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A simplified estimation of impedances for pile groups under vertical and horizontal vibrations

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ABSTRACT: An estimation of the impedances referred to rigid massless caps of pile groups is proposed in the simple procedure without much computational effort under harmonically vertical and horizontal vibrations and consistently in the static and dynamic states. The dynamic soil reaction and the interaction factor in consideration of the pile-soil-pile interaction are derived from the dynamic Winkler model by employing the dynamic Kelvin's solution of the soil. The simplified estimation is verified from comparison with the previous solutions of others. The proposed estimation is sufficiently accurate and readily useful in the practical situation. A convenient expression of the efficiency factor of pile groups in the static case is also presented in the closed form including the number of pile and the pile spacing of the group.

1 INTRODUCTION

For the purpose of predicting the dynamic responses of structures with pile group foundations exposed to seismic excitation, in recent years, a significant amount of work has been done on the impedances referred to the pile caps of soil-pile group systems, as the impedances become to play an important role in coupling to superstructures. It is desirable that the dynamic responses of structures are readily predicted in consideration of the complex pile-soil-pile dynamic interaction in the practical situation, because much computational effort is used up costly for more accurate estimation of the impedances by the useful numerical method such as the finite-element method or the boundary-element method. For that demand the simplified methods, which deal with the soil as the Winkler medium, are successfully developed from the dynamic plane-strain solution[1] of the soil by Novak[2] for single piles and by Nogami[3] for pile groups, and from the dynamic Kelvin's solution of the soil by Nozoe et al.[4] for single piles. The another simple method, where the impedances of pile groups is derived from the known impedances of single piles, is presented by Dobry et al.[5]. The simplified estimation of impedances for pile groups, however, is not performed consistently in the static and dynamic cases.

A simplified estimation of impedances for pile groups, in this paper, is proposed under vertical and horizontal vibrations by extension of the above simplified method of single piles based on the dynamic Kelvin's solution of the soil.

2 DESCRIPTION OF MODEL AND FORMULATION

An analysis model of a pile-soil-pile system sketched as cylindrical coordinates in Fig.1 is considered for a floating pile group installed in a surface stratum lying on the rigid bedrock under vertical and horizontal vibrations. The soil is dealt with as three-dimensional continuum to be elastic, homogeneous and isotropic with the linear hysteretic damping. Each of the identical pile in the group is assumed to be perfectly in contact with the soil during the motion.

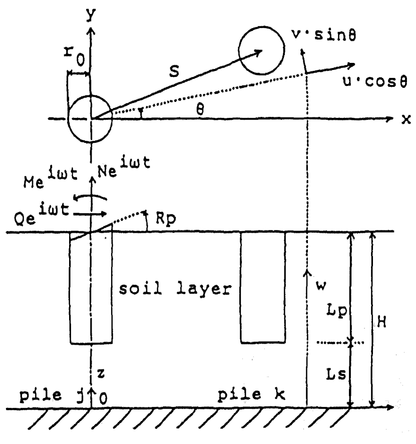

Fig.1 Model of pile-soil-pile system

For the pile j, the equations of motion and the constitutive relationships with respected to the vertical and horizontal displacements W_P and U_P, respectively, and the rotational angle R_P is expressed by

$$\frac{dN_{Pj}}{dz} - p_{Vj} = -\rho_P A_P \omega^2 w_P \,;\quad \frac{dW_{Pj}}{dz} = \frac{N_{Pj}}{E_P A_P}$$
$$\frac{dQ_{Pj}}{dz} - p_{Hj} = -\rho_P A_P \omega^2 u_P \,;\quad \frac{dU_{Pj}}{dz} = -R_{Pj}$$
$$\frac{dM_{Pj}}{dz} - Q_{Pj} = 0 \qquad ;\quad \frac{dR_{Pj}}{dz} = \frac{M_{Pj}}{E_P I_P} \tag{1}$$

where the time factor $\exp(i\omega t)$ in the harmonic excitation is abbreviated for convenience. N_{Pj} and Q_{Pj} are the axial and shear forces, respectively, and M_{Pj} is the bending moment. ρ_P=mass density, E_P= Young's modulus, A_P=cross sectional area, I_P=moment of inertia of the cross section. p_{Vj} and p_{Hj} are the tortal tractions per unit length of the soil along the shaft of the pile in the z and x directions, respectively. The pile j is also subjected to the harmonic excitations N_{PLj}, Q_{PLj} and M_{PLj} at the pile head, and the total soil reactions N_{Bj}, Q_{Bj} and M_{Bj} at the pile tip.

