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On identification of soil-structure interaction from recorded motions of buildings

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ABSTRACT: The effects of soil-structure interaction on the recorded frequencies of buildings are investigated, and methods for detection and identification of the soil-structure interaction from acceleration measurements are given. The method for detection is based on the causality of the building's response to base excitation. The method for identification gives dominant frequencies of the building and the foundation. Examples for the methods are given for a model building.

1 INTRODUCTION

Soil-structure interaction (SSI) can significantly alter the interpretation to be placed on the seismic recordings obtained in buildings. Although SSI is always present to some degree, it is generally assumed when SSI is small that the recorded motions at foundation level are not influenced by the motions of upper stories. Therefore the building can be assumed to be fixed-based and the foundation level recordings can be taken as the base excitation. When SSI is significant, this assumption is not valid because of the feedback from the building to the foundation and the surrounding soil medium. The feedback makes the building a closed-loop dynamic system, where the input (i.e., the foundation motion) and the output (i.e., the motion of the superstructure) are coupled. The identification of soil-structure interaction is relatively easy if, in addition to the records from the building, there are also records available from a nearby free-field site that is not influenced by the building's vibrations. In most cases, such free-field records are not available.

This paper investigates the identifiability of soil-structure interaction in buildings when there are no nearby free-field records available. The effects of soil-structure interaction on the recorded frequencies of the superstructure are discussed by using a 2-DOF (two degrees of freedom) model. A method is presented to detect the existence of SSI from the vibration recordings. The identifiability of soil-structure interaction is discussed, and a method is introduced to identify fundamental frequencies of the fixed-base building and the foundation. An example for the application of the method is given for a model building.

2 EFFECT OF SSI ON FREQUENCIES

The effect of SSI on the frequency content of the building records can be shown by studying the response of a 2-DOF system as shown in Figure 1. A

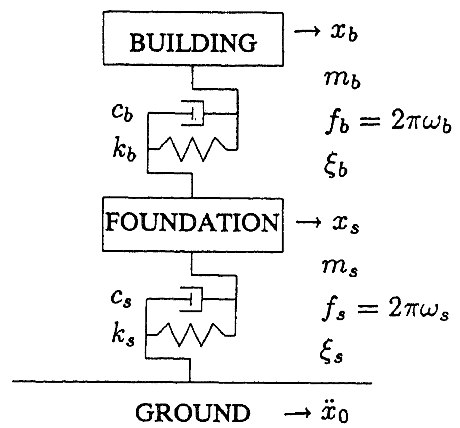


Figure 1. Two degrees of freedom system representing soil-structure interaction.

2-DOF system is a simple model for soil-structure interaction. The lower mass-spring-dashpot (m_s, k_s, and c_s) represents the soil-foundation system, whereas the upper mass-spring-dashpot (m_b, k_b, and c_b) represents the building. The model is based on the assumptions that both systems are linear, their motions are dominated by fundamental modes, and rocking motions are negligible.

Let x_s, x_b, and x_0 denote the absolute displacements of m_s, m_b, and the free-field, respectively, and $y_s = x_s - x_0$, and $y_b = x_b - x_s$ the relative displacements of m_s with respect to the free field, and of m_b with respect to foundation. With these, we can write the following equations for the motion of the 2-DOF system under base excitation $\ddot{x}_0$:

$$m_s\ddot{y}_s + c_s\dot{y}_s + k_s y_s - c_b\dot{y}_b - k_b y_b = -m_s\ddot{x}_0 \quad (1)$$

$$m_b(\ddot{y}_s + \ddot{y}_b) + c_b\dot{y}_b + k_b y_b = -m_b\ddot{x}_0 \quad (2)$$

