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An approximate analysis method of embedded foundations considering backfilled soil

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ABSTRACT: Taking the actual foundation construction operations into account, it is quite natural to think that the soil around the foundations may be disturbed and backfilled. In this paper, the authors attempt to present an approximate method of analysis that considers backfilled soil effects when calculating both impedance functions and input motions for an embedded cylindrical foundation. The employed approximate method of analysis is based on the assumption that the soil underlying the foundation base is an elastic half-space and that the overlying soil consists of independent elastic layers. It is shown that the employed approximate method may give fairly good results when compared with those of more rigorous hybrid methods.

1 INTRODUCTION

An optimal aseismic design of structures requires a detailed understanding of dynamic soil-structure interaction effects. The effects may be important particularly for large-scale structures. For linear elastic or viscoelastic soil systems, the dynamic soil-structure interaction problems can be separated into two basic types : a radiation problem and a scattering problem. The radiation problem may be characterized by impedance functions, or inversely, compliance functions, which represent the force-displacement relationship between the foundation and soil when the massless foundation is excited by external forces. The scattering problem may be characterized by foundation input motions, which are defined by motions of the massless foundation subjected to seismic excitations. Once these two basic problems have been solved, the response of the complete soil-foundation system can be evaluated without difficulty.

In this paper, an approximate analysis method that considers backfilled soil effects is presented to calculate both impedance functions and input motions for embedded foundations. A 3-dimensional hybrid method which combines finite element and boundary element methods has been developed by the authors to evaluate dynamic response characteristics of rigid embedded foundations considering backfilled soil effects. (see Yoshida et al.(1990), and Takahashi et al.(1992)) The hybrid method has the advantage of the effective properties of the two methods. The authors believe that the hybrid method is the most reliable and powerful approach to analyze these kinds of complicated problems. It is true, however, that the hybrid method requires large amounts of computational efforts. Therefore, there is a need for an alternative approximate method to yield reasonable results. The employed approximate method used to evaluate dynamic impedance functions for an embedded rigid foundation surrounded by backfilled soil is based on the approach by Novak et al. (1978), who assumed that the soil underlying the foundation base is an elastic half-space and that the overlying soil consists of independent elastic layers. In order to investigate the effects of the backfilled soil, the overlying elastic layers are assumed to have soil with different physical properties around the foundation than those of the surrounding soil. The concept figure of the approximate analysis method is illustrated in Fig. 1.

After the impedance functions for the embedded foundation have been obtained, the foundation input motions can be calculated by making use of the free field ground motion information. Iguchi (1982) has proposed an approximate method to evaluate the input motions of embedded foundations. The method is based on the assumption that the impedance functions of rigid embedded foundation are known. The input motions are evaluated by the weighted average values of the free field displacements generated along the contact area between the embedded foundation and the soil by adding a correction term, which is caused by the total forces and moments associated with the free field stresses generated along the contact area. The feature of this method is that it suffices to know only the geometry of the foundation and the free field motions, as long as the impedance functions are known in advance when the input motions are evaluated. It is confirmed by Pais and Kausel (1989) that Iguchi's method gives satisfactory results for engineering purposes. On the other hand, Luco (1986) has shown that input motions for an embedded foundation can be easily calculated if the distribution of contact tractions for unit motions are known. In the present study, since the impedance functions for the foundation base and for the side wall of the foundation are evaluated independently, it is not difficult to calculate the distribution of contact tractions when the foundation is subjected to vertically incident waves. Luco's formulation is employed in this study. Though the formulation is exact, the input motions obtained in this study are approximate because some compatibility conditions are violated in the calculation of the impedance functions even for the case of the foundtion without backfilled soil.

The formulation of the present approximate analysis method is described briefly in the next section. Numerical results and discussions are also presented to study the accuracy and the applicability of the present approximate method by comparing these results with those obtained by more rigorous methods.

2 METHOD OF ANALYSIS

Taking the actual foundation construction operations into account, it is quite natural to think that the soil around the foundations may be backfilled. The physical properties of the backfilled soil are expected to be different from those of the outer soil. In this section, an approximate soil-structure interaction analysis method that considers the backfilled soil effects is presented. Throughout this study, it is assumed that the backfilled soil has a constant width and exists only in the side direction of the foundation. An approximate method to evaluate dynamic impedance functions for an embedded rigid foundation is described first. The impedance functions for the side wall of the foundation and those for the bottom base of the foundation are evaluated separately, assuming that the soil underlying the foundation base is an elastic half space and that the overlying soil consists of independent elastic layers. The employed approximate method is an extension of the method proposed by Novak et al. (1978). The impedance functions for the whole foundation can be calculated easily if the condition of rigid body motions of the foundation is imposed. Once the impedance functions have been obtained, the foundation input motions can be calculated by making use of the free field ground motion information. This procedure will follow the formulation presented by Luco (1986).

