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Dynamic uplift analysis of liquid storage tanks

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ABSTRACT: A substructure method is developed for dynamic uplift analysis of liquid storage tanks with deformable base plate. The tank shell is discretized into a number of ring-shaped shell elements and the base plate is modelled by plate elements coupled with plane stress elements. The base ring of the shell, i. e. the perimeter of the base plate is assumed keeping in a plane all along during vibration. The interaction of the base plate with the foundation is taken into consideration by proper choice of impedance coefficients of the foundation soils. Numerical results indicate that the lift-off region of the base plate is crescent in shape which agrees well with the experimental results. Some useful conclusions can be drawn from the calculated examples.

1. INTRODUCTION

Liquid storage tanks are widely used in petrochemical industry and in nuclear power plants. During past strong earthquakes occured in Japan, USA, China etc. a great number of such tanks suffered severe damages. Through field seismic observation and laboratory experiments it was realized that uplifting of the base plate serves the main cause that leads to the damage of unanchored liquied storage tanks. However, in the literature, apart from very complicated modelling with a large number of finite elements (Barton 1987), most of the approaches were too simple to capture the fundamental behaviour of the response of the structure. The present paper aims to develop a more rational model to reflect the uplift mechanism of the tank subjected to earthquake loading.

In the analysis the followings are assumed.

1. The tank wall and the base plate are analyzed based on linear elastic theory. Further refinement of the approach is suggested to take into consideration geometric non-linearities by introducing corresponding finite elements.

2. The irrotational flow of non-viscous fluid is assumed.

3. The dynamic interaction effect of the base plate with the foundation is modelled by impedance coefficients of the underlying soils.

2. EQUATIONS OF MOTION OF THE SYSTEM

The equations of motion of the system is formulated by substructure method.

2.1 Equations of motion of the tank wall

The tank under investigation is shown in Figure 1. The shell is discretized into a number of ring-shaped strip elements. Take a typical element, the displacement vector at any point within it δ may be expressed by the displacement vector at its upper and lower nodal rings Δ.

$$\{\delta\} = [N]\{\Delta\} \tag{1}$$

where

$$\{\delta\} = [u\ v\ w]^T$$

$$\{\Delta\} = [u_1\ w_1\ \beta_1\ v_1\ u_2\ w_2\ \beta_2\ v_2]^T$$

u, v, w denote respectively the radial, tangential and vertical displacements at any point; while u_i, w_i, v_i, β_i denote respectively the displacements of the nodal ring in the x, z, y direction and the rotation about y-axis, subscript $i=1,2$ refer to the upper and lower rings respectively. The displacement of the ring along radial direction is idealized by cosmθ and sinmθ (m=1,2,…) distribution and N represent the shape functions. The coordinate system is choosed relative to the base ring of the shell.

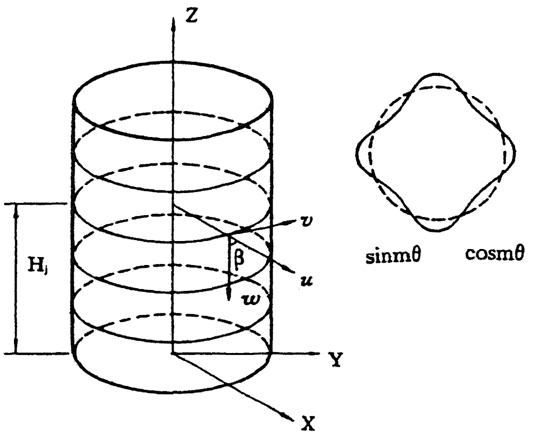

Fig. 1 Coordinate system of the tank shell

Applying Lagrange equations we get the equations of motion of the shell subjected to earthquake ground motion

$$[M_s]\{\ddot{\Delta}\} + [K_s]\{\Delta\} = \{F_p\} \qquad (2)$$

where M_s and K_s denote the mass and stiffness matrices; force vector F_p is determined in accordance with the dynamic pressure of the liquid in the tank. For simplifying derivation we omit hereafter tentatively the damping terms in the equation.