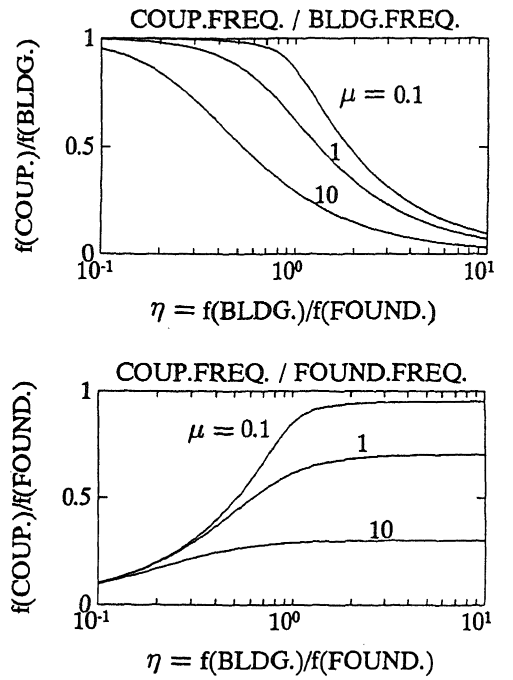


Figure 2. Variations of ω_1/ω_b (top figure) and ω_1/ω_s (bottom figure) with $\eta = \omega_b/\omega_s$ for $\mu = 0.1$, 1, and 10.

where a dot over a variable denotes the derivative with respect to time. The two natural frequencies of this coupled system are :

$$\omega_{1,2}^2 = \frac{1}{2}\Big[\omega_s^2 + (1+\mu)\omega_b^2 \mp \sqrt{[\omega_s^2 + (1+\mu)\omega_b^2]^2 - 4\omega_s^2\omega_b^2}\,\Big] \quad (3)$$

where $\mu = m_b/m_s$ is the mass ratio, and $\omega_s = \sqrt{k_s/m_s}$ and $\omega_b = \sqrt{k_b/m_b}$ are the natural frequencies of the foundation when no building is present, and of the building with non-moving foundation, respectively. In the paper, the radial frequency ω and the corresponding cyclic frequency f (i.e., $f = 2\pi\omega$) have been used interchangeably. We can write the following equations for the ratio of the fundamental frequency, ω_1, of the coupled system to the frequencies of the building and the foundation:

$$\left(\frac{\omega_1}{\omega_b}\right)^2 = \frac{1}{2\eta^2}\Big[1 + (1+\mu)\eta^2 \mp \sqrt{[1+(1+\mu)\eta^2]^2 - 4\eta^2}\Big] \quad (4)$$

$$\left(\frac{\omega_1}{\omega_s}\right)^2 = \eta^2\left(\frac{\omega_1}{\omega_b}\right)^2 \quad (5)$$

where $\eta = \omega_b/\omega_s$. The ratios are functions of μ and η only.
Variations of these ratios for η varying from 0.1 to 10, and three values of μ are plotted in Figure 2. The figure shows that the ratios are always less than one. In other words, the fundamental frequency with SSI is always lower than the individual frequencies of the building and the foundation. For flexible buildings and stiff soil conditions (i.e., $\eta << 1$) the fundamental frequency of the coupled system converges to that of the fixed-base building for all values of μ. For stiff buildings and soft soil conditions (i.e., $\eta >> 1$) the fundamental frequency converges to $\omega_s/\sqrt{1+\mu}$; and if μ is small, it converges to ω_s, the frequency of the foundation.

In order to see the effects of SSI on the frequency content of $\ddot{x}_s$ and $\ddot{x}_b$, which will be the recorde quantities, we will investigate the frequency response functions H_s and H_b of the foundation and the building, respectively. To calculate frequency response functions, we take $\ddot{x}_0 = e^{i\omega t}$ in equations (1) and (2), and let $\ddot{x}_s = H_s e^{i\omega t}$ and $\ddot{x}_b = H_b e^{i\omega t}$. Solving for H_s and H_b gives:

$$H_s = \frac{1}{\Delta}\Big[-i\omega^3 2\xi_s\omega_s - \omega^2(\omega_s^2 + 4\xi_s\xi_b\omega_s\omega_b) + i\omega(2\xi_s\omega_s\omega_b^2 + 2\xi_b\omega_b\omega_s^2) + \omega_s^2\omega_b^2\Big] \quad (6)$$

