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Seismic analysis of circular foundation plates of variable thickness

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ABSTRACT: The paper considers a seismic analysis of circular and annular plates of variable thickness resting on an elastic foundation. The plates are connected with a cone or a cylindrical shell. The results are obtained in closed form.

The paper concerns seismic analysis of foundation plates of such high rise structures as television and water towers, chimneys, tanks. The foundation slabs of such structures are considered as circular or annular plates with varying thickness, resting on an elastic subgrade. In the seismic design the consideration of antisymmetric loading $p = p \sin\theta$ is of great importance. The similar analysis of circular plates of variable thickness is very poorly developed in literature. The present work obtains rigorous solutions of these problems. The results are convenient and comparatively simple for computation.

This paper investigates circular foundation plates consisting of two parts. The central part bears cylinder or cone shell; this part is continuous and circular. It is considered as a rigid circular punch on an elastic foundation. The central part bounded with flexible ring plate of nonuniform thickness resting on an elastic subgrade. The paper utilizes the Winkler model of an elastic subgrade. This work deals with elastic analysis of above-mentioned foundation plates, according the conception of the seismic design for small and frequent earthquakes / 1 /.

The present work considers in detail an annular isotropic plate resting on an elastic Winkler foundation. The variation of the plate thickness can be represented by the law:

$$h = h_0 x^{4/3}, \quad x = \frac{r}{r_0}, \qquad (1)$$

where h_0, r_0 - constants.

Antisymmetric bending of a plate of nonuniform thickness on an elastic foundation with modulus is governed by the differential equation:

$$D_M \nabla^2 \nabla^2 w + \frac{dD_M}{dr}\left(2\frac{\partial^3 w}{\partial r^3} + \frac{2+\nu}{r}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2}\frac{\partial w}{\partial r} + \frac{2}{r^2}\frac{\partial^3 w}{\partial r \partial \theta^2} - \frac{3}{r^3}\frac{\partial^2 w}{\partial \theta^2}\right) + \frac{d^2 D_M}{dr^2}\left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r}\frac{\partial w}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + c w = q, \qquad (2)$$

where $\mathcal{D}_M$ – flexural rigidity of a plate, ν – Poisson's ratio, w – deflection, c – modulus of subgrade.

Let us take the deflection in the following form:

$$w(r,\theta)=w(r)\sin\theta \qquad (3)$$

Using (3) and (1), equation (2) becomes:

$$\frac{d^4w}{dx^4}+\frac{10}{x}\frac{d^3w}{dx^3}+\frac{(17+4\nu)}{x^2}\frac{d^2w}{dx^2}-\frac{3(3-4\nu)}{x^3}\frac{dw}{dx}+\frac{(9-12\nu)}{x^4}w+\frac{cr_0^4}{\mathcal{D}_0}w=\frac{qr_0^4}{\mathcal{D}_0}. \qquad (4)$$

This is Euler equation. Introducing a new variable z, defined by $x=e^z$, (4) reduces to the differential equation with constant coefficients.

The deflection of the plate is given by:

$$w=x^{-1}\left(A_1x^{\alpha_1}+A_2x^{\alpha_2}+A_3x^{\alpha_3}+A_4x^{\alpha_4}\right)\sin\theta,$$

$$\alpha_{1,2,3,4}=\pm\sqrt{2(2-\nu)\pm\sqrt{2\nu^2-\beta^4}}, \qquad (5)$$

constants A_1, A_2, A_3 and A_4 are found from the boundary conditions.

We note that $\alpha_1=-\alpha_2, \alpha_3=-\alpha_4$.

Analysing the influence of the loading by forces and moments distributed along the circumferences, we have to determine Cauchy functions $Z_1(a;x), Z_2(a;x), Z_3(a;x), Z_4(a;x)$, which properties were discussed in / 2,3 /.