The liquid pressure during vibration is expressed as

$$P = \sum_m \sum_n \sum_k N^k_{mn} P^k_{mn}$$

or

$$P = [N_p]\{p\} \qquad (3)$$

where p is a vector of pressure coefficients P^k_{mn}. According to the weighted residual method, the equation of motion of the fluid is formulated as

$$([M_c] + [M_g])\{\ddot{p}\} + [H]\{p\} = \{F_f\} \quad (4)$$

in which M_c and M_g represent respectively the compressibility and surface wave effect of the fluid; vector F_f is closely related to the normal acceleration components of the tank during vibration.

Coupled fluid — structure equations of motion are obtained by combing equations (2) and (4).

$$\begin{bmatrix} M_s & 0 \\ \rho S^T & M_f \end{bmatrix} \begin{Bmatrix} \ddot{\Delta} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K_s & -S \\ 0 & H \end{bmatrix} \begin{Bmatrix} \Delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (5)$$

In the above equation, ρ is the mass density of the fluid; S is a transformation matrix.

$$\{F_p\} = [S]\{p\}$$

and

$$[M_f] = [M_c] + [M_g]$$

For computational convenience equation (5) may be transformed into a symmetrical ones through some treatments. If the compressibility and surface wave effects of the fluid may be neglected, then the lower part of equation (5) becomes

$$\{p\} = -\rho[H]^{-1}[S]^T\{\Delta\}$$

and the upper part of equation (5) is further simplified to

$$[M]\{\ddot{\Delta}\} + [K_s]\{\Delta\} = \{0\} \qquad (6)$$

in which

$$[M] = [M_s] + [M_p]$$
$$= [M_s] + \rho[S][H]^{-1}[S]^T$$

2. 2 Equations of motion of the base plate

By introducing plate elements coupled with plane stress elements the mass matrix and stiffness matrix of the base plate are easily obtained. At each nodal point i we have five displacement components

$$\{\bar{x}_i\} = [u_i\, w_i\, \theta_{yi}\, \theta_{xi}\, v_i]$$

where u, w, v, θ_y, θ_x refer respectively the nodal displacement components in the x, z, y direction and the rotation components about y and x axes. Assume earthquakes take place in the direction of x-axis, the less important degree of freedom v may be eliminated by static condensation. Finally, we have reduced equations of motion of the base plate

$$[M_b]\{\ddot{x}^t\} + [K_b]\{x\} = \{0\} \qquad (7)$$

where

$$\{x\} = [x_1\, x_2 \cdots\cdots]^T$$

x^t corresponds to the total displacement vector and x relative to the free field ground motion.

The dynamic interaction effect of the base plate with the foundation is taken into consideration by means of impedance coefficients which are considered as frequency independent and are choiced in conformity with the predominant frequency of the earthquake ground motion.

$$F_x = K_x Au = (K_{11} + iC_{11})Au$$
$$F_z = K_z Aw = (K_{22} + iC_{22})Aw$$
$$M_y = K_\theta A\theta_y = (K_\theta + iC_\theta)A\theta_y$$

By incorporating foundation impedance coefficients the equations of motion of the base plate become

$$[M_b]\{\ddot{x}^t\} + [K_f]\{x\} = \{0\} \qquad (8)$$

Owing to the restraint of the plate itself, the perimeter of the base plate, i. e. the base ring of the shell may be assumed keep undeformable all along during vibration. According to this, the displacement vector x_s of all points of the base plate lying on its periphery may be expressed in terms of the reference displacement vector x_0 of the base ring of shell

$$\{x_s\} = [T_s]\{x_0\} \qquad (9)$$

where

$$\{x_0\} = [u_0\, w_0\, \theta_{y0}\, \theta_{x0}]^T$$

Performing coordinate transformation (9) eventually we get the equations of motion of the base plate in the partitioned form

$$[M^*]\begin{Bmatrix}\ddot{x}_0\\ \ddot{x}_i\end{Bmatrix} + [K^*]\begin{Bmatrix}x_0\\ x_i\end{Bmatrix} = -[M^*][J]\{\ddot{u}_g\} \qquad (10)$$