$$H_b = \frac{1}{\Delta}\Big[-\omega^2 4\xi_s\xi_b\omega_s\omega_b + i\omega(2\xi_s\omega_s\omega_b^2 + 2\xi_b\omega_b\omega_s^2) + \omega_s^2\omega_b^2\Big] \quad (7)$$

with

$$\Delta = \omega^4 - i\omega^3[2\xi_s\omega_s + 2(1+\mu)\xi_2\omega_2] - \omega^2[\omega_1^2 + (1+\mu)\omega_b^2 + 4\xi_s\xi_b\omega_s\omega_b] + i\omega(2\xi_s\omega_s\omega_b^2 + 2\xi_b\omega_b\omega_s^2) + \omega_s^2\omega_b^2 \quad (8)$$

where $i = \sqrt{-1}$, and ξ_s and ξ_b are the damping ratios defined as $\xi_s = c_s/2\omega_s$, $\xi_b = c_b/2\omega_b$.

Plots of transfer functions for four values of μ ($\mu = 0.01$, 0.5, 1.0, and 2.0), and for $f_s = 2\pi\omega_s = 0.7\,Hz$, $f_b = 2\pi\omega_b = 0.5\,Hz$, $\xi_s = 0.30$, and $\xi_b = 0.05$ are given in Figure 3. For low μ values (i.e., $\mu \leq 0.01$) the SSI is negligible, and transfer functions have peaks at their respective frequencies f_s and f_b. As the μ value increases, the effect of SSI becomes significant, and both transfer functions have a peak at the same frequency which is lower than the individual frequencies of the foundation and the building. This observation is in parallel to that concluded from Figure 2.

3 DETECTION OF SSI

Detection of SSI is the first step in the identification of a building susceptible to SSI. Normally, the motion of the building is recorded at the foundation level, top story, and several intermediate stories. If there is no SSI, we can identify the building by taking the recordings at the foundation level as the input, and the recordings at upper stories as the output. A building with no SSI represents a causal system,

