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Impedance functions of strip foundations on fluid-saturated porous media

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ABSTRACT: A two-dimensional boundary element formulation is developed for the analysis of steady-state vibration of a strip footing bonded to a fluid-saturated porous half plane. The governing equations of the medium are derived by applying the Fourier transform to Biot's equations of dynamic poroelasticity and replacing the fluid displacements by the fluid pressure. The boundary integral formulation is obtained through the application of the Galerkin method to the governing equations of the medium. The fundamental solutions pertaining to this foumulation, which define the solid displacement field and the fluid pressures due to point forces in the solid and a point source in the fluid, are obtained by a procedure developed by Kupradze. The results presented in this paper display the variations with frequency of the real and imaginary parts of the impedance functions of strip footings and the effect of soil permeability on these functions.

1 INTRODUCTION

Dynamic poroelasticity has applications in numerous branches of science and engineering, including geophysics, biomechanics and earthquake engineering. The three-dimensional theory of this problem was first developed by Biot (1956, 1962). According to this theory, a dynamic disturbance in a fluid-saturated porous medium generates one transverse (shear) wave and two longitudinal (pressure) waves. The shear and one of the pressure waves exhibit characteristics which are essentially similar to those of elastodynamics. The other pressure wave, on the other hand, is a highly-attenuated low-velocity wave which is associated with the out-of-phase motions of the constituents. More modern theories of continuum mechanics, such as the theory of mixtures (Truesdell and Toupin (1960) and Bowen (1976)) have resulted in equations with similar characteristics (see, e.g., Garg (1971), Prevost (1980), Bowen (1982) and Auriault (1980)).

Boit's equations are in terms of the solid and fluid displacement fields (u-w model). To use these equations more conveniently they are often recast in terms of the solid displacement field and the fluid pressure (u-p model) in a transformed domain. Such formulations have been utilized by Bonnet (1987), Boutin et al. (1987) and Kaynia (1990) to derive the fundamental solutions of dynamic poroelasticity and by Zienkiewicz and Shiomi (1984), Zienkiewicz et al. (1987) and Bougacha and Tassoulas (1991) to solve practical boundary value problems by the finite element method. More recently, Cheng et al. (1991) developed a boundary integral formulation for the steady-state vibrations of porous media and demonstrated its effectiveness by solving a number of soil dynamics problems.

To investigate the significance of pore water in soil-structure interaction problems a number of researchers have attempted to study the dynamic behavior of rigid foundations on fluid-saturated media. These studies have invariably been carried out by using (u-w) models. Gazetas and Petrakis (1987) numerically evaluated the compliance of a poroelastic half space for swaying and rocking motions of a rigid pervious strip footing. Halpern and Christiano (1986) derived Green's functions associated with steady-state harmonic concentrated forces applied to the solid and fluid phases at the surface of the half space and used them to numerically calculate the vertical compliance function of a rigid disk on a porous half space. A more complete solution of this problem, accounting for soil layering, has recently been presented by Philippacopoulos (1989).

The purpose of this paper is to present a (u-p) boundary integral formulation for poroelastodynamics and the implementation of a boundary element method for the calculation of steady-state impedance functions of strip foundations on a porous half plane. The boundary integral formulation, which is essentially similar to that developed by Suh and Tosaka (1989), is obtained by the application of the Galerkin method to the equations of poroelastodynamics, and the associated funda mental solutions are derived by the method developed by Kupradze (1979) for thermoelasticity.

2 GOVERNING EQUATIONS

Following the procedure outlined by Zienkiewicz et al. (1987) and Boutin et al. (1987), one can write the equations expressing respectively, the conservation of total momentum, the flow conservation for the fluid phase, the constitutive equation for a poroelastic solid and the generalized Darcy's law, as

$$\sigma_{ij,j}+f_i = \rho\ddot{u}_i+\rho_f\ddot{w}_i \quad (1)$$

$$\dot{w}_{k,k}+\alpha\dot{u}_{k,k}+\frac{1}{Q}\dot{p} = q \quad (2)$$

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij}-\alpha p\delta_{ij} \quad (3)$$