2.1 Impedance Functions for Side Wall

In order to evaluate impedance functions for the side wall of an embedded rigid foundation surrounded by backfilled soil, the system depicted in Fig.2 is considered in this study. A horizontal soil layer of unit depth and infinite extent is assumed to have a part of soil with different physical properties from those of the outer soil. An analytical method used to obtain impedance functions for this system has been proposed by Veletsos and Dotson (1986). One of the authors has also proposed the method in a different fashion. (see Kumagai et al. (1987)). An outline of the analysis method for a horizontal vibration mode is presented here.

The backfilled soil and the surrounding outer soil are assumed to be linear and viscoelastic, respectively. The equation of motion of the viscoelastic media in cylindrical coordinates can be written as follows (Novak (1978)) :

$$(\lambda_i+2\mu_i)\frac{\partial\Delta_i}{\partial r}\;\frac{2\mu_i}{r}\frac{\partial\omega_{zi}}{\partial\theta}=\rho_i\frac{\partial^2 u_i}{\partial^2 t} \qquad (1)$$

$$(\lambda_i+2\mu_i)\frac{\partial\Delta_i}{r\partial\theta}+2\mu_i\frac{\partial\omega_{zi}}{\partial r}=\rho_i\frac{\partial^2 w_i}{\partial^2 t} \qquad (2)$$

Where the subscript i stands for the backfilled soil (i=1) and the outer soil (i=2). Δ is the relative volume change and ω_z is the rotational component around z axis, ρ is the mass density, and λ and μ are the complex Lamé's constants defined by $\lambda_0(1+2hj)$and $\mu_0(1+2hj)$, in which h denotes the hysteretic material damping, and $j=\sqrt{-1}$. The solutions of Eqs. (1) and (2) can be obtained by means of two potential functions. Considering the conditions that the cross section of the foundation has to keep circular at $r=r_0$ and that displacements must vanish at $r\rightarrow\infty$, the solutions can be written as

$$u_1=\left(A_1\frac{\partial H_1^{(2)}(q_1r)}{\partial r}+\frac{1}{r}A_2H_1^{(2)}(s_1r)+A_3\frac{\partial H_1^{(1)}(q_1r)}{\partial r}+\frac{1}{r}A_4H_1^{(1)}(s_1r)\right)\cos\theta \qquad (3)$$

$$w_1=\left(-\frac{1}{r}A_1H_1^{(2)}(q_1r)-A_2\frac{\partial H_1^{(2)}(s_1r)}{\partial r}-A_3H_1^{(1)}(q_1r)-\frac{1}{r}A_4\frac{\partial H_1^{(1)}(s_1r)}{\partial r}\right)\sin\theta \qquad (4)$$

$$u_2=\left(A_5\frac{\partial H_1^{(2)}(q_2r)}{\partial r}+\frac{1}{r}A_6H_1^{(2)}(s_2r)\right)\cos\theta \qquad (5)$$

$$w_2=\left(-\frac{1}{r}A_5H_1^{(2)}(q_2r)-A_6\frac{\partial H_1^{(2)}(s_2r)}{\partial r}\right)\sin\theta \qquad (6)$$

Where, $q_i=\frac{\omega}{V_{Pi}\sqrt{1+2h_ij}}$ $\quad s_i=\frac{\omega}{V_{Si}\sqrt{1+2h_ij}}$. (7) (8)

Where $H^{(1)}{}_n$and $H^{(2)}{}_n$are Bessel functions of the first and second kind of order n. A_1 through A_6 are constants that will be determined by the conditions that the displacement amplitude of the vibration direction is unity at $r=r_0$ and that the displacements and stresses are continuous at the interface of the backfilled soil and the outer soil ($r=r_0+\Delta r$). Once the displacements have been determined, the stresses can be derived by using the displacement-stress relationship, and the horizontal impedance function can be obtained by integrating the stresses along the foundation. The final expression can be written as follows:

$$K=\mu_2K_{HS} \qquad (9)$$

Where K_{HS} is a dimensionless impedance function of $\mu 1/\mu 2$, $\lambda 1/\lambda 2$, $\rho 1/\rho 2$, $\Delta r/r0$, and a frequency. The actual expression of K_{HS} is quite complicated, but K_{HS} does not include any integration or series summation. So, the impedance function can be computed straightforwardly.