We calculate Wronkian for the solutions (5) and it's cofactors. Using the properties of Wandermond's determinant and Kramer's functions, we obtain:

$$Z_1(a;x)=\frac{a}{2(\alpha_3^2-\alpha_1^2)}\left\{\frac{\alpha_3^2}{\alpha_1}\left[(\alpha_1+1)a^{-\alpha_1}x^{\alpha_1-1}+(\alpha_1-1)a^{\alpha_1}x^{-\alpha_1-1}\right]-\frac{\alpha_1^2}{\alpha_3}\left[(\alpha_3+1)a^{-\alpha_3}x^{\alpha_3-1}+(\alpha_3-1)a^{\alpha_3}x^{-\alpha_3-1}\right]\right\},$$

$$Z_2(a;x)=\frac{a^2}{2(\alpha_3^2-\alpha_1^2)}\left\{\frac{(\alpha_3^2-3\alpha_1)}{\alpha_1}a^{-\alpha_1}x^{\alpha_1-1}-\frac{(\alpha_3^2+3\alpha_1)}{\alpha_1}a^{\alpha_1}x^{-\alpha_1-1}-\frac{(\alpha_1^2-3\alpha_3)}{\alpha_3}a^{-\alpha_3}x^{\alpha_3-1}+\frac{(\alpha_1^2+3\alpha_3)}{\alpha_3}a^{\alpha_3}x^{-\alpha_3-1}\right\},$$

$$Z_3(a;x)=\frac{a^3}{2(\alpha_3^2-\alpha_1^2)}\left\{-\frac{a^{-\alpha_1}(6+\alpha_1)}{\alpha_1}x^{\alpha_1-1}+\frac{a^{\alpha_1}(6-\alpha_1)}{\alpha_1}x^{-\alpha_1-1}+\frac{a^{-\alpha_3}(6+\alpha_3)}{\alpha_3}x^{\alpha_3-1}-\frac{a^{\alpha_3}(6-\alpha_3)}{\alpha_3}x^{-\alpha_3-1}\right\},$$

$$Z_4(a;x)=\frac{a^4}{2(\alpha_3^2-\alpha_1^2)}\left\{-\frac{a^{-\alpha_1}x^{\alpha_1-1}}{\alpha_1}+\frac{a^{\alpha_1}x^{-\alpha_1-1}}{\alpha_1}+\frac{a^{-\alpha_3}x^{\alpha_3-1}}{\alpha_3}-\frac{a^{\alpha_3}x^{-\alpha_1-1}}{\alpha_3}\right\}. \qquad (6)$$

The deflected surface of the plate which is free from any loading when $x>a_0$ is governed, using (11), by the equation:

$$w(x,\theta)=w_1(x)\sin\theta=\left[w_0Z_1(a_0;x)+\vartheta_0r_0Z_2(a_0;x)-\frac{M_0r_0^2}{\mathcal{D}_0a_0^4}Z_3(a_0;x)-\frac{Q_0r_0^3}{\mathcal{D}_0a_0^4}Z_4(a_0;x)\right]\sin\theta, \qquad (7)$$

where a_0 is the radius of the inner boundary; $w_0, \vartheta_0, M_0, Q_0$ – the deflection, the slope, the bending moment and the shearing force when $x=a_0$.

Now the paper considers the seismic analysis of the annular plate with thickness variation (1) connected with absolutely rigid central part (fig.1). The similar problem take place in the design of bottom slabs of cylindrical tanks.

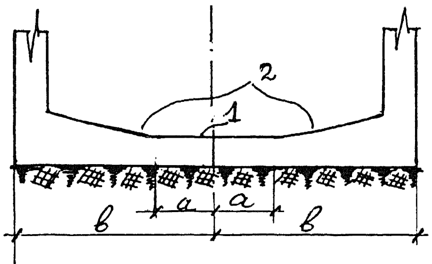

figure 1

1 central absolutely rigid part, loaded by an available load, reducing to a principal moment; it is considered as a rigid punch on an elastic foundation.
2 annular flexible plate of nonuniform thickness resting on an elastic foundation.