$$p_{,i} = -\frac{1}{k}\dot{w}_i-\rho_f\ddot{u}_i-m\ddot{w}_i \quad (4)$$

where p denotes the fluid pressure, u represents the displacemsnts of the solid skeleton and w denotes the average displacements of the fluid relative to the solid. λ and μ are the Lame constants for the solid skeleton. ρ_f is the mass density of the fluid, ρ is that of the solid-fluid mixture and $m = \rho_f/n$ is another mass parameter, with n denoting the porosity. α and Q are material parameters which describe the relative compressibilities of the constituents and are given by

$$\alpha = 1-K_d/K_s \quad (5)$$

$$1/Q = n/K_f+(\alpha-n)/K_s \quad (6)$$

where K_f and K_s denote the bulk moduli of the fluid and the solid granis, respectively, and K_d denotes that of the solid skeleton. Finally k is the coefficient of permeability of the medium and f and q denote the body force and the rate of fluid injection into the medium, respectively.

Assuming a steady-state harmonic vibration, for which the temporal variation of u(t) can be expressed as $\bar{u}\,e^{i\omega t}$, one can eliminate w from eqns (1)-(4) to arrive at the following coupled differential equations of dynamic poroelasticity:

$$(\lambda+\mu)\,\bar{u}_{j,ij}+\mu\bar{u}_{i,jj}-\alpha_1\bar{p}_{,i}+\omega^2\rho_1\bar{u}_i+\bar{f}_i = 0 \quad (7)$$

$$\xi\bar{p}_{,ii}-\frac{i\omega}{Q}\bar{p}-i\omega\alpha_1\bar{u}_{i,i}+\bar{q} = 0 \quad (8)$$

where $\xi = (i\omega m+1/k)^{-1}$, $\rho_1 = \rho-i\omega\rho_f^2\xi$ and $\alpha_1 = \alpha-i\omega\rho_f\,\xi$.

It is interesting to note that the transformed equations of poroelasticity (i.e., eqns. (7) and (8)) resemble those of generalized thermoelasticity (Suh and Tosaka (1989)).

3 BOUNDARY INTEGRAL FORMULATION

In the present paper the Galerkin method is employed to develop a boundary integral formulation. To this end, the eqns (7) and (8) are expressed as

$$L_{ij}\bar{U}_j = \bar{B}_i \quad (9)$$

where the matrix differential operator L and the force vector $\bar{B}$ for the plane strain case are given by

$$L_{ij} = \begin{bmatrix} \mu\Delta+(\lambda+\mu)D_1^2+\omega^2\rho_1 & (\lambda+\mu)D_1D_2 & -\alpha_1D_1 \\ (\lambda+\mu)D_2D_1 & \mu\Delta+(\lambda+\mu)D_2^2+\omega^2\rho_1 & -\alpha_1D_2 \\ -i\omega\alpha_1D_1 & -i\omega\alpha_1D_2 & \xi\Delta-\frac{i\omega}{Q} \end{bmatrix} \quad (10)$$

$$\bar{U}_j = \{\bar{u}_1 \quad \bar{u}_2 \quad \bar{p}\}^T \quad (11a)$$

$$\bar{B}_j = \{-f_1 \quad -f_2 \quad -\bar{q}\}^T \quad (11b)$$

Also $D_i = \partial/\partial x_i$ and Δ denotes the Laplacian.

Defining G^*_{ik} as the weighting tensor, one can write the following weighted residual statement for eqn (9):

$$\int_v (L_{ij}\bar{U}_j-\bar{B}_i)G^*_{ik}\,dv = 0 \quad (12)$$

Using eqns (10) and (11) in eqn (12) and integrating by parts, one obtains the following integral equation:

$$\int_v(A_{ij}G^*_{jk})\bar{U}_i\,dv(\mathbf{x})+\int_s(\bar{\sigma}_\alpha G^*_{\alpha j}-\bar{u}_\alpha\tau^*_{\alpha j})ds(x)$$

$$+\int_s(\xi\frac{\partial\bar{p}}{\partial n}G^*_{3j}-\xi\bar{p}\frac{\partial G^*_{3j}}{\partial n})ds(\mathbf{x})-\int_v\bar{B}_iG^*_{ij}\,dv(\mathbf{x})=0$$

$$(i,j,k=1,2,3 \quad , \quad \alpha=1,2) \quad (13)$$

where $\bar{U}_\alpha = \bar{u}_\alpha$ $(\alpha=1,2)$ and $\bar{U}_3 = \bar{p}$, and v and s denote the domain and its boundary, respectively. The traction vector $\bar{\sigma}_\alpha$ and the corresponding vector $\tau^*_{\alpha j}$, associated with G^*, are given by