2.2 Impedance Functions for Embedded Foundation

Impedance functions for the bottom mat of the embedded foundation are calculated by using a 3-dimensional boundary element method (Sato et al. (1983)). Effects of the existence of surface soil above the foundation bottom are neglected in this analysis. By utilizing the impedance components obtained independently for the side wall and for the base mat, the dynamic impedance functions for the embedded rigid foundation are expressed as follows :

$$[\mathrm{K}]=\begin{bmatrix} K_{HB}+K_{HS}H_0 & \frac{1}{2}K_{HS}H_0^2 \\ \frac{1}{2}K_{HS}H_0^2 & K_{MB}+K_{MS}H_0+\frac{2}{3}K_{HS}H_0^3 \end{bmatrix} \tag{10}$$

Where H_0 is the depth of the embedded foundation from the surface. The subscripts H and M denote horizontal and rocking components, respectively, and B and S stand for the base mat and the side wall components, respectively. The reference point is chosen at the center of the bottom of the foundation, and only sway and rocking components are considered in this study. The above expression is derived easily when the rigid motions of the foundation are assumed. It should be noted that the compatibility condition between the surface layer and the elastic half space is not satisfied fully in this approximate method. A relaxed contact condition between the foundation and soil is also assumed in this approximation.

2.3 Foundation Input Motions

Following Luco(1986), the foundation input motion vector $\{U_0{}^*\}$ can be written by,

$$\{U_0^*\}=[K]^{-1}\int_s \left([\Gamma(x)]^{\mathrm{T}}\{U_g\}-[\alpha(x)]^{\mathrm{T}}\{T_g\}\right)\mathrm{d}S \tag{11}$$

Where $[\Gamma(x)]$ is the contact traction matrix between the foundation and soil when the foundation is subjected to unit motions. $\{Ug\}$ and $\{Tg\}$ are the displacement and traction vectors of the free field motions (in absence of cavity for the foundation), respectively. $[\alpha(x)]$ represents a rigid-body motion influence matrix. S denotes the interface between the foundation and soil. In this study, each term of $[\Gamma(x)]$ can be calculated without difficulty by using impedance functions for the base mat and for the side wall. Unfortunately, it is not simple to evaluate the free field motions in the case of the foundation with backfilled soil. Since it is beyond the scope of this approximate method to obtain the free field motions exactly in this case, it is assumed that the outer soil of the surface layer has the same physical properties as those of the underlying half space and that the existence of the backfilled soil is neglected when the free field motions are computed. The second assumption can make it possible to calculate the free field motions. It should be mentioned that only the bottom of the foundation can contribute to the evaluation of the second term of the right hand in Eq. 11. when a relax contact condition is employed and the foundation is subjected to vertically incident S waves.

3 NUMERICAL RESULTS AND DISCUSSIONS

3.1 Impedance Functions for Side Wall

Numerical results are presented to clarify dynamic characteristics of the side wall of an embedded foundation surrounded with backfilled soil. The impedance functions for the side wall of unit depth are shown in Figs. 3 and 4 against the non-dimensional frequency $a_0(=\omega r_0/V_{s2})$, in which ω is the circular frequency and V_{s2} is the shear wave velocity of the outer soil. These results are normalized by the shear modulus $\mu 2$ of the outer soil for the horizontal component, or by the product $\mu 2 r_0{}^2$ for the rocking component. The results for a homogeneous layer (in the case of $r_0=0$, or properties of the backfilled soil being the same as those of the outer soil) are also depicted in the figures for comparison. Poisson's ratios and material damping ratios of the backfilled and outer soil are set to be 1/3 and 0.03, respectively. Figure 3 shows the impedance functions for the various width values of backfilled soil when the shear wave velocity ratio is fixed, and figure 4 shows those for the various shear wave velocity ratios when the width of backfilled soil is fixed. From these figures, it is clear that in the case of no backfilled soil, the real part of the impedance functions shows almost constant values for the non-dimensional frequency and imaginary part shows almost constant increasing values for the frequency. These phenomena suggest that the impedance functions may be replaced by the constant stiffness and damping that do not depend on the frequency. On the other hand, if the backfilled soil is considered in the analyses, it is observed that the impedance functions depend strongly on the non-dimensional frequency caused by reflected waves from the interface between the backfilled soil and the outer soil. In general, the imaginary part of the impedance functions decreases when the shear wave velocity ratio becomes small.

