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Concrete frame under incident seismic wave

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ABSTRACT: A concrete elastic frame system founded on several rigid plates at the soil surface is loaded by an incident seismic wave. The wave pattern defined in terms of vertical acceleration is entered outside the frame system and approaches the frame with the velocity of the Rayleigh wave. The rigid body boundary conditions at the foundations are simultaneously satisfied by also considering the interaction through the frame and the soil. Two different kinds of soil are investigated, homogenous and inhomogenous soil. The response of the system is given at the rigid foundations and the elastic frame for a realistic earthquake pattern (Friaul 1968). In addition, transfer functions are determined which relate the response of the foundations and the frame to the incident wave at distinct points.

1 INTRODUCTION

The determination of the dynamic stiffness for rigid foundations on the surface of the soil has been the object of many publications in the past two decades, cf. Luco (1982), Wolf (1985). Different kinds of numerical procedures were introduced to solve this mixed boundary value problem. In this paper the boundary value problem is extended to multiple foundations including a concrete frame system, Fig.1. The frame is founded on four rigid plates and the interactive coupling through the soil is considered as well as the influence of the elasticity of the frame. The loading function is defined in terms of vertical acceleration of a seismic wave and approaches the foundation system with the velocity of a surface wave (Rayleigh-wave). The applied seismic load function is the acceleration pattern from the Friaul earthquake of 1968. To control the motion, the wave was entered at a distance of 100 m from the foundation system. The time delay of the seismic excitation in the contact area foundation-soil is considered by a sufficiently fine discretization in space and time. For all calculations presented herein the computer-system DYBAST, Sarfeld & Savidis (1992) was applied. This computer code is based on analytical and finite element theories with special consideration of 3-dimensional soil structure interaction.

2 FRAME-FOUNDATION-SOIL SYSTEM

Fig.1 describes the system to be analyzed. The concrete frame has a Young's-modulus $E_{st} = 3{\cdot}10^4$ MN/m^2, a mass density $\varrho_{st} = 2.5$ Mg/m^3 and a damping ratio $\beta_{st} = 0.01$. The block foundations (2.00 m x 2.00 m x 0.60 m) are assumed to be rigid. The cross section area of the four concrete columns is 0.40 m x 0.40 m, while the area of the horizontal beams is 0.40 m x 0.60 m. The concrete columns are equally spaced with 10 m in both directions x and y. The frame has a height of 5 m. The calculations are carried out for two different types of soil:

- homogenous soil
- inhomogenous soil

In case of a homogenous soil a constant shear-wave velocity $v_{S0} = 100$ m/sec and a mass density $\varrho = 2.0$ Mg/m^3 are assumed. The Poisson's ratio $\nu = 0.25$ and the damping ratio $\beta = 0$. The system response will be determined at five dinstinct points (1,2,3,4,5) marked with black circles in Fig. 1. The term 'a(t)' indicates the acceleration time function of the seismic wave.

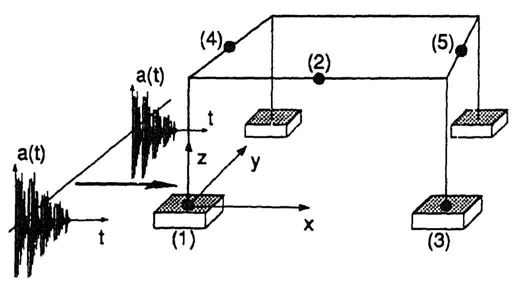

Figure 1: System under consideration

For the inhomogenous soil the shear-wave velocity is a function of the depth. At the surface the shear wave velocity $v_{S0} = 100$ m/sec. The mass density $\varrho = 2.0$ Mg/m^3, the Poisson's ratio $\nu = 0.25$ and the damping ratio $\beta = 0$. The depth-variation of the soil stiffness is defined by the following depth function of the shear modulus:

$$G(z) = G_0 + (G_\infty - G_0)[1 - \exp(\alpha z)], \quad \text{with} \quad 0 < G_0 \leq G_\infty \tag{1}$$

where G_0 and G_∞ are the shear moduli at the surface and at infinite depth, respectively. The parameter α is a constant of the dimension of inverse length. This model is chosen so as to describe uniformly deposited cohesionless soils, Vrettos & Prange (1991). According to Vrettos (1991) we introduce the degree of inhomogeneity

$$\Xi_0 = 1 - \frac{G_0}{G_\infty}, \tag{2}$$

and the inhomogeneity gradient parameter

$$\theta = \frac{\Omega}{\alpha v_{S0}}. \tag{3}$$

The homogenous soil represents then the limiting case $\Xi_0 = 0$. In the following analysis these parameter are fixed to $\Xi_0 = 0.7$ and $\theta = 7.65$. These values give a shear wave velocity of 182 m/s for $z \to \infty$. Ω is the circular frequency of the excitation.

