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Effect of friction and restitution on rocking response

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ABSTRACT: The paper examines the limitations of two assumptions commonly imposed on the analysis of rocking systems: sufficient friction to prevent sliding and the occurrence of perfectly plastic impacts with the foundation. In this study the response is evaluated numerically using a program which considers the full range of 2-D motions: rest, slide, rock, slide-rock and free-flight. Response is computed for an ensemble of appropriately scaled artificial earthquake accelerograms, assuming infinite friction and a range of coefficients of restitution. Motion is thereby restricted to pure rocking and free-flight. Results are presented in statistical form and include: occurrence of overturning, distribution of peak rotation and minimum friction required to sustain the motion. Results show that a lower bound on the friction necessary to sustain pure rocking is $.75B/H$, in which B is the width and H the height of the body. Also, a non-zero coefficient of restitution perhaps acts to stabilize the system and reduce the tendency toward overturning.

1 INTRODUCTION

The rocking response of free standing rigid bodies to base excitation has been a topic of interest to researchers for some time. The motivation for much of this work can generally be ascribed to one of two ideas: estimating the peak amplitude of earthquake ground acceleration based on the observed overthrow of free-standing objects, or minimizing the damage to building contents (e.g., hospital equipment, computer equipment, museum artifacts) and other free-standing objects during earthquakes.

Rocking systems have been studied analytically and numerically for a wide variety of ground excitations. In every instance, however, two assumptions have been imposed on the analysis: sufficient friction to prevent sliding during and throughout the motion, and perfectly plastic impacts with the foundation (i.e., bouncing is prohibited). These two assumptions prevent a transition to any other type of motion (e.g., pure sliding, slide-rock or free-flight) and thereby simplify the analysis. The extent to which these assumptions are valid has yet to be investigated.

The paper examines the limitations of the two aforementioned assumptions, in order to establish the range of validity of existing solutions. Response is evaluated numerically using a program developed specifically for analyzing the generalized behavior of free-standing object to base excitation. In the numerical scheme, consideration can be given to the full range of 2-D motions: rest, slide, rock, slide-rock and free-flight. In this study, the response is computed assuming infinite friction and for a range of coefficients of restitution. The motion is thereby restricted to pure rocking and free-flight. Earthquake-induced response is computed for an ensemble of appropriately scaled artificial earthquake accelerograms. Results are presented in statistical form and include: distribution of peak rotation, minimum friction required to sustain the motion and the occurrence of overturning.

2 DEFINITIONS

The system under consideration is shown in Figure 1. A symmetric rigid block of width $2B$, height $2H$, mass m and mass moment of inertia I rests on a moving foundation. The distance between either corner in contact with the foundation, 0 or 0', and the mass center is denoted by R, and the angle measured between R and vertical when the body is at rest is denoted by θ_c (i.e., $\tan\theta_c = B/H$). Coulomb

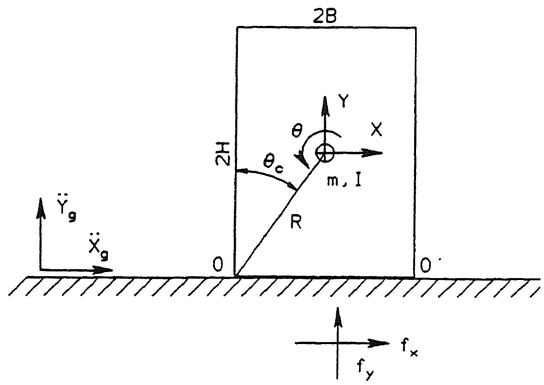

Figure 1: Problem definition

friction acts between the block and foundation, the coefficients of static and dynamic friction are denoted by μ_s and μ_k. Also, the block has a slightly concave bottom, so that contact is made only at the corners when in a vertical position. The horizontal and vertical displacement of the mass relative to the foundation are denoted by $x(t)$ and $y(t)$ respectively, angular rotations are denoted by $\theta(t)$, positive in the counter-clockwise direction. The motion of the foundation is described by the accelerations $\ddot{x}_g(t)$ and $\ddot{y}_g(t)$. In this study, $\ddot{y}_g(t) = 0$.

3 GOVERNING EQUATIONS OF MOTION

A general formulation for analysis of the response of free-standing rigid bodies to base excitation has been presented by Shenton and Jones (1991). The formulation includes the nonlinear equations of motion for a slide, rock and slide-rock mode, as well as small angle approximations for the rock and slide-rock modes. The formulation also includes a model for impact following rock, slide-rock and free-flight motion. The relations governing impact were derived using classical principles (Kane and Levinson 1985), with the effects of friction and restitution considered. Only those equations which are pertinent to the present study are summarized below.