For the soil in contact with the pile j, the total tractions p_{Vj} and p_{Hj}, the total displacements W_j and U_j along the pile shaft, and N_{Bj} and Q_{Bj} at the pile tip in the z and x directions, respectively, can be expressed by

$$p_{Vj} = \Sigma\, p_{Vjk} \,;\; W_j = \Sigma\, W_{jk} \,;\; N_{Bj} = \Sigma\, N_{Bjk}$$
$$p_{Hj} = \Sigma\, p_{Hjk} \,;\; U_j = \Sigma\, U_{jk} \,;\; Q_{Bj} = \Sigma\, Q_{Bjk} \tag{2}$$

where Σ is the meaning of the sum over all of the pile (k=1, 2, ···, n) in the group. M_{Bj} is omitted because it is negligibly small.

By solving Eqs. 1 to take into account of Eqs. 2 and the continuity condition of the displacements between the pile j and the soil, i.e. $W_{Pj}=W_j$ and $U_{Pj}=U_j$, the impedance matrix [K] referred to the rigid massless cap of the pile group is obtained from the following definition:

$$\begin{Bmatrix} N_c \\ Q_c \\ M_c \end{Bmatrix} = \begin{bmatrix} K_{VV}, & 0\ , & 0 \\ 0\ , & K_{HH}, & K_{HR} \\ 0\ , & K_{RH}, & K_{RR} \end{bmatrix} \begin{Bmatrix} W_c \\ U_c \\ R_c \end{Bmatrix} \tag{3}$$

where $K_{HR}=K_{RH}$. N_c, Q_c and M_c are the dynamic loadings acting at the middle point of the cap, and then W_c, U_c and R_c become the corresponding responses of the cap.

3 SIMPLIFICATION OF ANALYSIS MODEL

In the above boundary-value problem, the exact estimation of the pile j-soil-the pile k interaction requires the huge computational process. For the purpose of an simplified estimation of the impedances for the pile group, the soil is assumed as the Winkler medium. Then the tractions p_{Vjj} and p_{Hjj} and the soil reactions N_{Bjj} and Q_{Bjj} occurring on the active pile j due to the displacements W_{jj} and U_{jj} of the soil on itself can be approximately expressed as follows:

$$p_{Vjj} = K_{CV}(1+2i\xi)W_{jj}$$
$$p_{Hjj} = K_{CH}(1+2i\xi)U_{jj}$$
$$N_{Bjj} = K_{BV}(1+2i\xi)W_{jj} \tag{4}$$
$$Q_{Bjj} = K_{BH}(1+2i\xi)U_{jj}$$

where ξ =ratio of hysteretic damping for soil. It is the first step to a simplification in the dynamic Winkler model that we are able to find K_{CV}, K_{CH}, K_{BV} and K_{BH} of the dynamic Winkler springs to be constant with depth as the appropriate spring constants to reflect the pile behavior.

The Winkler spring constants K_{CV} and K_{CH} along the shaft of pile are derived from the approximate solutions for a tentative floating pile made of soil in an elastic half medium. Those solutions are obtained approximately under the dynamic loadings distributed along the pile shaft triangularly (suffix m=1) and uniformly (suffix m=2), which represent the two typical pile behaviors, by superposition of the dynamic Kelvin's solutions based on the mirror reflection and the dynamic plane-strain solutions in vertical and horizontal vibrations. The Winkler spring constants along the pile shaft ($r=r_o$) are defined as