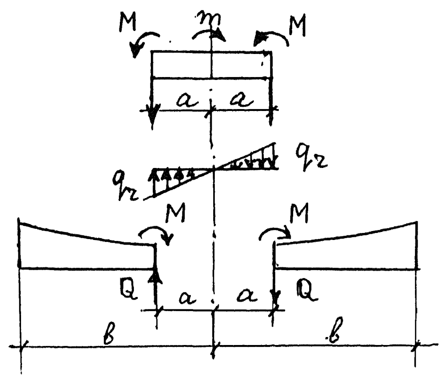

Figure 2

Using the notation M and Q as forces of interaction of circular punch and rigid plate. These forces are distributed accordingly along the outer boundary of the punch and inner boundary of the plate corresponding to the laws: $M\sin\theta$ and $Q\sin\theta$ (fig.2); m is the principal moment, q_r - the intensity of the reaction of the subgrade. For the consideration of interaction of parts 1 and 2 let us write the equations of the method of forces:

$$\left.\begin{aligned} M\delta_{11}+Q\delta_{12}+\Delta_{1p}=0, \\ M\delta_{21}+Q\delta_{22}+\Delta_{2p}=0 \end{aligned}\right\} \qquad (8)$$

where M and Q boundary moment and force $Q=Q_r-\frac{1}{x r_0}\frac{dH}{d\theta}$, coefficients δ_{11}, δ_{12} - accordingly slope and deflection of inner edge of the ring plate subjected to the action of unit moments distributed along the inner boundary $1\cdot\sin\theta$; δ_{12}, δ_{22} - slope and deflection of inner edge of the plate subjected to the action of unit loads distributed along the inner boundary of the plate $1\cdot\sin\theta$. We can calculate these values using (11), (12). And $\Delta_{1p}=\frac{q_r}{Ca}$, $\Delta_{2p}=\frac{q_r}{C}$ - slope and deflection of circular punch. The intensity q_r is calculating from the conditions of equilibrium:

$$\sum M=m+Q\cdot 2a^2+M\cdot 4a+\frac{4}{3}a^3 q_r \,. \qquad (9)$$

Numerical example

This example analyses briefly the flexible annular plate with thickness variation (1) resting on an elastic foundation. The inner boundary $a=1$ is free, the outer one $b=5$ is clamped.

The following two problems are to be considered:

1) inner boundary loaded by moments $M=M_0\sin\theta$, $M_0=1$;
2) inner boundary loaded by forces $Q=Q_0\sin\theta$, $Q_0=1$.

Satisfying the boundary conditions, magnitudes ω_0 and ϑ_0 can be computed. Then we can determine the full stressed and strained state.

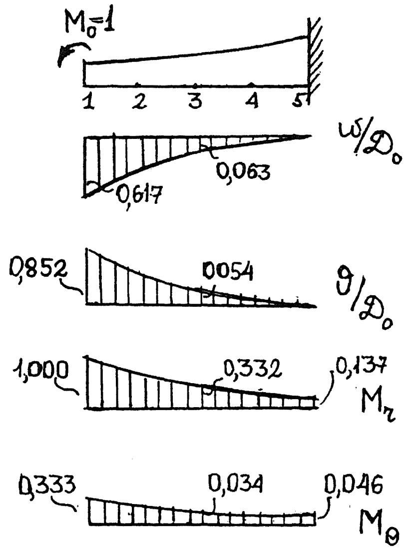

figure 3

Figure 3 shows the deflections $\frac{w}{\mathcal{D}_0}$, slopes $\frac{\vartheta}{\mathcal{D}_0}$, bending moments M_r and M_θ. It is possible to calculate shearing forces Q_r and Q_θ. Problem 2 was solved in the similar way.

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