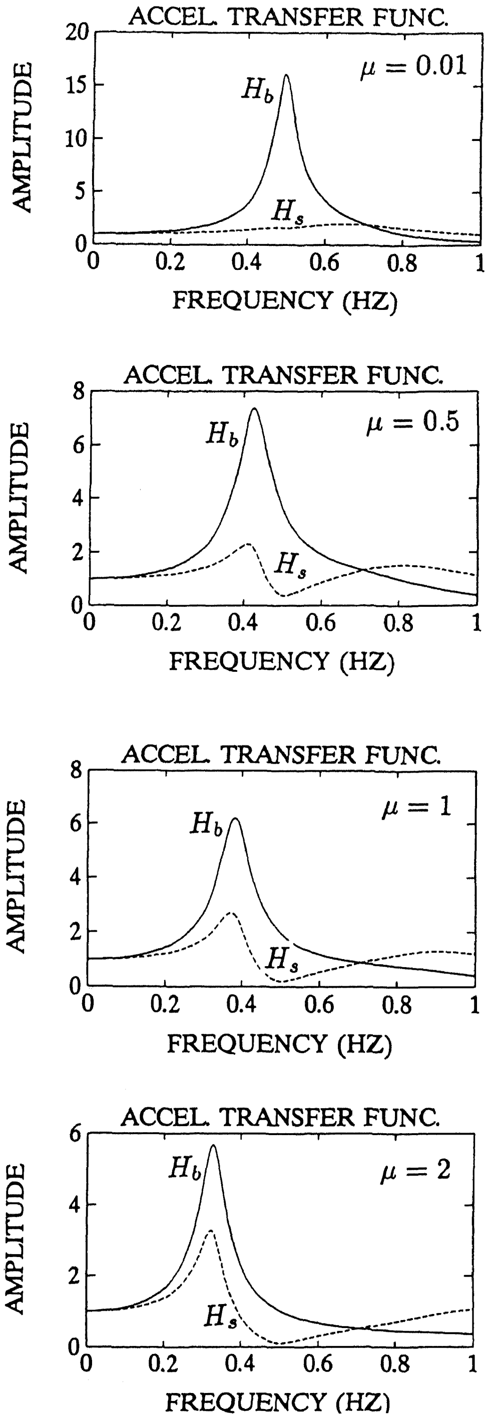


Figure 3. Transfer functions H_b and H_s for four values of μ, for $f_s = 2\pi\omega_s = 0.7\,Hz$, $f_b = 2\pi\omega_b = 0.5\,Hz$, $\xi_s = 0.30$, and $\xi_b = 0.05$

because the input at time t_i can only influence the output at times $t \geq t_i$. Since there is no coupling between input and output, such systems are termed open-loop systems. When there is SSI, the system becomes non-causal because the upper story motions influence the foundation motion. Since input and output are coupled, such systems are termed closed-loop systems.

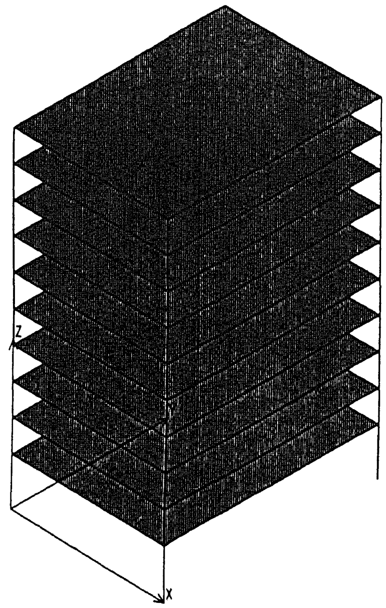
The causality condition provides a convenient tool to detect the SSI. For causal systems the impulse response of the system is zero for $t < 0$, whereas for non-causal systems it is not. Therefore, we investigate the impulse response of the building from the records of the foundation and the upper stories. If the impulse response shows significant amplitudes at negative times in comparison to those at positive times, we conclude that there is SSI.

We will give an example for this criterion by using calculated responses of a 10-story building shown in Figure 4. The frequencies and response time histories to two directional base excitation of the building were calculated twice, one assuming a fixed-base condition, and the other assuming SSI represented by horizontal springs at the foundation level. The base excitation is the two horizontal components of the accelerations recorded at Piedmont Junior High School, a rock site, during the magnitude 7.1 Loma Prieta earthquake (CDMG, 1991). For each direction and each case, dominant frequencies of the building are given in Figure 4; time histories of the excitation (i.e., ground acceleration), and calculated acceleration responses at the top story and the foundation level are given in Figure 5. Note that for no SSI, foundation and ground accelerations are identical. Impulse response functions were calculated by taking the foundation accelerations as the input, and the top-story accelerations as the output, and using the method given in Ljung (1987). A prewhitening filter was applied to the input so that it becomes as close to a white noise as possible. The same filter was also applied to the output. The impulse response function is obtained by scaling the cross-correlation function of the filtered input and output. This approach allows to see the values of the impulse response for negative times. Figure 6 shows calculated impulse response functions in the weak (y) direction for the fixed-base and the SSI cases. It is clear from the figure that when there is SSI, the impulse response function has amplitudes for $t < 0$ comparable to those for $t > 0$. For no SSI, the amplitudes for $t < 0$ are significantly smaller than those for $t > 0$.

Another method to detect SSI is to identify the building as if there were no SSI and then investigate the cross-correlation of the input with the residuals (i.e., the difference between the recorded output and the output of the identified model) of the identification. If there is SSI, the cross-correlation would show large amplitudes at negative lags. A more detailed discussion of this method can be found in Şafak (1988).