The equation of motion for the only non-trivial mode of interest here, i.e., rocking, is

$$\left(I + mR^2\right)\ddot{\theta} = mR\cos(\theta_c - |\theta|)\ddot{x}_g - S(\theta) mR \sin(\theta_c - |\theta|)(\ddot{y}_g + g) \quad (1)$$

in which $S(\theta) = +1$ for $\theta > 0$ and $S(\theta) = -1$ for $\theta < 0$.

Equation (1) is valid provided $\theta \neq 0$; when $\theta = 0$, an impact occurs between the body and foundation. Impact also occurs as the body comes in contact with the foundation following free-flight. Assuming the duration of the impact is short, the impulsive forces are large, and changes in position and orientation are neglected, the post-impact velocities (denoted by a subscript "2") are defined in terms of the pre-impact velocities (denoted by a subscript "1") for impact following free-flight, as

$$\dot{\theta}_2 = \delta_i \dot{\theta}_1;\ \dot{x}_2 = -\delta_i R C \dot{\theta}_1$$
$$\dot{y}_2 = \eta R S S_{\theta^-}(\delta_i + e) - e\eta\dot{y}_1 \quad (2)$$

where $\eta = +1$,

$$\delta_i = 1 - \frac{3}{4}C^2(1+\lambda_x) - \frac{3}{4}S^2(1+e)(1 - S_{\theta^-}\lambda_y) \quad (3)$$

and

$$\lambda_x = \frac{\dot{x}_1}{RC\dot{\theta}_1};\ \lambda_y = \frac{\dot{y}_1}{RS\dot{\theta}_1}$$
$$S = \sin(\theta_c - |\theta|);\ C = \cos(\theta_c - |\theta|) \quad (4)$$

In (2) and (3), e is the classical coefficient of restitution, which relates the pre- and post-impact linear velocities at the point of impact (corner(s) 0 or 0'), and is in the range $0 < e < 1.0$. Equation (2) is valid provided there is sufficient friction to prevent sliding during the impact, i.e.,

$$\mu_s \geq \left| \frac{C/S(\delta_i + \lambda_x)}{S_{\theta^-}(\delta_i + e) - (1+e)\lambda_y} \right| \quad (5)$$

The relations governing impact during a rock mode are obtained by setting $\theta = 0$, $\lambda_x = -1$, $\lambda_y = -S_{\theta^-}$ and $\eta = -1$ in (2) - (5), with the additional constraint that $\delta_i > -e$.

The impact model described in (2) through (5) guarantees that the velocity of any point in contact with the foundation during impact is greater than or equal to zero after impact (a constraint which must be satisfied for impact following a rock or slide-rock motion). The equations, however, do not explicitly guarantee a decrease in system kinetic energy following impact. This is not an anomaly, but rather indicates a fault of existing impact theory as pointed out by Brach (1984).

4 EARTHQUAKE TIME HISTORIES

An ensemble of 25 artificial earthquake accelerograms was generated for use in the time history analysis. A modified Kanai-Tajimi filter of the form (Clough and Penzien 1975)

$$S(f) = S_0 \frac{1 + (2\xi_g f/f_g)^2}{(1 - (f/f_g)^2)^2 + (2\xi_g f/f_g)^2} \times \frac{(f/f_2)^4}{(1 - (f/f_2)^2)^2 + (2\xi_2 f/f_2)^2} \quad (6)$$

was used to produce a power spectral density (PSD), where S_0 represents the level of white noise and ξ_g, f_g, ξ_2 and f_2 are the filter parameters. In this case, the parameters were chosen to correspond to a "firm soil", with $\xi_g = 0.62$, $f_g = 2.46$ Hz, $xi_2 = 0.62$, and $f_2 = 0.26$ Hz (Kung and Pecknold 1982). The ensemble realizations were generated using frequency increments $\Delta\omega = 0.05$ Hz distributed over the range 0.05 – 25 Hz, with random phase components, distributed uniformly over the range $[0, 2\pi]$.