3 ANALYSIS PROCEDURE

The mathematical procedure to analyze the problem described above can be splitted into two parts:

- concrete frame
- foundation-soil system.

First, the frame is idealized with the classical finite element method. Three dimensional beam elements are applied to build up the stiffness matrix $\mathbf{K}_{st}$, the damping matrix $\mathbf{C}_{st}$ and the mass matrix $\mathbf{M}_{st}$. This leads to the following equation of motion for the frame structure:

$$\mathbf{M}_{st}\ddot{\mathbf{u}}_{st} + \mathbf{C}_{st}\dot{\mathbf{u}}_{st} + \mathbf{K}_{st}\mathbf{u}_{st} = \mathbf{p}_{st}(t), \tag{4}$$

where the vector $\mathbf{u}_{st}$ represents the displacements of the frame and $\mathbf{p}_{st}(t)$ describes an arbitrary time dependent load function acting on the frame. The index '*st*' denotes the concrete frame structure.

The second partial problem is the solution of the dynamic interaction of the rigid foundation with the soil. The procedure to solve this dynamic problem results in a mixed boundary value problem for displacements and stresses in the freefield and the foundation area. Both, the boundary conditions (rigid body) for the foundations and the equations of motion for the soil and the foundation must be simultaneously satisfied, including also the interactive coupling through the soil for all foundations. The method used here discretizes the unknwon stresses at the contact area of the foundations and the soil with a stepwise constant stress distribution. To obtain the response of the soil for a constant stress distribution numerical integration based on Green's functions is used. Details are given by Savidis & Sarfeld (1980) and Sarfeld (1992).

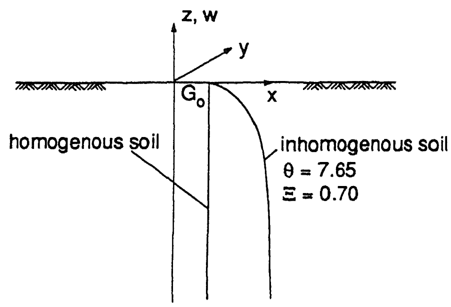

Figure 2: Shear modulus depth-profiles

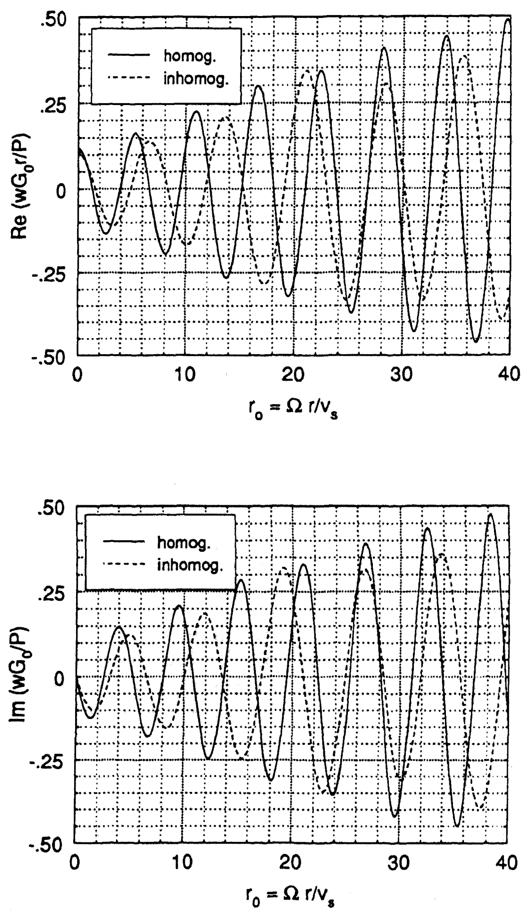

Figure 3: Vertical Green's function

Real and imaginary parts of the vertical Green's functions (vertical displacement w due to vertical point load P) at the surface of the homogenous and the inhomogenous soil are shown in Fig. 3 as a function of the distance from the source r. The curves emphasize the different behaviour of the inhomogenous profile. Details on calculation of the Green's function are given by Sarfeld (1992) and Vrettos (1991).