4 IDENTIFICATION OF SSI

Identification of SSI involves extracting the individual dynamic characteristics of the fixed-base building and the foundation from the recordings of the



NO SSI : $f_x = 2\pi\omega_x = 1.15\,Hz.$

$f_y = 2\pi\omega_y = 0.71\,Hz.$

WITH SSI : $f_x = 2\pi\omega_x = 0.60\,Hz.$

$f_y = 2\pi\omega_y = 0.52\,Hz.$

Figure 4. 10 story model building and its fundamental frequencies in x and y directions with and without soil-structure interaction.

building. In addition to those from the building, if there are recordings from a nearby free-field site (i.e., a site whose motion is not influenced by the vibrations of the building), the identification of SSI is fairly straightforward. We take the free field recordings as the input, and foundation and upperstory recordings as the output, and identify the soil-foundation-structure system as a whole using open-loop identification techniques. In most cases, we do not have such free field records. The problem is then how to extract the dynamic characteristics of the building and the foundation from the foundation and upper story recordings alone.

To solve this problem, we will investigate the ratio $R = H_b/H_s$ of the 2-DOF system of Figure 1. From equations (6) and (7), and by defining the nondimensional frequency ratios $r_s = \omega_s/\omega$ and $r_b = \omega_b/\omega$, we can write for R

$$R = \frac{-4\xi_s\xi_b r_s r_b + i(2\xi_s r_s r_b^2 + 2\xi_b r_b r_s^2) + r_s^2 r_b^2}{-i2\xi_s r_s - r_s^2 - 4\xi_s\xi_b r_s r_b + i(2\xi_s r_s r_b^2 + 2\xi_b r_b r_s^2) + r_s^2 r_b^2} \qquad (9)$$

First, note that R is independent of $\mu = m_b/m_s$. Also, note that if $\xi_s = \xi_b = 0$, then $R = r_b^2/(r_b^2 - 1)$,

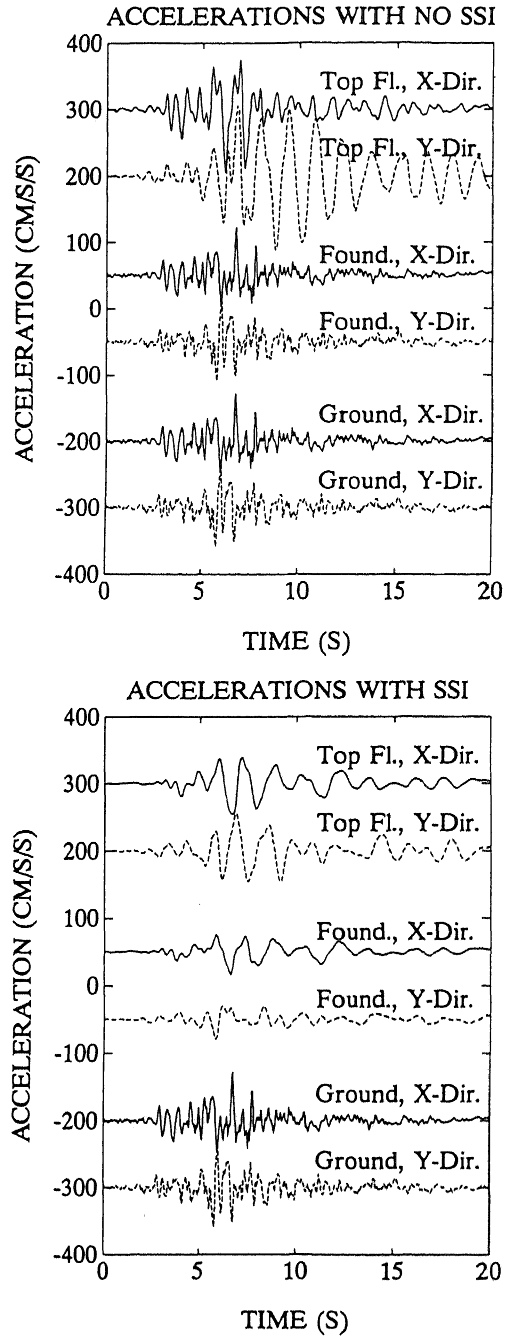


Figure 5. Ground accelerations, and calculated acceleration responses at the top story and the foundation level with no SSI (top figure; note that ground and foundation accelerations are identical) and with SSI (bottom figure).