$$\bar{\sigma}_\alpha = \bar{\sigma}_{\alpha\beta} n_\beta = \{(\lambda \bar{u}_{k,k} - \alpha_1 \bar{p})\delta_{\alpha\beta} + \mu(\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha})\} n_\beta \quad (14)$$

$$\tau^*_{\alpha j} = \{(\lambda G^*_{kj,k} + i\omega\alpha_1 G^*_{3j})\delta_{\alpha\beta} + \mu(G^*_{\alpha j,\beta} + G^*_{\beta j,\alpha})\} n_\beta \quad (15)$$

Also A is the adjoint operator of L (eqn (10)) with entries similar to those of L except that $A_{31} = -L_{13}$, $A_{32} = -L_{23}$, $A_{13} = -L_{31}$ and $A_{23} = -L_{32}$.

If G^*_{jk} satisfies the following equation

$$A_{ij} G^*_{jk} + \delta_{ik}\delta(\mathbf{x}-\mathbf{y}) = 0 \quad (16)$$

then the first ferm in eqn (13) can be replaced by $-c_{kj}\bar{U}_k(\mathbf{y})$ where $c_{kj} = \frac{1}{2}\delta_{kj}$ for a smooth boundary and **x** and **y** represent field point and singularity point, respectively. Incorporating this result in eqn (13) and assuming zero body force, $\bar{B}$, ond obtains the following Somigliana-type integral equations:

$$c_{kj}\bar{U}_k + \int_S \bar{u}_\alpha \tau^*_{\alpha j}\, ds + \int_S \xi \bar{p} \frac{\partial G^*_{3j}}{\partial n}\, ds = \int_S \bar{\sigma}_\alpha G^*_{\alpha j}\, ds + \int_S \xi \frac{\partial \bar{p}}{\partial n} G^*_{3j}\, ds \quad (17)$$

which can be expressed in matrix form as:

$$\underset{\sim}{c}\,\bar{\underset{\sim}{U}} + \int_S \underset{\sim}{P}^* \bar{\underset{\sim}{U}}\, ds = \int_S \underset{\sim}{U}^* \bar{\underset{\sim}{P}}\, ds \quad (18)$$

where $\underset{\sim}{c}$ is a diagonal matrix and

$$\underset{\sim}{U}^* = \begin{bmatrix} G^*_{11} & G^*_{21} & \xi G^*_{31} \\ G^*_{12} & G^*_{22} & \xi G^*_{32} \\ G^*_{13} & G^*_{23} & \xi G^*_{33} \end{bmatrix}, \quad \bar{\underset{\sim}{U}} = \begin{Bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{p} \end{Bmatrix} \quad (19)$$

$$\underset{\sim}{P}^* = \begin{bmatrix} \tau^*_{11} & \tau^*_{21} & \xi\, \partial G^*_{31}/\partial n \\ \tau^*_{12} & \tau^*_{22} & \xi\, \partial G^*_{32}/\partial n \\ \tau^*_{13} & \tau^*_{23} & \xi\, \partial G^*_{33}/\partial n \end{bmatrix}, \quad \bar{P} = \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \partial\bar{p}/\partial n \end{Bmatrix} \quad (20)$$

To develop a boundary element formulation one needs to solve eqn (18) numerically. This can be achieved by discretizing the boundary into N segments (elements) and using interpolation functions to define the unkowns in terms of the corresponding nodal values. In the present study constant element, with one node in the middle of each element, has been adopted. In this case one needs to write eqn (18) for the N nodes to arrive at a system of algebraic equations of order 3N in the form:

$$\underset{\sim}{H}\,\bar{\underset{\sim}{U}} = \underset{\sim}{G}\,\bar{\underset{\sim}{P}} \quad (21)$$

Introducing the boundary conditions one can solve eqn (21) for the unknown boundary quantities.