$$K_{CV} \fallingdotseq 2\pi r_o k_V \,;\; k_V = \tau_m / W_m$$
$$K_{CH} \fallingdotseq \pi r_o k_H \,;\; k_H = \sigma_m / U_m \tag{5}$$

where τ and σ are tractions in the z and x directions, respectively. By the displacements $u\cos\theta$, $v\sin\theta$ and w of the soil in the r, θ and z directions, respectively, $U=(u-v)/2$, $V=(u+v)/2$ and $W=w$. Then the solutions in the horizontal vibration result in $U_{jj}=U(r_o)+V(r_o)\cos 2\theta \fallingdotseq U(r_o)$ and represent nicely the circular condition on the pile circumference like a real pile, because $V(r_o)$ becomes negligibly small over deeper depth from the ground surface by employing the plane-strain solution besides the dynamic Kelvin's solution, which is used only in the previous estimation[4]. The coefficients of traction k_V and k_H become large near the pile head and tip in the exact results, but are regarded to be roughly constant along depth as those of the approximate solutions. The coefficients of traction at the ground surface in the solutions for the triangular and uniform distributions would be available as a representative value.

Therefore, an approximation of the solutions for the triangular and uniform distributions is performed at the ground surface under the slenderness ratio $L_P/r_o \geqq 10$ in the explicit form. That is

1) Triangular distribution:

[Lower frequency range: $a_P \leqq 2$]

$$\frac{W_1}{A_1 r_o} = -\left[\ln\left(\frac{2L_P}{r_o}\right) - \frac{1}{4(1-\nu)} - 1\right] + \frac{1}{24}a_P^2 + \frac{1}{3}\, i\, a_P\left(1 - \frac{1}{60}a_P^2\right)$$

$$\frac{U_1}{B_1 r_o} = -\frac{3-4\nu}{8(1-\nu)}\left[\ln\left(\frac{2L_P}{r_o}\right) + \frac{1}{2(3-4\nu)} - 1\right] + \frac{1}{32}a_P^2\left(1 - \frac{1}{36}a_P^2\right) + \frac{1}{6}\, i\, a_P\left(1 - \frac{1}{30}a_P^2\right)$$

$$\frac{\tau_1}{A_1\mu}=1-\frac{3-2\nu}{2(1-\nu)}\cdot\frac{r_o}{L_p},\qquad \frac{\sigma_1}{B_1\mu}=1-\frac{3-4\nu}{4(1-\nu)}\cdot\frac{r_o}{L_p} \tag{6a}$$

[Higher frequency range: $a_p>2$]

$$\frac{W_1}{A_1 r_o}=\frac{W_o}{r_o}+\frac{1}{a_p^2}\{\exp(-i b_p)+\frac{L_p}{r_o}[(1-i a_o)\exp(-i a_o)-\exp(-i b_o)]\}$$

$$\frac{U_1}{B_1 r_o}=\frac{U_o}{r_o}+\frac{1}{4a_p^2}\{2\exp(-i a_p)-\frac{L_p}{r_o}[(1+i a_o)\exp(-i a_o)-(1-i b_o)\exp(-i b_o)]\}$$

$$\frac{\tau_1}{A_1\mu}=\frac{\tau_o}{\mu}-\frac{r_o}{L_p}\exp(-i a_o)-\frac{2}{a_p^2}\cdot\frac{L_p}{r_o}[(1+i a_o)\exp(-i a_o)-(1+i b_o)\exp(-i b_o)]$$

$$\frac{\sigma_1}{B_1\mu}=\frac{\sigma_o}{\mu}-\frac{r_o}{2L_p}[\exp(-i a_o)+\exp(-i b_o)]+\frac{1}{a_p^2}\cdot\frac{L_p}{r_o}[(1+i a_o)\exp(-i a_o)-(1+i b_o)\exp(-i b_o)] \tag{6b}$$

2) Uniform distribution:

[Lower frequency range: $a_p\leqq 2$]

$$\frac{W_2}{A_2 r_o}=-[\ln(\frac{2L_p}{r_o})-\frac{1}{4(1-\nu)}]+\frac{1}{8}a_p^2+\frac{2}{3}i a_p(1-\frac{1}{30}a_p^2)$$

$$\frac{U_2}{B_2 r_o}=-\frac{3-4\nu}{8(1-\nu)}[\ln(\frac{2L_p}{r_o})+\frac{1}{2(3-4\nu)}]+\frac{3}{32}a_p^2(1-\frac{5}{108}a_p^2)+\frac{1}{3}i a_p(1-\frac{1}{15}a_p^2)$$

$$\frac{\tau_2}{A_2\mu}=1,\qquad \frac{\sigma_2}{B_2\mu}=1 \tag{7a}$$

[Higher frequency range: $a_p>2$]

$$\frac{W_2}{A_2 r_o}=\frac{W_o}{r_o}+\frac{1}{a_p^2}[2\exp(-i a_p)-(1+i b_p)\exp(-i b_p)]$$

$$\frac{U_2}{B_2 r_o}=\frac{U_o}{r_o}+\frac{1}{2a_p^2}[\exp(-i b_p)-i a_p\exp(-i a_p)]$$

$$\frac{\tau_2}{A_2\mu}=\frac{\tau_o}{\mu},\qquad \frac{\sigma_2}{B_2\mu}=\frac{\sigma_o}{\mu} \tag{7b}$$

3) Plane-strain solution:

$$\frac{W_o}{r_o}=\frac{\pi}{2}i H_0^{(2)}(a_o),\qquad \frac{\tau_o}{\mu}=-\frac{\pi}{2}i a_o H_1^{(2)}(a_o)$$

$$\frac{U_o}{r_o}=\frac{\pi^2}{32}b_o^2[H_2^{(2)}(b_o)H_0^{(2)}(a_o)+H_2^{(2)}(a_o)H_0^{(2)}(b_o)]$$

$$\frac{\sigma_o}{\mu}=-\frac{\pi^2}{16}b_o^2 a_o[H_2^{(2)}(b_o)H_1^{(2)}(a_o)+\frac{\kappa_H}{\kappa_V}H_2^{(2)}(a_o)H_1^{(2)}(b_o)] \tag{8}$$

where A_m and B_m ($m=1,2$) are arbitrary constants. $\kappa_V=\omega/V_V$, $\kappa_H=\omega/V_H$, V_V and V_H are wave velocities of soil for dilatation and distortion, respectively. μ = shear modulus of soil, ν = Poisson's ratio of soil. $a_o=\kappa_H r_o$, $b_o=\kappa_V r_o$, $a_p=\kappa_H L_p$, $b_p=\kappa_V L_p$, $H_i^{(2)}$ is the Hankel function of the second kind of the i-th order. When the pile length tends to infinity, the above solutions agree with the plane-strain solutions of Novak et al.[1].

The dynamic coefficients of traction for arbitrary distribution by superposing the solutions for the triangular and uniform distributions. That is

$$k_V=-\frac{\tau_1+\tau_2}{W_1+W_2},\quad k_H=-\frac{\sigma_1+\sigma_2}{U_1+U_2} \tag{9}$$

and

$$A_1=B_1=\frac{L_o}{H_o},\qquad A_2=B_2=1-\frac{L_o}{H_o} \tag{10}$$

where the effective length $L_o=\min(L_p, L_c)$ and the effective depth $H_o=\min(H, L_c)$ are introduced and L_p in the above solutions is replaced by L_o. Also the critical length is defined in the static state ($\omega=0$) as follows:

$$L_c=3/\sqrt{K_{cV}/(E_p A_p)}\ \text{for vertical};\quad =3/\sqrt[4]{K_{cH}/(4E_p I_p)}\ \text{for horizontal} \tag{11}$$

Because the result for the triangular distribution is suitable for end bearing piles or long-flexible piles ($L_p>L_c$) and the result for the uniform distribution is also suitable for rigid floating piles in half media, Eqs. 10 are given in a simple superposition. Moreover, the static coefficients of traction can be expressed in the following simple form:

[$L_o/r_o\geqq 10$]

$$\frac{1}{k_V}=\frac{r_o}{\mu}[\ln(\frac{2L_o}{r_o})-\frac{1}{4(1-\nu)}-\frac{L_o}{H_o}]/[1-\frac{3-2\nu}{2(1-\nu)}\cdot\frac{r_o}{H_o}]$$

$$\frac{1}{k_H}=\frac{r_o}{\mu}\cdot\frac{3-4\nu}{8(1-\nu)}[\ln(\frac{2L_o}{r_o})+\frac{1}{2(3-4\nu)}-\frac{L_o}{H_o}]/[1-\frac{3-4\nu}{4(1-\nu)}\cdot\frac{r_o}{H_o}] \tag{12}$$

The proposed coefficients of traction of Eqs. 12 are useful within 10 % error from comparison with the exact solutions of others. The dynamic Winkler spring constants along the pile shaft can be obtained by Eqs. 5 and 9 readily and consistently in the static and dynamic states. Although the critical length L_c is determined with a few iteration from Eqs. 11 and 12 by $L_o=H_o=L_c$, we give the following regression of L_c as a useful estimation.

$$\frac{L_c}{r_o}=\frac{5}{2}\left[\frac{E_pA_p}{\mu A_s}\right]^{\frac{7}{15}} \text{ for vertical}$$
$$=\frac{36}{11}\left[\frac{E_pI_p}{E_sI_s}\right]^{\frac{13}{46}} \text{ for horizontal} \qquad (13)$$

where E_s=Young's modulus of soil, $A_s=\pi r_o^2$ and $I_s=\pi r_o^4/4$. The error of Eqs. 13 is within 2 % in $E_pA_p/\mu A_s=35\sim750$ and 1 % in $E_pI_p/E_sI_s=15\sim10^4$ for $\nu\geqq0.3$.

On the other hand, the Winkler spring constants K_{sv} and K_{sH} at the pile tip are proposed in the simple expression. Those are derived from the static state.

$[L_s/r_o\geqq5]$

$$K_{sv}=\frac{2.56\mu r_o}{1-\nu}(1+\frac{2r_o}{L_s})$$
$$K_{sH}=\frac{6\mu r_o}{2-\nu} \qquad (14)$$

where $L_s=H-L_p$. Eqs. 14 are calibrated from Kausel's semi-analytical expressions for a rigid circular foundation on a stratum over a rigid bedrock in consideration of the effect of the tractions along the pile shaft and valid within 10 % error from comparison with the exact results of others. The dynamic springs are also used approximately by Eqs. 14, because of the small influence of N_{sj} and Q_{sj} on the impedance for the slender piles ($L_p/r_o\geqq10$).

The passive pile j is influenced by the active pile k through the soil. It is the second step to a simplification that p_{vjk}, p_{Hjk}, N_{sjk}, Q_{sjk}, W_{jk} and U_{jk} of the soil on the pile j made of soil, namely the soil column j, which occur due to the displacements W_{kk} and U_{kk} of the active pile k, can be obtained to reflect the soil motion. Though the responses on the circumference of the soil column j generally vary, these responses can be estimated approximately at the axis of this soil column for $a_o\leqq\pi/6$ in the case of longer wavelength propagated in comparison with the diameter of pile. Therefore, W_{jk} and U_{jk} at $r=S_{jk}$ and $\theta=\theta_{jk}$, where S_{jk} is axis-to-axis pile spacing and θ_{jk} is angle between the line of the two piles and the x direction, can be expressed as follows:

$$W_{jk}=T_{vjk}W_{kk}$$
$$U_{jk}=T_{Hjk}U_{kk} \qquad (15)$$

The dynamic interaction factors T_{vjk} and T_{Hjk} are derived from the same solutions as those of the coefficients of traction along the pile shaft. For simplification T_{vjk} and T_{Hjk} to be constant with depth are approximately estimated at the ground surface as a representative value.

$$T_{vjk}=W(S_{jk})/W(r_o)$$
$$T_{Hjk}=[U(S_{jk})+V(S_{jk})\cos2\theta_{jk}]/U(r_o) \qquad (16)$$

where $T_{vjj}=T_{Hjj}=1$ are also defined. As a result, the variation of the passive responses W_{jk} and U_{jk} with depth becomes analogous to the variation of the active responses W_{kk} and U_{kk}.