which is independent of r_s (or ω_s) and has a peak at $r_b = 1$ (or at $\omega = \omega_b$). It can further be shown that R is not affected by r_s and ξ_s, and always has its peak at or near $r_b = 1$ for small values of ξ_b (Şafak,

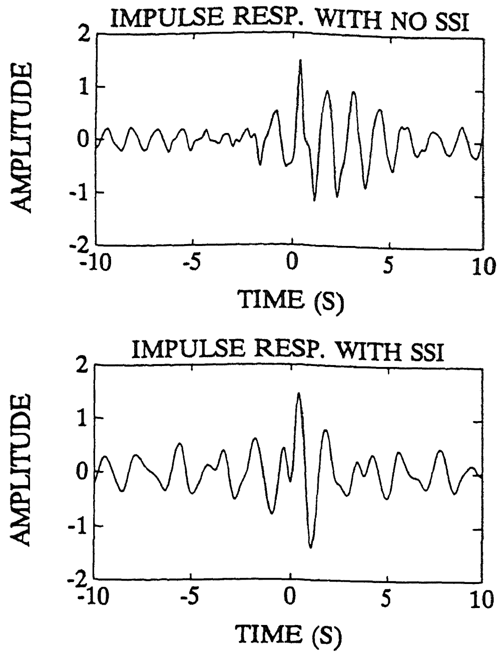


Figure 6. Impulse response functions obtained from top story and foundation accelerations with no SSI (top figure) and with SSI (bottom figure).

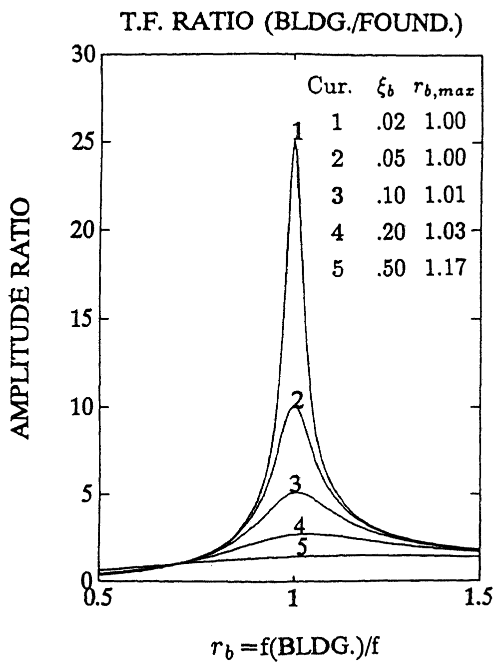


Figure 7. Variation of $|R|$ with r_b for four values of ξ_b, and r_b values corresponding to peaks (r_s and ξ_s do not affect the curves).

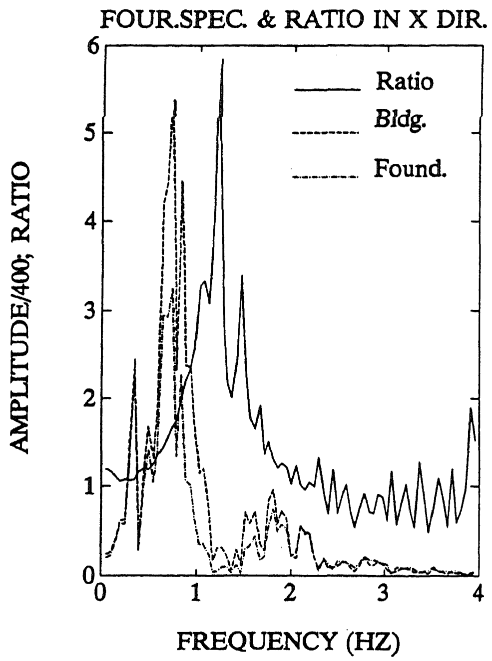
1992). This observation suggests that R is not influenced by the dynamic characteristics of the foundation, and always peaks at or near the fixed-base building frequency for buildings with small damping. To show this graphically, we plot $|R|$ in Figure 7 for r_b varying from 0.5 to 1.5, and for four values of ξ_b. Variations in r_s and ξ_s do not change the curves. Also given in the figure is the r_b value corresponding to the peak of each curve. As the figure shows, $|R|$ has a peak at $r_b = 1$ (or $\omega = \omega_b$) for realistic values of building damping ratios (i.e., $\xi_b < 0.20$). Even at $\xi_b = 0.50$, the shift of the peak frequency from the frequency of the fixed-base building is only 17 percent.

