement s_{Y2}.

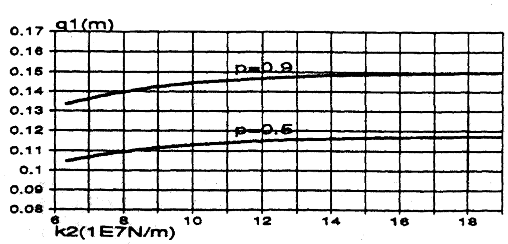

Figure 4a:response of the isolator versus the structure's initial stiffness

When varying a single parameter, all the remaining ones are kept at their mean values.For brevity, the dependence of q_1 and $\overline{q}_2$ from the central frequency of the motion and from the stiffness of the isolator is not shown with the aid of diagrams, given that these effects are quantitatively modest.In fact, the response q_1 and $\overline{q}_2$ decreases with increasing ω_g, which is easily explainable since it corresponds to an increasing separation between the frequency of the motion and of the structure.Note however that for the lowest considered value for ω_g (to which corresponds a value of 0.004 of the distribution function) the corresponding increase in $q_1, \overline{q}_2$ does not exceed 10%.Besides, it is found that the response of the isolator decreases with increasing k_1 (max 20%), while that of the structure decreases when k_1 decreases (max 40%).Looking now at fig.4a) one can observe that for the considered range of variation the response of the isolator is almost insensitive to the stiffness (and hence to the period) of the superstructure, a fact which might not be unexpected.

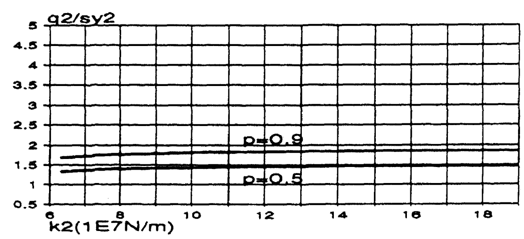

Figure 4b:response of the structure versus its initial stiffness

Less predictable would have been the almost flat dependence of $\overline{q}_2$ from the stiffness of the superstructure (fig.4b)).In fact if the values of the actual displacement :q_2 were plotted instead of the ductility ratio $\overline{q}_2$, the diagrams would show a decrease of q_2 with increasing k_2, a fact that could be seen as consistent with the corresponding progressive decoupling between superstructure and isolator.From fig.4b) one learns further that with the strength of the structure held constant, the demand of ductility on it is not influenced by variations in the stiffness.The obvious conclusion cannot be other than the ductility demand on the structure is essentially governed by its own strength , a fact which is clearly confirmed by the following fig.5b) which shows how sensitive $\overline{q}_2$ actually is to a reduction of F_{Y2} with respect to its mean value and, in the opposite direction, how far one must go to get a high probability of an entirely elastic response.Finally, fig.5a) indicates that the strength of the superstructure does not affect noticeably (except when it takes very low values) the response of the isolator, a result that is not difficult to accept.

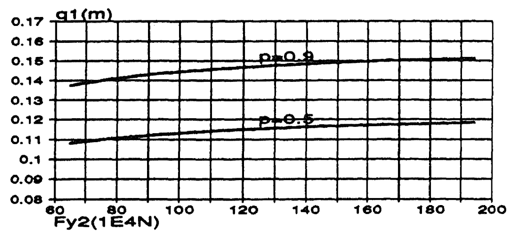

Figure 5a:response of the isolator versus the structure's yield strength

There are two general indications of different nature that can be derived from the sensitivity analysis discussed so far .One is the substancial stability of the response quantities q_1 and $\overline{q}_2$ with respect to major design variables such as the central frequency of the input motion and the stiffness of both the isolator and the structure : this is already a result by itself, and a favourable one since it allows to concentrate the efforts for calibration in a more restricted space of variables.This space appears to contain essentially the strength of the structure, whose role as key parameter was certainly, though not exclusively, anticipated.

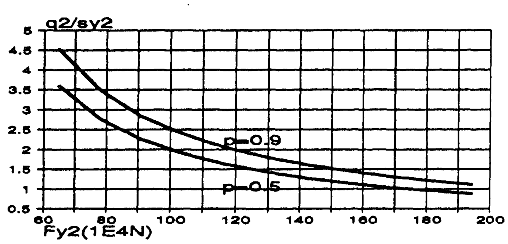

Figure 5b:response of the structure versus its yield strength

The second indication is that the adopted design procedure does give less than complete protection against the exceedance of the design control parameters, i.e.the maximum displacement of the isolator and the yielding of the structure.In the example discussed in this study the value of q_1 to be adopted for the design of the isolator was 9 cm, while from figs.4) to 5), one can note that for the average values of all the parameters involved there are 50% probabilities that q_1 exceeds 11.5 cm, and 10% that it may exceed about 15 cm.Under the same conditions, there are 50% probabilities that the structure enters into yielding with a ductility factor grater than about 1.4, and 10% that this factor may be in excess of 1.8.It is behind the scope of the present work to discuss whether this amount of protection is adequate or not; it pertains to it, however, to point out that greater protection, if desired, calls for the introduction of appropriately calibrated safety factors, whose order of magnitude can already be quantified based on the present results.As a final remark concerning the sensitivity analysis, one might observe that the parametric analysis has been conducted within ranges that appear to be rather restricted, since ±2.5 times the standard deviation means a maximum variation (for a normal variate with a c.o.v.of 0.2) of 50% above and below the mean value.This choice is justified in view of the subsequent reliability analysis, if one considers that the probability of being outside 2.5 standard deviations from the mean is practically negligible for all common probabilistic models, but it has an important implication : large uncertainties and/or errors (of whatever origin) on the mean values are not covered by the present approach (indeed isolation should better be renounced when uncertainties are too pervasive).