On the other hand, the tractions p_{vjk} and p_{Hjk} along the shaft of the soil column j due to the active pile k are directly estimated from the equations of motion such as Eqs. 1 for the pile when the motions are the responses W_{jk} and U_{jk}. That is

$$p_{vjk}=E_sA_s\frac{d^2W_{jk}}{dz^2}+\rho_sA_s\omega^2W_{jk}$$
$$p_{Hjk}=-E_sI_s\frac{d^4U_{jk}}{dz^4}+\rho_sA_s\omega^2U_{jk} \qquad (17)$$

where ρ_s=soil density.

The soil reaction N_{sjk} and Q_{sjk} at the tip of the soil column j due to the action of the pile k are obtained in the same way.

$$N_{sjk}=E_sA_s\frac{dW_{jk}}{dz}\ ;\ Q_{sjk}=-E_sI_s\frac{d^3U_{jk}}{dz^3} \qquad (18)$$

where the inertial force of soil is neglected.

4 RESULTS OF IMPEDANCES AND DISCUSSIONS

The pile-soil-pile interaction problem arrives at the eigen-value problem with respected to the independent responses W_{kk} and U_{kk} of the soil in contact with the pile k ($k=1,2,\cdots,n$) in the group. The solutions of the pile j can be obtained in the following expression.

$$W_{pj}=\Sigma\gamma_{vj\ell}W_{o\ell}(\lambda_{v\ell}z)$$
$$U_{pj}=\Sigma\gamma_{Hj\ell}U_{o\ell}(\lambda_{H\ell}z) \qquad (19)$$

where Σ is the sum of $\ell=1$ to n. $\lambda_{v\ell}$ and $\lambda_{H\ell}$ are the eigen values of the ℓ-th order. $\gamma_{vj\ell}=\Sigma T_{vjk}\alpha_{k\ell}$, $\gamma_{Hj\ell}=\Sigma T_{Hjk}\beta_{k\ell}$, where Σ is the sum of k=1 to n, and $\alpha_{k\ell}$ and $\beta_{k\ell}$ are the k-th components in the eigen vectors of the ℓ-th order. $W_{o\ell}$ and $U_{o\ell}$ are the general solutions of the ℓ-th order and the including integral constants are determined from the boundary conditions of the pile head and tip systematically. Finally, the impedances referred to rigid massless caps of pile groups are estimated in the simple procedure from Eq. 3.

The simplified estimation of the impedance matrix for pile groups is verified by the numerical analysis from comparison with the previous solutions of others. The numerical analysis is performed for the square groups of n piles capped rigidly and the closest pile spacing $S=4r_o$ as sketched in Fig. 2. The comparison of the results of the impedance matrix for pile groups in an homogeneous half medium by the proposed method, the exact solution of Kaynia & Kausel (1982) and the simple method of Dobry & Gazetas[8)] are shown for the real part= $Re(_nK)$ and the imaginary part=$Im(_nK)/(2a_o)$ normalized by the static $Re(_1K)$ of the single pile with the dimensionless frequency a_o in Fig. 3. The analytical parameters are as follows:
Poisson's ratio $\nu=0.4$ and the hysteretic damping ratio $\xi=0.05$ of soil, the slender-

ness ratio $L_p/r_0=30$, the Young's modulus ratio of pile to soil $E_p/E_s=10^3$ and the density ratio of pile to soil $\rho_p/\rho_s=4/3$.