The accelerograms produced, $a(t)$, were then temporally modulated ($\ddot{x}_g(t) = c(t)a(t)$) using a function of the form (adapted from Grigoriu et al. 1988)

$$|c(t)|^2 = 0.038 \exp[-16(t - 14.5)^2/25] + 0.0073 \exp[-16(t - 13.5)^2/400] \quad (7)$$

which effectively resulted in 30 second time histories with a peak occurring at around 15 sec. The peak acceleration for the record was chosen as one of the variables in the computation (see Section 5, below).

A consistent set of acceleration, velocity and displacement records with a 0.02 sec time interval was then generated by repeated high-pass filtering at 0.07 Hz, followed by integration. This procedure eliminated low frequency drift errors entering the records through the generation procedure and the integration process. The velocity and displacement are needed in this analysis to track the motion of the foundation during free flight. A sample from the ensemble (acceleration, velocity and displacement) is shown in Figure 2.

5 METHOD OF ANALYSIS

For the present study all analyses begin with quiescent initial conditions: motion begins when the amplitude of ground acceleration is sufficient to initiate rocking, i.e., $|\ddot{x}(t)| > B/H$. Once initiated, Eq.(1) is solved using a fifth and sixth order Runge-Kutta routine from the International Mathematics and Statistics Library (IMSL), until a change in sign of θ is noted. At this time the numerical procedure systematically iterates to locate the exact time

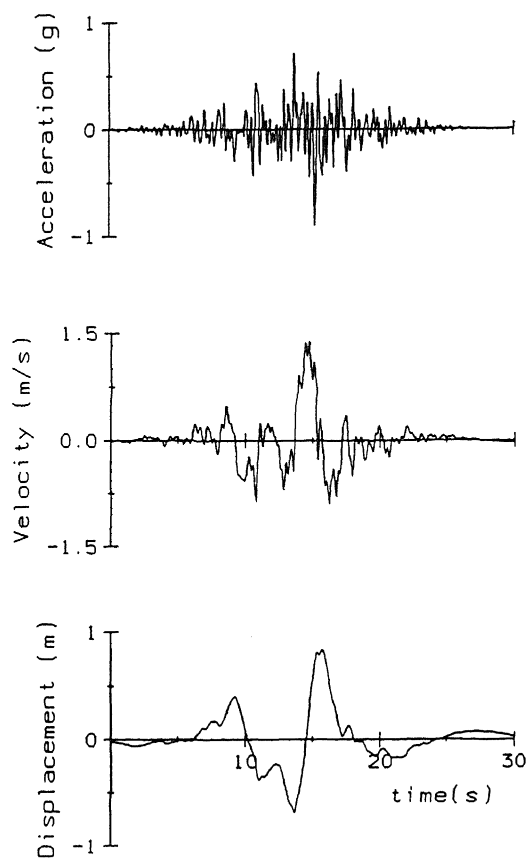

Figure 2: Typical acceleration, velocity, and displacement record set

when $\theta = 0$. Next, the relations governing impact are applied, the ensuing mode is determined and the solution proceeds to the next impact. If the ensuing motion is free-flight, as is usually the case when $e \neq 0$ and the post-impact velocities are of sufficient magnitude, motion is computed using closed-form expressions until the impact with the foundation is detected.

The response has been computed for a range of system parameters: base width $2B = 0.5$ m; aspect ratio $H/B = 2, 4, 6$; coefficient of restitution $e = 0.0$, 0.3, 0.6, 0.9; and peak amplitude of ground acceleration $A_g = 0.3, 0.6, 0.9, 1.2$. For each combination of parameters the response has been computed for the 25 artificial earthquakes in the ensemble. Computations proceed for the full 30 seconds of the artificial record or until overturning is indicated (defined by $\theta = \pi/2$). A number of peak response quantities are recorded following the analysis, these include: peak angle of rotation, friction required to sustain rocking through the motion (i.e., the ratio of horizontal to vertical reaction force), and the friction required to prevent sliding during impact from either a rock or free-flight motion (i.e., equation (5)). A default time step of 0.02 seconds has been used in the analyses, automatically reduced as necessary to capture the impact condition.

6 RESULTS

A few general comments are first in order regarding the outcome of the analyses. The calculations for $e = 0$ are not computationally intensive since bouncing and free-flight are precluded. With increasing e, however, the analyses become increasingly intensive as the body bounces and "rings-down" following every impact from a rock mode.