where M^*, K^* are the transformed matrices of the base plate; x_i is the displacement vector of the inner points of the plate; u_g is the free-field ground displacement vector induced by the earthquakes; elements in each columm of matrix $[J]$ corresponding to displacement component of u_g are unity, other elements are zero

$$\{u_g\} = [u_{gx}\, u_{gz}]^T \qquad (11)$$

2. 3 Equations of motion of the system

Based on equations (6), (9), (10) and (11), the total displacement vector of the jth nodal ring of the tank shell with respect to a fixed axis when the structure is subjected to the earthquake ground motion may be expressed as

$$u_j^t = u_j + u_0 + u_{gx} + h_j\theta_{y0}$$
$$w_j^t = w_j + w_0 + u_{gz}$$
$$\beta_j^t = \beta_j + \theta_{y0}$$
$$v_j^t = v_j$$

or in matrix form

$$\{\Delta^t\} = \{\Delta\} + [A]\{x_0\} + [I_u]\{u_g\} \qquad (12)$$

where

$$[I_u] = \begin{bmatrix}1000 & 1000 & \cdots\\ 0100 & 0100 & \cdots\end{bmatrix}^T$$

$$[A] = [I_{a1}\, I_{a2} \cdots\cdots]^T$$

$$[I_{aj}] = \begin{bmatrix}1 & 0 & h_j & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

h_j is the height of the jth nodal ring above the base (Fig. 1).

According to equations (6) and (12) the equations of motion of the tank shell (not including base ring) are expressed in the form

$$[M]\{\ddot{\Delta}\} + [M][A]\{\ddot{x}_0\} + [K_s]\{\Delta\} = -[M][I_u]\{\ddot{u}_g\} \qquad (13)$$

Forces exerted on the base ring by the tank shell and the impounded liquid may be written as

$$\{F_I\} = [A]^T[M][\ddot{\Delta}] + [A]^T[M][A]\{\ddot{x}_0\} + [A]^T[M][I_u]\{\ddot{u}_g\} \tag{14}$$

According to equation (10) the equation of motion of the base plate including dynamic liquid presure are written in the form

$$\begin{bmatrix} M_{rr}+M_{ar} & 0 \\ 0 & M_{ii}+M_{ai} \end{bmatrix} \begin{Bmatrix} \ddot{x}_0 \\ \ddot{x}_i \end{Bmatrix} + \begin{bmatrix} K_{rr} & K_{ri} \\ K_{ir} & K_{ii} \end{bmatrix} \begin{Bmatrix} x_0 \\ x_i \end{Bmatrix}$$

$$= - \begin{bmatrix} M_{rr}+M_{ar} & 0 \\ 0 & M_{ii}+M_{ai} \end{bmatrix} [J]\{\ddot{u}_g\} \tag{15}$$

where M_{ar} and M_{ai} are the added mass matrices of the fluid.

Assembling equations (13) to (15) and including damping terms yield the equations of motion of the system

$$[M_t]\{\ddot{\Delta}_t\} + [C_t]\{\dot{\Delta}_t\} + [K_t]\{\Delta_t\} == \{F_t\} \tag{16}$$

where

$$[M_t] = \begin{bmatrix} M & MA & 0 \\ A^TM & A^TMA+M_{rr}+M_{ar} & 0 \\ 0 & 0 & M_{ii}+M_{ai} \end{bmatrix}$$

$$[K_t] = \begin{bmatrix} K_s & K_{sr} & 0 \\ K_{rs} & K_{ss}+K_{rr} & K_{ri} \\ 0 & K_{ir} & K_{ii} \end{bmatrix}$$

$$\{\Delta_t\} = [\Delta^T \; x_0^T \; x_i^T]^T$$

$$\{F_t\}_1 = [F_1 \; F_2 \; F_3]^T\{u_g\}$$

$$[F_1] = [M][I_u]$$

$$[F_2] = [A]^T[M][I_u] + [M_{rr}+M_{ar}][J_u]$$

$$[F_3] = [M_{ii}+M_{ai}][J_d]$$

Equation (16) is solved by step by step numerical time integration technique. During the process, once any nodal point of the base plate is lifting off from the foundation the impedance coefficients at that point turn to zero, the stiffness matrix of the base plate is adjusted correspondingly and vice versa. The computational efforts may be reduced if the adjusted terms of the stiffness matrix is moved to the right hand side as a fictitious load.