3.2 Impedance Functions for Embedded Foundation

Impedance functions for an embedded rigid foundation can be evaluated by combining the obtained side wall impedance functions and impedance functions for the foundation base mat independently obtained by the boundary element method (Sato (1983)). In order to study the validity of the present approximate method of analysis, comparisons are made with the results of a more rigorous method. An embedded rigid foundation without backfilled soil is chosen for comparison, first.

Figure 5 shows impedance functions for a rigid foundation embedded to the depth of $H_0=0.59r_0$. The material damping constant and Poisson's ratio are set to be 0.03 and 1/3, respectively. The impedance values are shown against non-dimensional frequency and are normalized by the shear modulus of the half space and

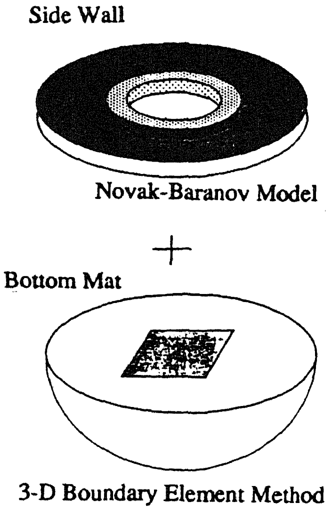
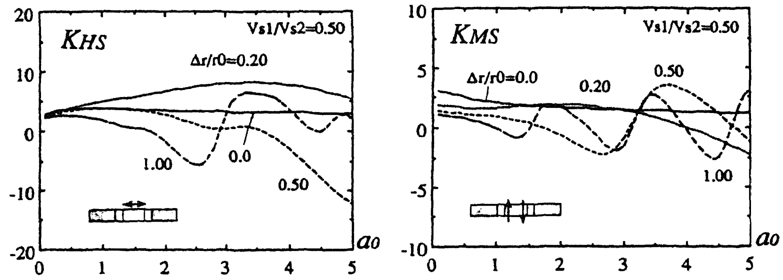
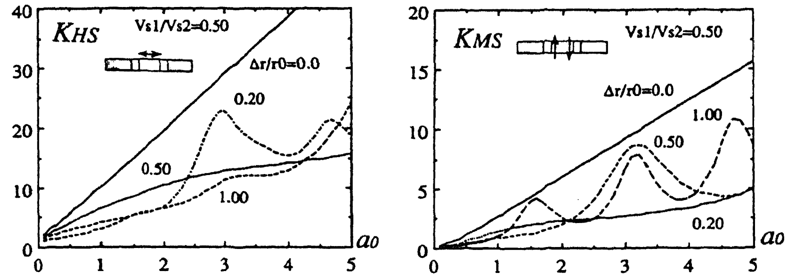


Fig.1 Coneptional Figure



(a) Real Part of Horizontal Component (c) Real Part of Rocking Component



(b) Imaginary Part of Horizontal Component (d) Imaginary Part of Rocking Component

Fig.3 Impedance Functions for Side Wall

($\rho 1=\rho 2$, $h1=h2=0.03$, $\nu 1=\nu 2=1/3$)

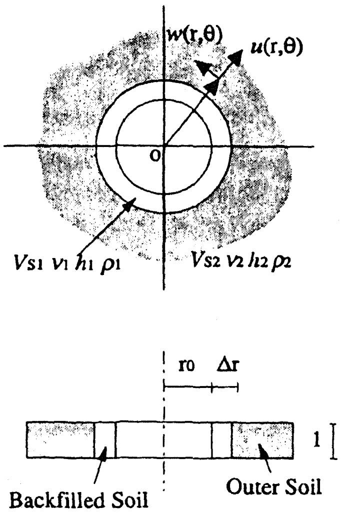
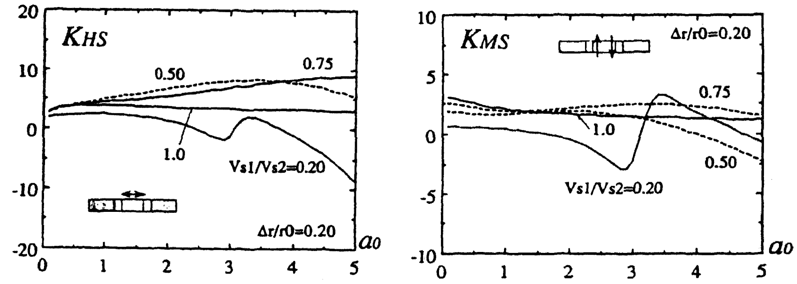
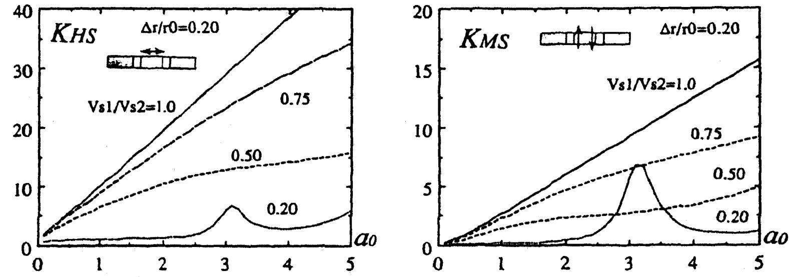