Of the 1200 time history analyses conducted, 584 or 49% resulted in overturning being predicted. The majority of these occurred for the aspect ratio 4 and 6. It is noted, however, that the version of the program used in this study is not capable of handling a transition from rocking to free-flight, which is indicated when the normal reaction force vanishes during a rock mode. In the absence of vertical ground acceleration, the likelihood of such an event was assumed low. This in fact occurred in 320 or 27% of the calculations. In such an event, the calculation was halted and the angle of rotation at the last time step recorded. In a vast majority of these cases the angle of rotation was several times θ_c for the body, indicating that the block was most likely on the way to overturning. As such, these cases were counted as overturning and are included in the total statistics accordingly.

Results are presented in Tables 1, 2 and 3 for aspect ratios 2, 4 and 6 respectively. Presented in each table are: number of analyses which predicted overturning; for those analyses which did not end in overturning, average peak rotation normalized by θ_c, average friction required to sustain rocking, and average friction required to prevent sliding during impact. Table 1 does not include entries for $A_g = 0.3$, as this amplitude of ground acceleration is insufficient to initiate rocking for an $H/B = 2$.

Before discussing the implications of these results in detail, it is important to place the statistical results presented above in an appropriate context. One of the major goals of this study was to ascertain the levels of friction required to sustain a rock/free-flight motion (or simply rock in the case $e = 0$) in order to judge the necessity for a slide-rock analysis. Sliding was therefore prevented in all the analyses, and the peak friction force mobilized is tabulated. It will be seen – and discussed shortly – that in many cases the friction required *far exceeds* that which may reasonably be expected in real situations. In cases such as this, it is then clear that

Table 1: Statistics for $H/B = 2$

e	A_g	# OT	θ_{max}/θ_c ave	μ_{mot} ave	μ_{imp} ave
0.0	0.6	0	0.0155	0.524	0.353
	0.9	8	0.434	0.575	0.353
	1.2	14	0.753	0.613	0.353
0.3	0.6	0	0.0149	0.524	3.01
	0.9	6	0.305	0.569	14.9
	1.2	17	0.404	0.631	11.4
0.6	0.6	0	0.0242	0.523	111
	0.9	1	0.214	0.572	87.4
	1.2	7	0.503	0.595	91.8
0.9	0.6	0	0.0128	0.521	71.3
	0.9	0	0.166	0.547	55.4
	1.2	2	0.273	0.557	80.6

Table 2: Statistics for $H/B = 4$

e	A_g	# OT	θ_{max}/θ_c ave	μ_{mot} ave	μ_{imp} ave
0.0	0.3	0	0.102	0.262	0.185
	0.6	18	0.595	0.309	0.185
	0.9	21	0.697	0.354	0.185
	1.2	25	-	-	-
0.3	0.3	0	0.0858	0.262	3.54
	0.6	16	0.554	0.303	5.76
	0.9	22	0.575	0.315	1.85
	1.2	24	0.656	0.424	3.19
0.6	0.3	1	0.124	0.262	20.5
	0.6	15	0.543	0.297	17.8
	0.9	19	0.638	0.312	74.9
	1.2	24	0.820	0.311	2.46
0.9	0.3	0	0.0657	0.261	5480
	0.6	9	0.475	0.230	237
	0.9	19	0.523	0.333	9550
	1.2	23	0.600	0.324	42.0

the simulation is *not* modeling a physically realistic situation, and further interpretation of the results should be performed with caution. Relaxation of the friction condition may indeed have a large or small effect on the ensuing motion. This effect was not investigated herein, but is currently under study and will be reported separately later.

Table 3: Statistics for $H/B = 6$

e	A_g	# OT	θ_{max}/θ_c ave	μ_{mot} ave	μ_{imp} ave
0	0.3	5	0.306	0.194	0.124
	0.6	21	0.478	0.218	0.124
	0.9	24	0.941	0.338	0.124
	1.2	23	0.848	0.287	0.124
0.3	0.3	6	0.322	0.195	69.9
	0.6	21	0.535	0.222	3.36
	0.9	24	1.07	0.232	2.56
	1.2	24	0.628	0.325	1.57
0.6	0.3	6	0.372	0.192	6.37
	0.6	19	0.504	0.221	64.6
	0.9	22	0.665	0.245	3.92
	1.2	24	0.774	0.265	46.7
0.9	0.3	3	0.344	0.191	160
	0.6	22	0.468	0.198	6.78
	0.9	24	0.814	0.219	26.1
	1.2	25	-	-	-

With the above caveats, a number of significant observations can be made relative to the data presented above.