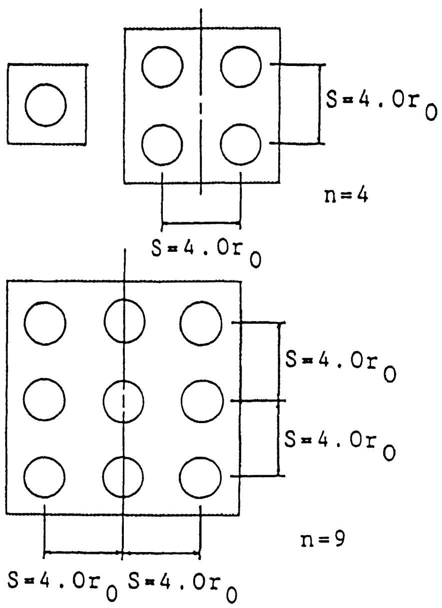

Fig.2 Block plan of pile group

The present results of the single pile (1×1) are sufficiently accurate from the results of the exact solution with frequency. In the present results of pile groups (2×2, 3×3, 4×4), the real parts of the impedances agree well with those of the exact solutions overall, but the results of the imaginary part are inferior to the results of the real part. The results of the proposed method are more accurate than the results of the simple method in the higher frequency range. The simplified estimation of the impedances for groups of end bearing piles has been verified from comparison with the previous solutions of others in the reference 6). Therefore, the proposed estimation is available for both end bearing and floating piles of groups and the dynamic characteristics of pile groups can be estimated readily and consistently in the static and dynamic states.

Meanwhile the group efficiency of the stiffness ${}_nK/{}_1K$ in the static case is investigated for a single pile to a group of 100 piles. Poulos[7] has been found that the deflection of the pile group tends to be proportionally to the root of the number of pile for the deflection of single pile and this consideration is also reasonable for the stiffness in the analogy of the area of the surface foundation to the number of pile in the square group from the suggestion of Kusakabe. The results of the group efficiency are shown with $\sqrt{n}$ in Fig.4 for the same