Fig.2 Notation of System



(a) Real Part of Horizontal Component (c) Real Part of Rocking Component



(b) Imaginary Part of Horizontal Component (d) Imaginary Part of Rocking Component

Fig.4 Impedance Functions for Side Wall

($\rho 1=\rho 2$, $h1=h2=0.03$, $\nu 1=\nu 2=1/3$)

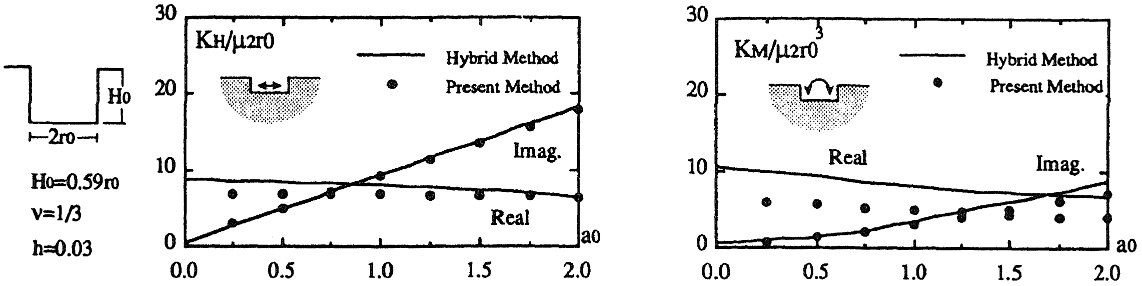


Fig.5 Impedance Functions for Embedded Foundation Without Backfilled Soil

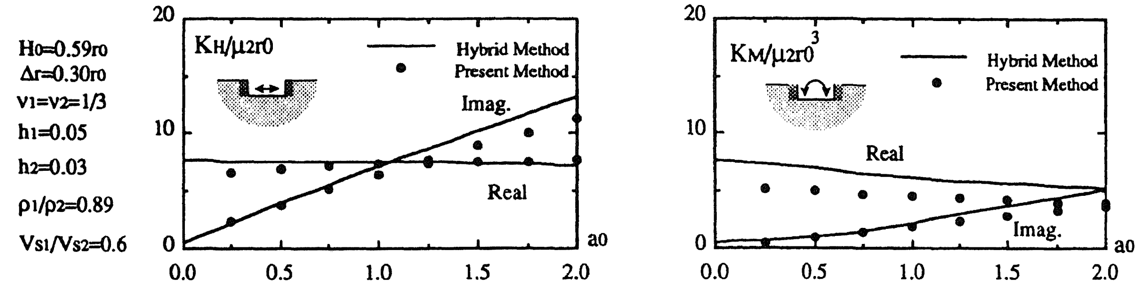


Fig.6 Impedance Functions for Embedded Foundation With Backfilled Soil

the radius of the foundation. Results obtained by Takahashi et al. (1992) using a hybrid method that combines finite element and boundary element methods are also plotted in the figure. Since the results of the hybrid method are obtained for a square foundation, the impedance values are converted to those for an equivalent cylindrical foundation. It is obvious from this figure that the present approximate method may give reasonable values, especially for the horizontal impedance. Some discrepancies in the rocking impedance may come from the fact that in the approximate method, some compatibility conditions are violated and a relaxed contact condition between the foundation and the surrounding soil is employed while a welded contact condition is assumed in the hybrid method. It is expected that the discrepancies may decrease when the embedment becomes smaller.