Considering the definitions of H_s and H_b, R can be calculated as the ratio of the Fourier spectrum of the top story accelerations to that of the foundation accelerations. The dominant peak of this ratio gives the fundamental frequency of the building for the fixed-base case. The fundamental frequency of the foundation can be determined from Eq. 3. For this, we first determine ω_1, the dominant frequency of the top story records, and make an assumption for the value of μ based on the design specifications for the building and the characteristics of the surrounding soil. Putting the values of ω_1, ω_b, and μ in Eq. 3, we obtain ω_s. It can be shown that if m_s is negligible (i.e., $\mu \to \infty$), the following relationship holds: $\omega_1^{-2} = \omega_b^{-2} + \omega_s^{-2}$ (Wolf, 1985).

We will give an example for the method, again by using the calculated response of the 10-story building given in Figures 4 and 5. From the results for SSI case, we calculate R as the ratio of the Fourier amplitude spectrum of the top-story acceleration response to that of the the foundation acceleration response. The amplitudes of R are shown in Figure 8 for each direction, along with the Fourier amplitude spectra of top-story and foundation accelerations. Note that for both directions the peaks of $|R|$ are at the natural frequencies of the fixed-base building, $1.15\,Hz$ in the x direction and $0.71\,Hz$ in the y direction. These frequencies differ from the dominant frequencies of the top story response, which are $0.60\,Hz$ in the x direction and $0.52\,Hz$ in the y direction.

5 CONCLUSIONS

Soil-structure interaction can significantly alter the characteristics of seismic records in buildings. Dominant frequency with SSI is always smaller than that of the fixed-base building and the foundation. The existence of SSI can be detected by investigating the impulse response functions between the foundation and upper-story accelerations. If the amplitudes of the impulse response function at negative times are comparable to those for positive times, the building is affected by SSI. The ratio of the Fourier spectrum of an upper-story acceleration record to that of the foundation acceleration record shows the dominant frequency of the fixed-base building. Once this frequency is determined, the dominant frequency of the foundation can be calculated from analytical relations derived from a simple model of SSI.



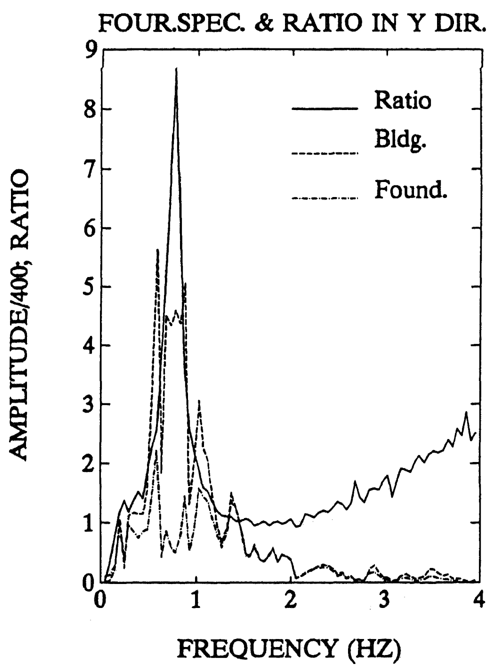


Figure 8. Fourier amplitude spectra (divided by 400) and their ratio ($|R|$) of the top story and foundation accelerations in x (top figure) and y (bottom figure) directions for SSI case.

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