Further refinement of the approach may be achieved by introducing geometriaclly nonlinear quasi-conforming shell elements (Guan 1991) to construct the stiffness matrices of the structure.

3. NUMERICAL RESULTS AND DISCUSSION

To evaluate the effectiveness of this analysis procedure, results of analysis of a free-base steel cylindrical tank impounding water subjected to sinusoidal earthquake excitation are presented. The parameters used in the calculation are as follows: height of the tank 354cm; radius 58cm; thickness of the wall 0.08cm; thichness of the base plate 0.24cm; depth of water 283cm. Material properties of steel are: $E=2.1\times10^5 MPa$; $\nu=0.3$; $\rho=8000kg/m^3$.

The exciting frequency is 10 rad/sec with maximum acceleration equal to 250cm/sec. A total of 800 time steps are calculated with step length equal to 0.02sec.

The tank shell is discretized into 10 ring-shaped elements and the base plate 28 triangular elements with 22 nodal points.

The impedance coefficient of the foundation in the z direction K_z is selected as $49N/cm^3$, and the coefficients in the x direction and rotation are calculated by approximate formulas in terms of K_z.

The time history of uplift of point No. 1 (intersection of x-axis with periphery of the base plate) is shown in Fig. 2. positive value means that point No. 1 is lifting off from the foundation.

Fig. 3 demonstrates the uplifting process of the base plate. Black dots indicate the position of points currently lifting off of the foundation. It is clear that the calculated uplifting region of the base plate is crescent in shape which agrees well with the experimental results (Clough 1979).

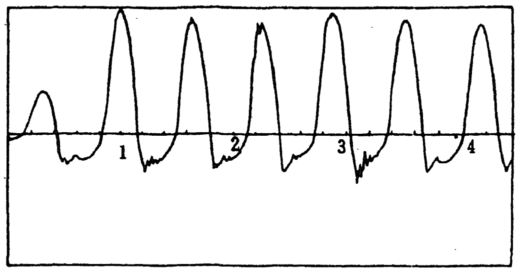

Fig. 2 Time history of uplifting of point No. 1

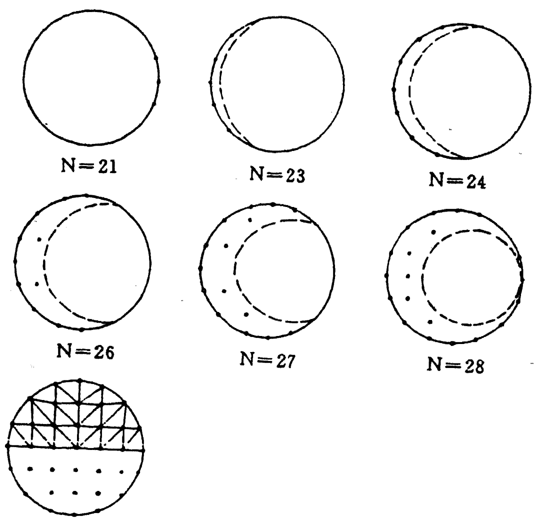

Fig. 3 Uplifting of base plate

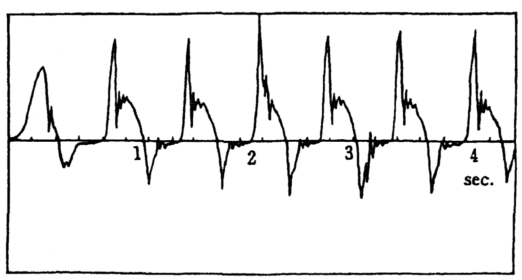

Fig. 4 Moments developed at the base plate, uplifting included (Max. value 356 N·cm/cm)

Fig. 4 and Fig. 5 compare the bending moments *My* developed at the base plate with uplifting effects included or neglected. It can be seen that uplifting remarkably increases the radial stresses of the base plate.