4 FUNDAMENTAL SOLUTION

The fundamental solution pertaining to the present boundary integral formulation is the solution of eqn (16). The elements of the fundamental solution tensor for the two-dimension are given by (for the details of the derivation see, e.g., Kupradze (1979))

$$G^*_{\alpha\beta} = \sum_{j=1}^{3} \{\psi_j(r)\delta_{\alpha\beta} - \chi_j(r).r_{,\alpha} r_{,\beta}\} \quad (22)$$

$$G^*_{\alpha 3} = \sum_{j=1}^{3} i\omega r_{,\alpha} \phi_j \lambda_j K_1(\lambda_j r) \quad (23)$$

$$G^*_{3\beta} = \sum_{j=1}^{3} r_{,\beta} \phi_j \lambda_j K_1(\lambda_j r) \quad (24)$$

$$G^*_{33} = \sum_{j=1}^{3} \gamma_j K_0(\lambda_j r) \quad (25)$$

where $r = |\mathbf{x}-\mathbf{y}|$, $r_{,\alpha} = \partial r/\partial x_\alpha$ and

$$\psi_j(r) = \frac{\delta_{3j}}{2\pi\mu} K_0(\lambda_j r) - \eta_j \lambda_j \frac{1}{r} K_1(\lambda_j r) \quad (26)$$

$$\chi_j(r) = -\eta_j \lambda_j^2 K_2(\lambda_j r) \quad (27)$$

K_0, K_1 and K_2 are modified Bessel functions of the second kind of order zero, one and two respectively, λ_j are defined by the following relations

$$\lambda_3^2 = -\rho_1 \omega^2/\mu \quad (28)$$

$$\begin{cases} \lambda_1^2 + \lambda_2^2 = k_1^2 + \frac{i\omega}{\xi}\left(\frac{1}{Q} + \frac{\alpha_1^2}{\lambda+2\mu}\right) & (29) \\ \lambda_1^2 \lambda_2^2 = \frac{i\omega}{Q\xi} k_1^2 & (30) \end{cases}$$

where $k_1^2 = -\omega^2 \rho_1/(\lambda+2\mu)$, and η, ϕ and γ are given by

$$\eta_j = \frac{(-1)^j (1 - i\omega\lambda_j^{-2}/Q\xi)(\delta_{1j}+\delta_{2j})}{2\pi(\lambda+2\mu)(\lambda_2^2 - \lambda_1^2)} + \frac{\delta_{3j}}{2\pi\omega^2\rho_1} \quad (31)$$

$$\phi_j = \alpha_1 \frac{(-1)^j(\delta_{1j}+\delta_{2j})}{2\pi\xi(\lambda+2\mu)(\lambda_2^2 - \lambda_1^2)} \tag{32}$$

$$\gamma_j = \frac{(-1)^j}{2\pi\xi} \frac{(\lambda_j^2 - k_1^2)(\delta_{1j}+\delta_{2j})}{(\lambda_2^2 - \lambda_1^2)} \tag{33}$$

5 NUMERICAL RESULTS

The boundary integral formulation developed in this paper has been used to obtain a set of results for the impedance functions of pervious strip foundations on a porous half plane. The material properties used for the half plane are as follows:

$\lambda = 0.274\times10^7$ KN/m^2 , $\mu = 0.585\times10^7$ KN/m^2

$Q = 0.997\times10^7$ KN/m^2 , $\alpha = 0.83$, $n = 0.195$

$\rho_f = 1000$, $\rho = 2270$ and $m = 5130$ Kg/m^3

The drained elastic properties (λ and μ) correspond to a medium with $E=1.357\times10^7$ KN/m^2 and $\nu=0.16$. These properties are those measured by Yew and Jogi (1978) that have been converted to match the parameters appearing in eqns (1)-(4).

The quantities of interest in this study are the impedance functions (vertical, horizontal and rocking) of rigid strip foundations. Each of these functions is a frequency-dependent complex quantity, the real part of which represents the stiffness and the imaginary part represents the damping of the foundation. The presented results display the variation of the real and imaginary parts of the impedances as a function of the non-dimensional frequency $a_0 = \omega d/C_s$, where d is the width of the foundation and C_s is the shear wave velocity of the medium.

Figure 1 (a and b) shows the variations of the vertical impedance of a strip foundation for three value of permeability: 0.002, 0.02 and 0.2 m/sec. The results in the figure suggest that whereas the vertical stiffness (Fig 1a) decreases with permeability the damping tends to increase (Fig 1b).

Similar trends are displayed by the horizontal and rocking impedance functions, as shown in figures 2 and 3. These figures portray the variations of the horizontal and rocking impedances for three values of $k = 0.002$, 0.01 and 0.02 m/sec.