Assembling the influences of all soil elements leads to the flexiblitiy matrix of the soil $\mathbf{F}_s$. The inversion of the flexibility matrix results in the frequency dependent complex stiffness matrix $\mathbf{K}_s$ of the soil

$$\mathbf{K}_s(i\Omega)\mathbf{u}_s = \mathbf{p}_s(i\Omega) \quad (5)$$

where $\mathbf{p}_s$ is the vector of forces applied to the soil. The index 's' indicates the soil and i is the imaginary unit defined as $\sqrt{-1}$. The rigid body boundary conditions are formulated by a transformation matrix $\mathbf{T}$. Performing the transformation $\mathbf{T}$ on eq. (5) the effective complex stiffness matrix $\mathbf{K}_{sf}$ for the foundation-soil system is obtained in the frequency domain. The relation including also the mass matrix of the foundations is written as

$$\left[\mathbf{T}^T\mathbf{K}_s(i\Omega)\mathbf{T} - \Omega^2\mathbf{M}_f\right]\mathbf{u}_f = \mathbf{T}^T\mathbf{p}_s(i\Omega) \quad (6)$$

or

$$\left[\mathbf{K}_{sf}(i\Omega) - \Omega^2\mathbf{M}_f\right]\mathbf{u}_f = \mathbf{p}_f(i\Omega), \quad (6a)$$

where the superscript 'T' denotes transposed, $\mathbf{M}_f$ is the mass matrix of the foundations and $\mathbf{p}_f$ is the load vector acting on the foundations. $\mathbf{u}_f$ is the displacement vector associated to the degrees of freedom of the rigid body motion.

Performing the Fourier transform on eq. (4), the equations (4) and (6a) can be assembled at their common interfaces so that we obtain

$$\left[\mathbf{K}_t(i\Omega) + \mathbf{C}_t - \Omega^2\mathbf{M}_t\right]\mathbf{u}_t = \mathbf{p}_t(i\Omega). \quad (7)$$

This system of linear equations can be solved in the frequency domain for a given $\mathbf{p}_t$. The index 't' denotes the entire system 'frame - foundation - soil'. Finally, the inverse Fourier transform leads to the solution in the time domain.

4 INDIRECT SEISMIC WAVE EXCITATION

For the description of a travelling wave along the surface of the soil, an arbitrary wave time function propagating in the positive n_j-direction is defined as,

$$f(x,y,z=0,t) = f(t - \frac{1}{c}x_j n_j) \quad (8)$$

where t is time, c is the velocity of propagation, $x_j = \{x, y, z = 0\}$ the position vector and n_j the unit vector defining the direction of propagation. The Fourier transform $\mathcal{F}\{f\}$ of this equation yields the following relation

$$\mathcal{F}\{f(x_j,t)\} = \exp\left[-k_j x_j i\right] F(i\Omega), \quad (9)$$

where k_j represents the wave vector with the components $\Omega/c\{n_x, n_y, 0\}$ and $F(i\Omega)$ is the Fourier transform of the wave function (8) for $x_j = \{0,0,0\}$. The expression $\exp\left[-k_j x_j i\right]$ can be interpreted as the transfer function in time of the wave function (8) along the surface of the soil.

Equation (9) is used to express the wave influence function, in order to determine the displacement vector due to the indirect wave excitation. This vector $\mathbf{u}_s^e$ is evaluated for all points 'n' of the soil elements at the contact area foundation-soil.

$$\mathbf{u}_s^e(x_s, y_s, \Omega) = \begin{pmatrix} \exp\left[-(k_x x_1 + k_y y_1)i\right] \\ \vdots \\ \exp\left[-(k_x x_s + k_y y_s)i\right] \\ \vdots \\ \exp\left[-(k_x x_n + k_y y_n)i\right] \end{pmatrix} F(i\Omega)$$

$$\{s = 1 \ldots n\} \quad (10)$$

The index 's' represents the soil whereas the index 'e' represents the excitation. For an indirect wave excitation the displacement vector of the soil elements is defined as

$$\mathbf{u}_s = \mathbf{u}_s^r + \mathbf{u}_s^e, \quad (11)$$

where $\mathbf{u}_s^r$ is the vector of the relative displacement. By setting the right hand side of eq. (5) equal to zero and substituting the relative displacement vector $\mathbf{u}_s^r$ in equation (5) we obtain

$$\mathbf{K}_s(i\Omega)\mathbf{u}_s = \mathbf{K}_s(i\Omega)\mathbf{u}_s^e = \mathbf{p}_s(i\Omega). \quad (12)$$