- The friction required to sustain the rock motion (excluding impact) is relatively insensitive to the coefficient of restitution, e. This through-motion demand increases moderately with magnitude of peak ground acceleration, and decreases as H/B increases. This observation is consistent with physical behaviors: taller blocks will more readily sustain a pure rock mode than their more stocky counterparts. Significant levels of friction are required to sustain rocking for an $H/B = 2$ for all A_g considered, and for an $H/B = 4$ with high A_g. Some of these levels are physically unrealistic.

  Although the statistics are not reported, the standard deviation for the computed through-motion μ is very low, suggesting perhaps a strong dependence on the peak ground acceleration. This effect requires further investigation and is currently under study.

- The friction demands during impact are not so well behaved, and in some cases are extremely large. It is clear from these results that it is highly probable that sliding will occur during impact for non-zero e, for all aspect ratios. In that case the true post-impact motion will be either slide, slide-rock or free-flight.

  For the special case $e = 0$ the tabulated impact-friction is independent of A_g and can be computed from the simple relation (Shenton and Jones 1991)

$$\mu_{imp} = \frac{3\cos\theta_c \sin\theta_c}{1 + 3\cos^2\theta_c} \tag{8}$$

  Assuming small θ_c, this relation can be approximated by $\mu_{imp} = .75B/H$. These values of friction are generally bounded above by the through-motion μ required, but represent an absolute lower bound on the friction required to prevent sliding, regardless of the characteristics of the ground acceleration.

  As e increases the impact-friction demand grows rapidly. Note that the scatter in the high-e cases can most likely be attributed to the rather small number of cases (i.e., those which did not overturn) used in the averaging.

  It is apparent from these observations that only in special cases (e.g., large H/B, low e, etc.) should the no-slide assumption be made. Further investigation is necessary, as noted above, to determine the effect of the sliding on the ensuing performance of the system.

- It is clear that taller blocks are more prone to overturning than smaller blocks (as would be expected); realistic friction values may be expected to change the reported numbers slightly. In most cases, again as would be expected, the likelihood of overturning is increased with increasing PGA. What is not clear from the data presented herein, that is, without careful consideration of the effects of finite friction, is the dependence of overturning susceptibility on coefficient of restitution. The data indeed suggest that higher e values produce a lower proclivity toward overturning. This may be due to the fact that nonzero-e cases are actually in contact with the ground for a smaller length of time (higher bounces, longer periods of free-flight, etc), and so are effectively isolated from many of the high-amplitude acceleration pulses.

  This assertion is given further weight by noting the behavior of the peak angle of rotation, θ_{max} (in particular for the $H/B = 2$, the aspect ratio with fewest overturns). As the coefficient of restitution is increased, this variable exhibits similar trends, i.e., θ_{max} generally decreases as e increases. Note that this parameter is normalized by θ_c, so care must be exercised when comparing blocks of differing aspect ratios.

Further investigation – including the consideration of finite μ – is necessary to completely validate this assertion.

7 CONCLUSIONS

The preceding paper has presented the results from a series of simulations of the response of free-standing blocks to an ensemble of artificial earthquake records. The test series was designed primarily to address the accuracy of the common assumption of "sufficient friction to prevent sliding".

The results suggest that this assumption is certainly reasonable for tall blocks, when based on the through-motion friction demand, but becomes less so as the aspect ratio is decreased. Indeed, for blocks of aspect ratio 2 and 4, the friction demands during rocking are such that sliding is likely under realistic conditions. A lower bound on the minimum friction required to sustain pure rocking is given approximately by $.75B/H$.

The results also suggest that the friction demands during impact for non-zero e are unrealistically large. Thus, sliding during impact is inevitable, and consequently a post-impact mode which includes sliding highly likely. The extent to which this sliding will affect the overall motion of the block is beyond the scope of this study, and will be addressed in later works.

The tendency for tall blocks to be more prone to overturning was confirmed in the simulation, as well as the destabilizing effect of increased levels of ground acceleration. An interesting result suggested by the data is that increasing the coefficient of restitution actually *stabilizes* the system, perhaps by reducing the effective time that the block is in contact with the ground. It is emphasized, however, that further analysis with finite friction is necessary to confirm the realism of this assertion.

The behavior of rigid blocks to ground excitation remains a fascinating and challenging topic of research. While much has been learned about this fundamental problem over the past decades, there are still many unresolved issues. Work is continuing by the current authors in assessing the full ranges of motion these systems can exhibit, exploring the effects of changes in the governing parameters, and ultimately developing guidelines for practice relative to systems of this type.

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