In order to examine the influence of pore water on the dynamic behavior of a soil mass the horizontal impedance of a highly permeable medium is compared in Fig.4 with that of a dry (one phase) mediuum with the same λ, μ, and ρ. The figure suggests that, in the frequency range of interest, the two media display similar characteristics. This feature, which was also observed for the other impedances (not shown here), suggest that the available results for the impedances of dry media might be used to infer the impedances of highly permeable saturated soil media.

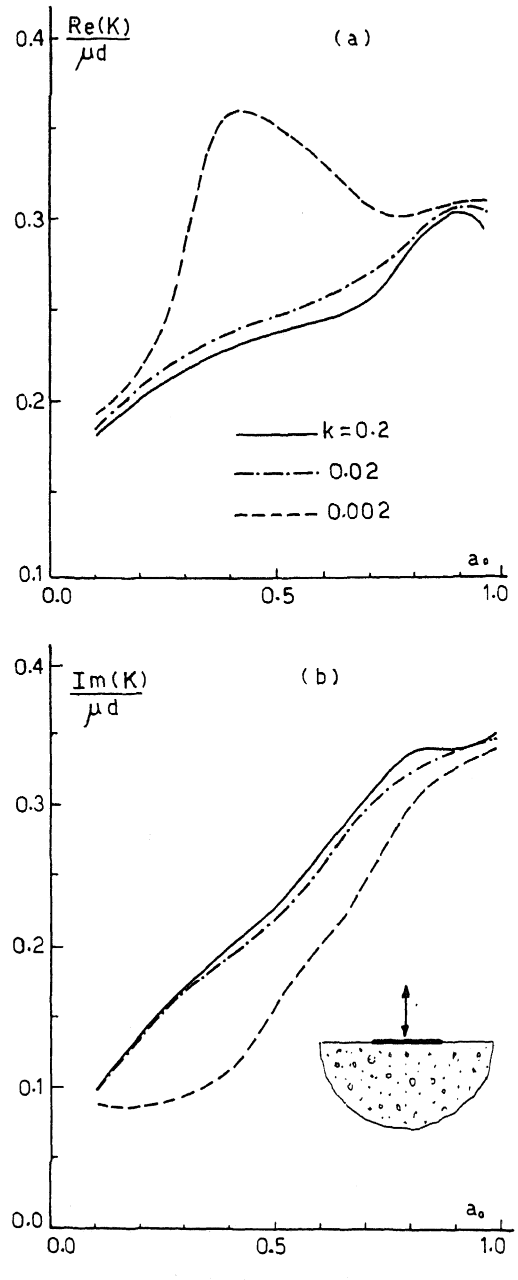
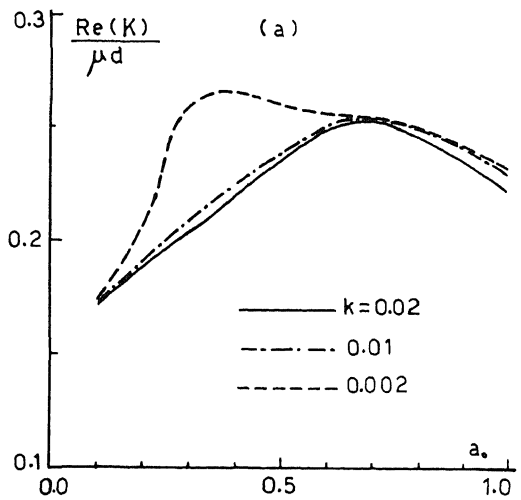