Now, the load vector $\mathbf{p}_s$ corresponding to the soil elements is introduced for a given wave excitation. Multiplying $\mathbf{p}_f$ by the transposed rigid body matrix $\mathbf{T}$ leads to the following expression for the excitation vector of the entire system

$$\mathbf{p}_f(i\Omega) = \mathbf{T}^T\mathbf{K}_s(i\Omega)\mathbf{u}_s^e. \quad (13)$$

The system of equations (7) is completly assembled together with the seismic wave excitation so that the resulting response includes the stiffness of the frame as well as the interaction of the soil with the foundation and the structure.

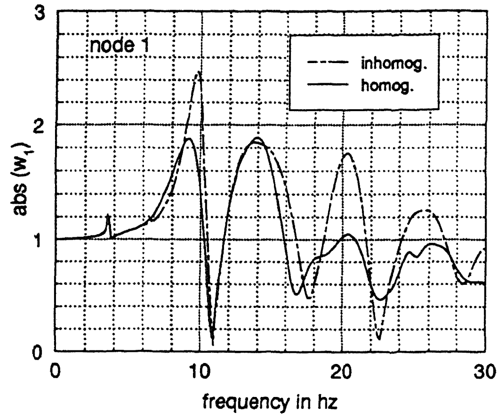

Figure 4: Vertical amplification. Found. node 1

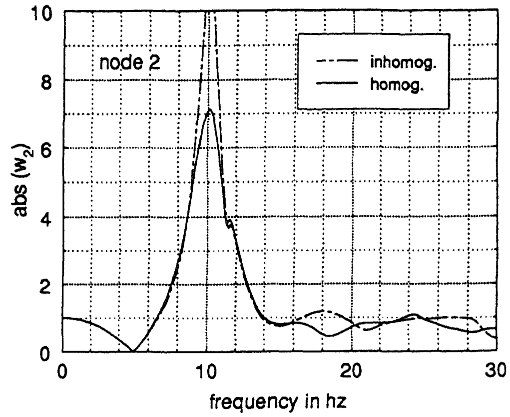

Figure 6: Vertical amplification. Frame node 2

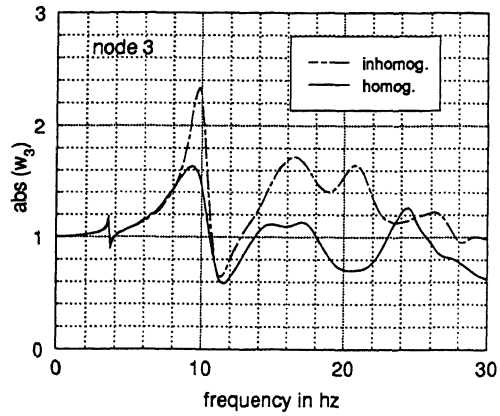

Figure 5: Vertical amplification. Found. node 3

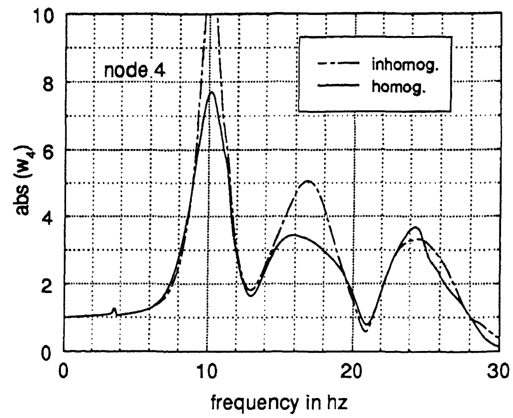

Figure 7: Vertical amplification. Frame node 4

5 RESULTS AND DISCUSSION

The behaviour of a concrete frame system founded on the soil depends on the elastic parameters of soil and frame as well as on the geometry of the system.