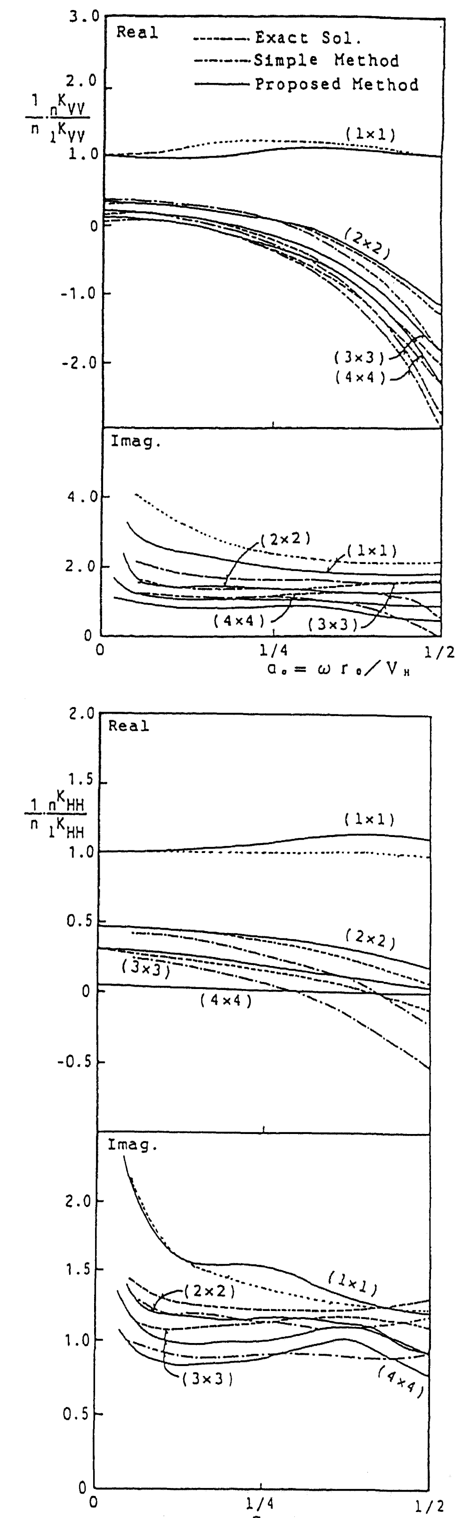

Fig.3 Variations of impedances with frequency

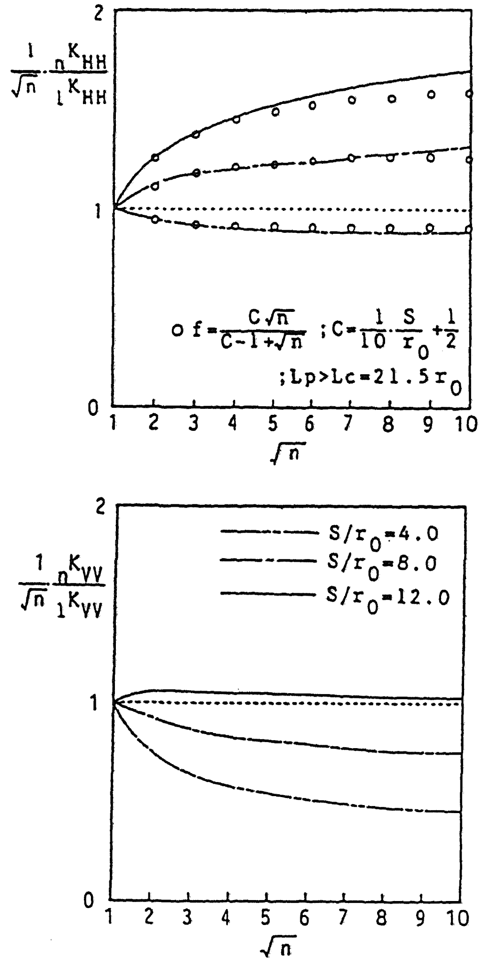

Fig. 4 Efficiency factor of pile group with $\sqrt{n}$

parameter of Fig. 3. It is seen that $_nK$ are dependent on $\sqrt{n}$ remarkably as n increases and are influenced by the closest pile spacing S. A convenient expression of the efficiency factor of pile groups is presented for K_{HH} of long-flexible piles ($L_p > L_c = 21.5 r_o$). That is

$$f=\frac{C\sqrt{n}}{C-1+\sqrt{n}} \;;\; C=\frac{1}{10}\cdot\frac{S}{r_o}+\frac{1}{2} \qquad (20)$$

The above estimation is verified in Fig. 4, and is valid for the number of pile and the pile spacing of pile groups.

5 CONCLUSIONS

A simplified estimation of impedances for pile groups has been proposed physically and consistently in the static and dynamic states. The proposed estimation is sufficiently accurate and readily useful in the practical situation. A convenient estimation of the efficiency factor of pile group in the static case is also proposed in the closed form including the number of pile and the pile spacing of the group. This application to the preliminary design and so on would offer a good insight into the interaction between pile-soil-pile systems and structures without much computational effort even if a pile group consists of 100 piles.

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REFERENCES

1) M. Novak, T. Nogami and F. Aboul-Ella 1978. Dynamic Soil Reaction for Plane Strain Case, ASCE, Vol. 104, No. EM4: 953-959.
2) M. Novak 1974. Dynamic Stiffness and Damping of Piles, Canadian Geotechnical J., Vol. 11, No. 4: 574-598.
3) T. Nogami 1985. Flexural Responses of Grouped Piles under Dynamic Loading, Earthquake Engineering and Structural Dynamics, Vol. 13: 321-336.
4) H. Nozoe, Y. Gyoten, K. Mizuhata and T. Fukusumi 1988. Estimation of Impedance and Transfer Functions for End Bearing and Floating Piles, Proc. of 9th WCEE, Vol III: 563-568.
5) R. Dobry and G. Gazetas 1988. Simple Method for Dynamic Stiffness and Damping of Floating Pile Groups, Geotechnique, Vol. 38, No. 4: 557-574.
6) H. Nozoe , K. Kusakabe and T. Fukusumi 1990. A Physical Approximation for the Pile-Head Impedances of Soil- Pile Group Systems, Proc. of 8th JEES: 1257-1262.
7) H. G. Poulos 1979. Group Factor for Pile-Deflection Estimation, ASCE, Vol. 105, No. GT12: 1489-1509.