7 RELIABILITY EVALUATION FOR DIFFERENT DESIGN CRITERIA

A full probabilistic analysis has been performed for the two response variables $q_1, \overline{q}_2$, accounting not only of the stochasticity of the excitation, but also of the randomness of the input and system parameters described in Sect.5.Operatively, this has been accomplished through the use of conditional probabilities.In general terms, if $\underline{\Theta}$ is the vector of the conditioning variables $\underline{\Theta}^T \equiv \{\omega_g, k_1, k_2, F_{Y2}\}$, first the distribution of the response has been evaluated as a function of $\underline{\Theta}$: $F_Q(q/\underline{\Theta})$ for discrete values of the vector components.The marginal distribution has then been numerically obtained from:

$$F_Q(q) = \int_\Theta F_Q(q/\Theta) f_\Theta(\underline{\Theta}) d\underline{\Theta} \qquad 14)$$

where $f_\Theta(\underline{\Theta})$ is the probability density function of $\underline{\Theta}$.Guided from the outcome of the sensitivity analysis, it has been decided to disregard the parameter ω_g, owing to its very limited effect on the response.By the same reasoning, k_2 might have been eliminated as well (see fig.4a) and 4b)) but finally this variable was retained.Looking at the figures 4) and 5), however, the effect of the unconditioning cannot be expected to be significant for q_1 at all, while for $\overline{q}_2$ it all depends on the variability of F_{Y2}.The results of this final step of the study are displayed in the figs.6) and 7) for the 50% peak fractiles of q_1 and $\overline{q}_2$, respectively, with the addition of a few 90% fractile points.

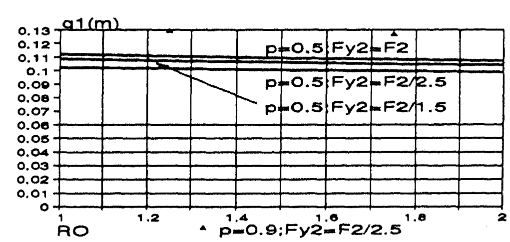

Figure 6:response of the isolator versus ρ

The diagrams give the variation of the two parameters as a function of a factor ρ which has the effect of proportionally increasing the c.o.v.'s of the three random variables considered.For example, ρ=2 means that the result is relative to a c.o.v.equal to 0.20 for k_1 and to

0.40 for k_2 and F_{Y2}.Looking first at the values for $\rho=1$, one sees that the unconditioning leaves the 50% fractiles almost unchanged (see figs.from 4) to 5)) with respect to the values obtained for the average values of k_1, k_2 and F_{Y2}: this result does not come much to surprise, since we are dealing with the central (median or mean) values of the distributions of the response.Less easy to anticipate is perhaps the fact that even for an increase of up to two times of the c.o.v.of the basic variables the central values are not significantly altered.

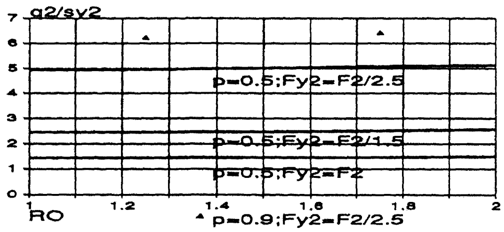

Figure 7:response of the structure versus ρ

But the result of main interest from figs.8) and 9) is the effect of the reduction of the strength of the structure, down from the value required to resist elastically the response spectrum.Reduction factors R_w equal to 1.5 and 2.5 have been adopted.While the effect on q_1 is not of much consequence (reductions of the order of 10% for R_w passing from 1 to 2.5), that on the required ductility is quite drastic, since the demand increases more than proportionally with R_w.For R_w=2.5, there is a probability of 50% that $\overline{q}_2$exceeds the value of 5, and still 10% that this value can be more than 6.This sensitivity has been already pointed out, though in different contexts, in other studies: see for ex.Vestroni et alii(1991).One might wonder whether it does make enough sense to provide an isolation of a sort that, upon the occurence of the design event, leads to a ductility demand higher that one would normally accept for a non isolated structure.

8 CONCLUSIONS

A systematic investigation carried out to find the parameters having the major influence on the response of the buildings isolated at their base by means of HDLRB has brought the following main points to the fore.The commonly used deterministic design procedure based on a model consisting of a secant elastic viscous isolator and an elastic superstructure, both to be dimensioned according to an elastic response spectrum possibly with a reduction factor for the design of the structure, leaves the system substancially unprotected, should the postulated design event occur, against the exceedance of the calculated design thresholds.To exemplify, if the superstructure is designed to resist at yield the elastic response spectrum without reduction, there are significant probabilities that it will actually be subjected to a ductility demand of the order of 2.To remedy this, partial safety factors should be introduced in the design procedure, for both the isolator and the structure.In the case examined, a factor of about 1.5 to increase the design strength of the structure and the design displacement of the isolator would be adequate for the purpose.The criterion for isolation adopted in the present study: a factor of 3 between the periods of the isolated and the rigid-base structure, has proven to be sufficient for covering even important deviations of the controlling parameters from their design values, namely the central frequency of the input motion and the two stiffnesses of the isolator and of the structure.The variations in the fractile peak responses associated with these deviations were actually found to be of minor importance.On the other hand, the strength conferred to the structure has confirmed to play a dominant role in controlling the ductility demand, a role greater and less predictable than in the case of non isolated structures.This result might favour the conclusion that base-isolation of the type discussed in the present study would more aptly resorted to in cases where the exploitation of the structure's ductility can be avoided.Otherwise, the critical sensitivity of this type of systems to the strength of the structure should be carefully accounted for, and force-reduction factors only derived on the basis of probabilistic calibration studies of the kind indicated in this paper.

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