Fig.1 Variations of a) real and b) imaginary parts of the vertical impedance



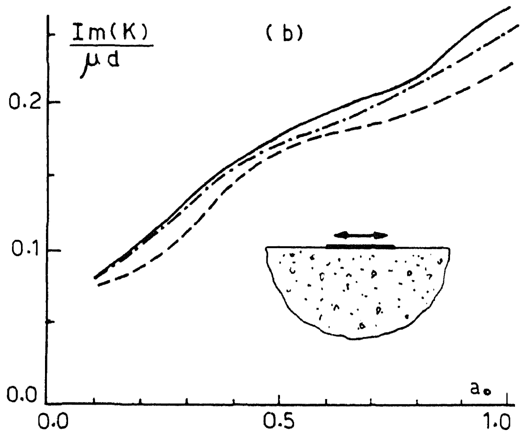


Fig.2 Variations of a) real and b) imaginary parts of the horizontal impedance

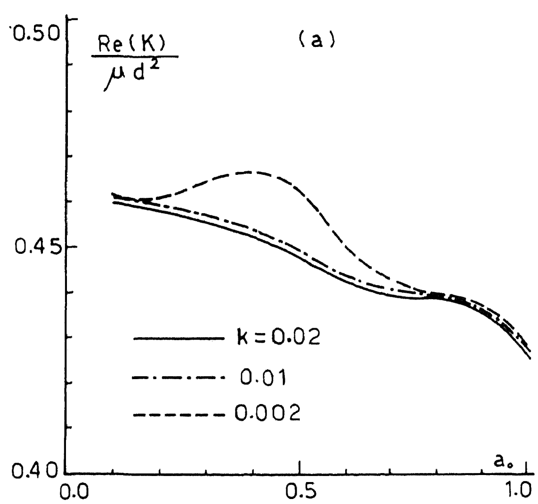


Fig. 3(a)

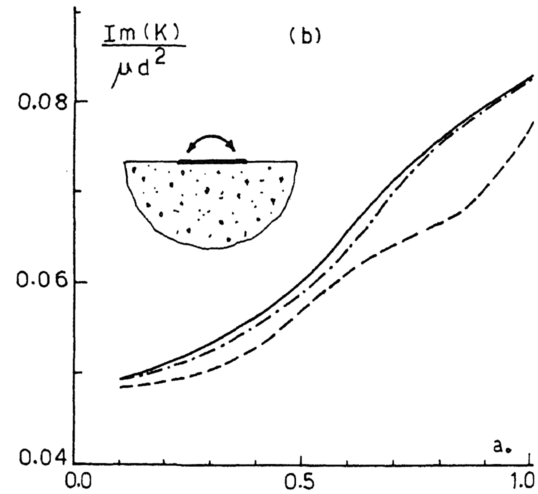


Fig.3 Variations of a) real and b) imaginary parts of the rocking impedance

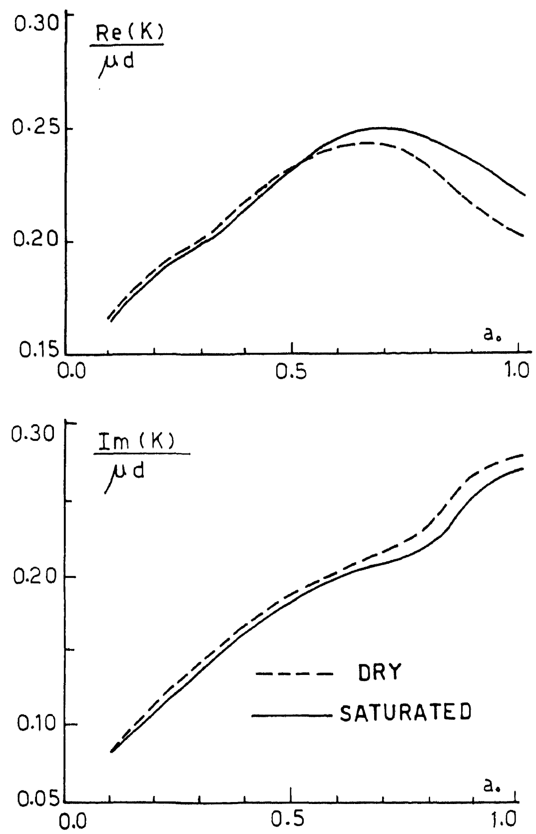


Fig.4 Comparison between horizontal impedances of dry and saturated soil media

6 CONCLUSIONS

In this paper a boundary element formulation for dynamic poroelasticity was presented. The integral equation was obtained by applying the weighted residual method to the equations of dynamic poroelasticity and the required fundamental solution was derived by the Kupradze method. The formulation was used to calculate the impedance functions of a rigid strip foundation under steady-state vibrations. The limited presented results suggested that for a fluid-saturated medium as permeability increases the stiffness of the foundation decreases while the damping increases. Also, the stiffness characteristics of a highly permeable saturated medium is essentially similar to those of the corresponding dry medium.

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