In Fig. 6, normalized impedance functions are plotted when the embedded foundation is surrounded by backfilled soil. The width of the backfilled soil is $0.3r_0$, the shear velocity ratio to that of the half space is set to be 0.6, and the material damping of the backfilled soil is 0.05. Results computed for the similar model by Takahashi et al (1992) are also depicted in the figure for comparison. It is concluded that the horizontal impedance functions of the present approximate method agree well with those of the more rigorous hybrid method. It is also clear from this figure that the differences in the rocking component are reduced by the introduction of backfilled soil.

3.3 Foundation Input Motions

In this section, numerical calculations and discussions are extended to the case of foundation input motions. Input motions for an embedded rigid foundation are computed when the foundation is subjected to vertically incident S waves.

Figure 7 shows foundation input motions for cylindrical foundations of various embedment ratios (H_0/r_0). It is assumed in this figure that the foundation is embedded in a half space and that there is no backfilled soil around the foundation. Poisson's ratio and the material damping of the half space are also 1/3 and 0.03, respectively. The values are normalized by the displacement at the free surface and are multiplied by the radius of the foundation in the rocking case. Results by Mita (1986) are also indicated in the figure. Mita calculated the foundation input motions for cylindrical foundations by means of a hybrid method which combines finite element and boundary integration equation methods. From this figure, it is obvious that the results of the present approximate method coincide fairly with those of Mita, independent of the embedment ratios. Some differences can be seen in the rocking component when the embedment ratio becomes greater. This tendency may correspond to the fact that the present approximate method is apt to give the smaller rocking impedance values when the foundation is embedded deeply.

Figure 8 indicates input motions for a foundation without backfilled soil and a foundation with backfilled soil. The parameters used in this analysis are the same as those used in the calculation of impedance functions (Fig.6). Results by Takahashi et al. (1992) are also depicted in the figure. As seen with the impedance functions, it is recognized that the introduction of backfilled soil makes the input motions better in spite of the approximation used in the calculations of the free field motions.

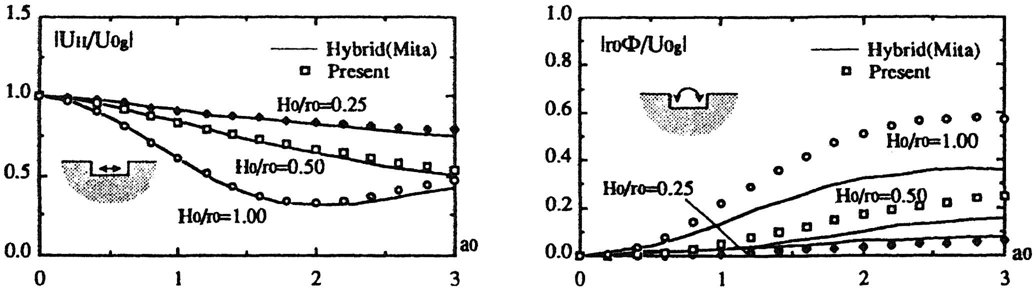


Fig. 7 Foundation Input Motions For Embedded Foundation

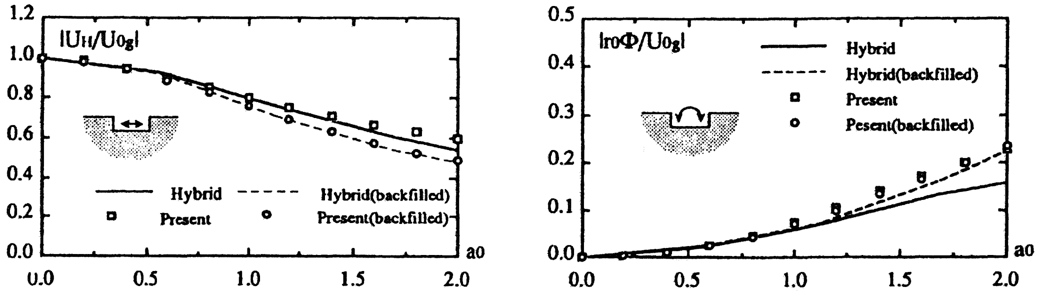


Fig.8 Foundation Input Motions For Embedded Foundation

4 CONCLUSIONS

In this paper, an approximate method of analysis that considers backfilled soil effects when calculating both impedance functions and input motions for an embedded cylindrical foundation has been presented. The approximate method is based on the assumption that the soil underlying the foundation base is an elastic half space and that the overlying soil consists of independent elastic layers. It may be confirmed that the employed approximate method may give fairly good results in comparisons to those of more rigorous hybrid methods. It should be noted, however, that the problem for the case of rather deeply embedded foundations and the problem of non-vertically incident waves are to be studied in the future.

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