For a seismic excitation the response of the system in time domain is quite complex and it is difficult to clearly identify all the dynamic characteristics. First, transfer functions are determined to enlighten the dynamic response of the entire system. The transfer function are directly related to the incident wave excitation. The absolute vertical displacements w are calculated for the output nodes of the frame and the foundation. The graphs in Fig. 4 and 5 show the vertical displacement of the foundation (nodes 1, 3) for the homogenous and the inhomogenous soil. Both foundations show a strong amplification at the frequency of 10 Hz. A similar behaviour, but even more pronounced, is found for the frame nodes (2, 4, 5), Fig. 6 - 8. This indicates, that an eigenfrequency for the horizontal beams exists at 10 Hz. This has a great influence on the response of the foundations. Thus, in the frequency range around 10 Hz the dominant vibration arises from the elasticity of the frame.

All graphs of transfer functions show a lower damping behaviour for the inhomogenous soil compared to the homogenous one. Hence, the energy dissipation due to radiation damping is not so effective for the inhomogenous soil. The vibration response of the founations above 10 Hz shows a strong interaction with the soil. For the inhomogenous soil the stiffness behaviour remains still higher. The frame amplitudes for this frequency range result from the elasticity of the beams so that the differences between the homogenous and inhomogenous soil become smaller.

Next, an analysis based on a seismic wave from the earthquake in Friaul 1968 is applied. About 100 m distance from the frame system the seismic wave is initiated. The propagation velocity is 100 m/s so that the time delay to reach the first foundation

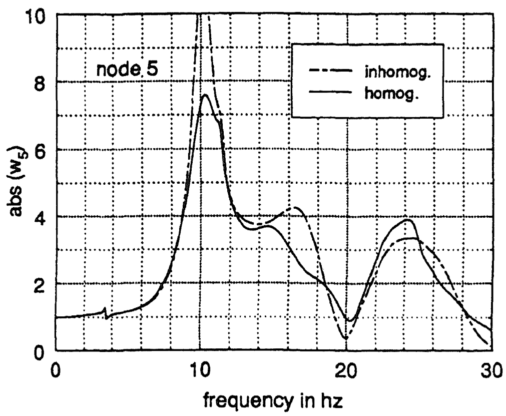

Figure 8: Vertical amplification. Frame node 5

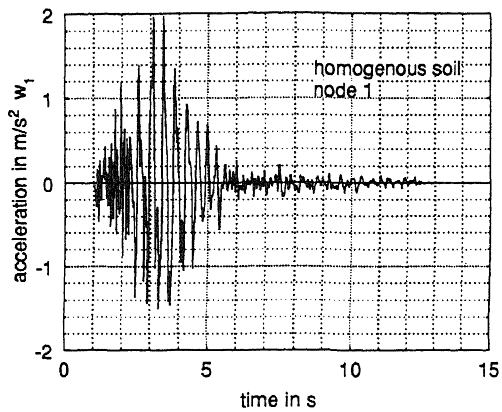

Figure 10: Time history. Foundation node 1

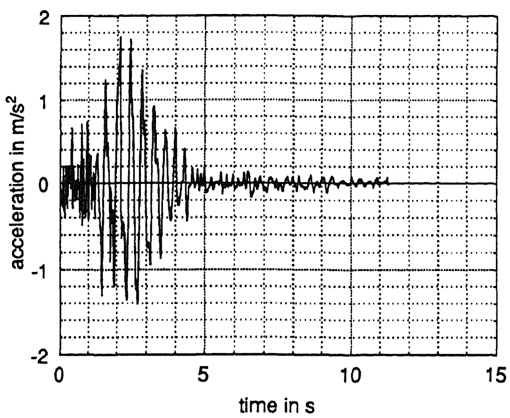

Figure 9: Time history. Friaul earthquake

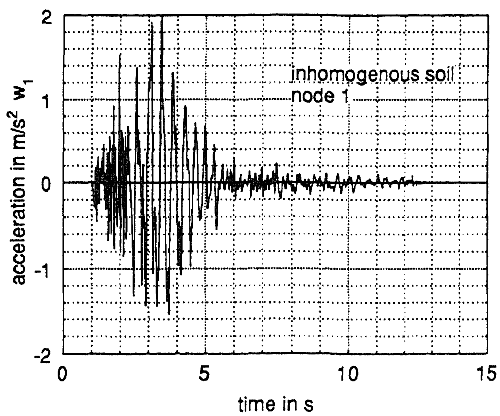

Figure 11: Time history. Foundation node 1

(node 1) is 1 sec. Fig 9 shows the time dependent function of the acceleration of the seismic wave. The dynamic response of the foundation (node 1) is plotted in Fig. 10 for the homogenous soil and in Fig. 11 for the inhomogenous soil. The results of the frame (node 4) are illustrated in Fig. 12 and 13 for the two soil profiles. The response of the foundation in Fig. 10 and 11 is quite similar for both soils. The fact, that the inhomogenous soil is stiffer has no essential influence on the response of the time history acceleration. The comparison of the response of the foundation to the seismic input function shows only a slightly magnification only of 10 %. Due to the soil-foundation interaction the damping effect of the semi infinite soil produces no significant amplification.

Quite different is the dynamic response for the frame. The influence of the resonant frequency at 10 Hz dominates the vibration, and the time history has been strongly modified. The overall dynamic behaviour of the frame is defined by its own elasticity.

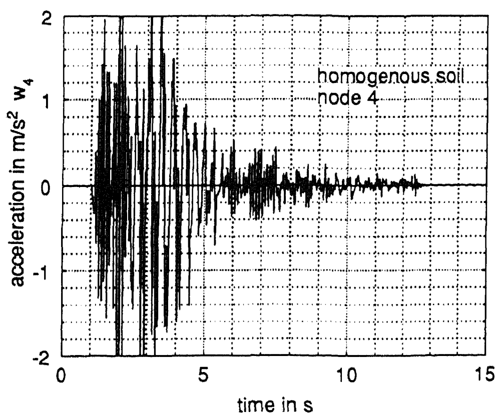

Figure 12: Time history. Frame node 4

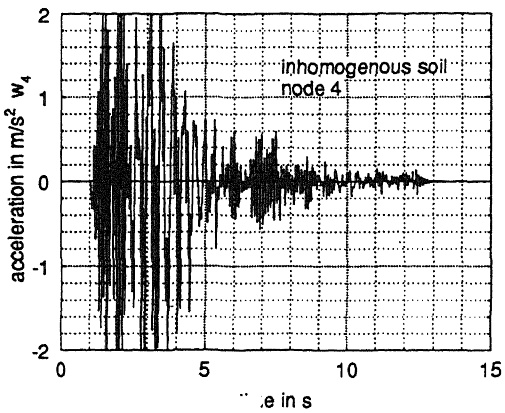

Figure 13: Time history. Frame node 4

However, the interaction with the soil reduces the amplitudes, both in the frequency and in the time domain.

For the frame, the differences in the response for the homogenous and inhomogenous soil are neglisible.

6 CONCLUSIONS

The analysis of a concrete frame including the interaction frame-foundation as well as soil-structure interaction is presented for an incident seismic wave excitation. A numerical procedure was developed to solve the entire system 'frame - foundation - soil' by considering the rigid body boundary conditions of the foundations and the elasticity of the soil. Full interaction for all components of the model can be achieved to perform studies with various structure - foundation - soil systems.

The determination of the transfer functions show the strong influence of the frame on the response of the foundation system, especially at the resonant frequency of the frame. The radiation damping of the soil has a reducing effect on the amplitudes of the foundation and the frame. Moreover, a time history analysis showed, that the vibration of the frame strucure is dominant and is mainly affected by its own elasticity.

The investigations showed the importance of considering the interaction effects through the soil for the entire system. More studies are required to obtain further informations about the dynamic behaviour for coupled structure - foundations - soil systems.

REFERENCES

Luco, J.E. 1982. Linear soil-structure interaction: a review. In S.K. Datta (ed.) Earthquake groundmotion and its effects on structures: 41-57. ASME, AMD-Vol. 53.

Sarfeld, W. 1992. Numerische Verfahren zur Boden-Bauwerk Interaktion. Dr.-Ing. Thesis, Technical University Berlin.

Sarfeld, W., Savidis, S. and Faust, B. 1992. DYBAST, a computer system for the dynamic calculation of bases and structures. Internal report, Technical University of Berlin.

Savidis, S. and Sarfeld, W. 1980. Verfahren und Anwendung der dreidimensionalen dynamischen Wechselwirkung. Vorträge der Baugrundtagung, Mainz: 47-78.

Vrettos, C. 1991. Time-harmonic Boussinesq problem for a continuously nonhomogeneous soil. Earthquake Eng. Struct. Dyn. 20: 961-977.

Vrettos, C. and Prange, B. 1991. Evaluation of in situ effective shear modulus from dispersion measurements. J. Geotech. Eng. ASCE 116: 1581-1585.

Wolf, J.P. 1985. Dynamic soil structure interaction. Englewood Cliffs: Prentice Hall.