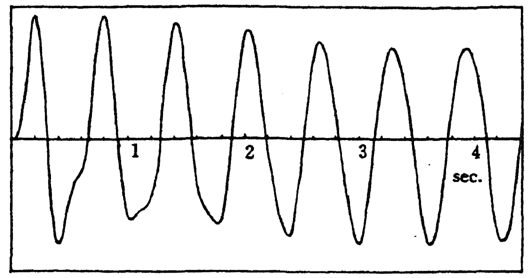

Fig. 5 Moments My developed at the base plate, uplifting neglected (Max. value 56 N·cm/cm)

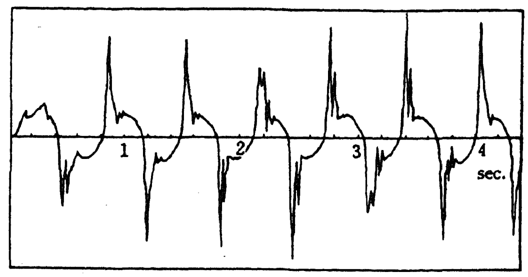

Fig. 6 Axial force N_z developed at the tank wall, uplifting included (Max. value 149 N·cm/cm)

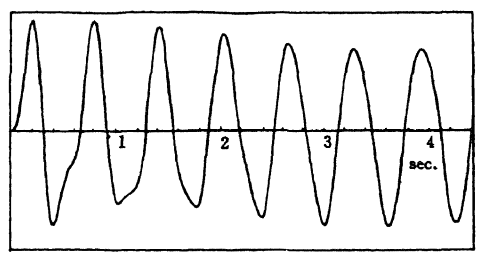

Fig. 7 Axial force N_z developed at the tank wall uplifting neglected (Max. value 54 N·cm/cm)

Fig. 6 and Fig. 7 compare the axial force N_z developed at the tank wall with uplifting effects included or neglected. Uplifting also tends to increase the axial stresses of the tank wall.

Fig. 8 shows bending moment at the tank wall including uplift effect, the same conclusion can be drawn when compared with the results obtained by neglecting uplifting.

In order to study the influence of the stiffness of underlying foundation soil on the dynamic response of

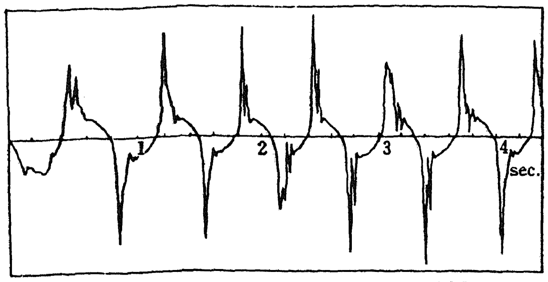

Fig. 8 Bending moment M developed at the tank wall, uplifting included (Max. value 149 N·cm/cm)

Fig. 9 Axial force N_z developed at the tank wall (Max. value 571 N·cm/cm)

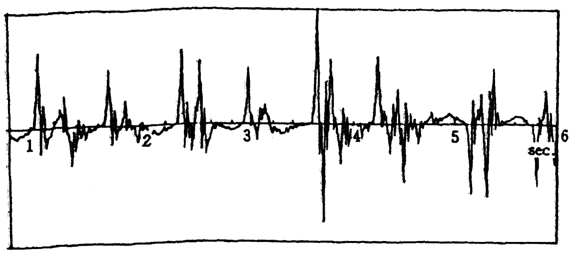

Fig. 10 Bending moment M developed at the tank wall (Max. value 0.13 N·cm/cm)

tank wall, numerical calculations of another example with stiffness of the soil enlarged to ten times that of the previous example were performed. Other parameters remain unchanged. The axial force and moment response of the tank wall are given in Fig. 9 and Fig. 10 Comparing with Fig. 6 and Fig. 8 it was found that the maximum compressive stresses developed at the tank wall during vibration increase to more than three times what it was in the previous example. Moreover, higher frequency components become more and more abundant. Hence, place the tank on softer ground is preferable.

4. CONSLUSIONS

A more rational approach for dynamic uplift analysis of liquid storage tanks with deformable base plate is presented. The calculated uplifting region of the base plate is crescent in shape which agrees well with the experimental results.

From numerical results the following conclusions are obtained.

1. The period of vibration of the tank increases a little bit during the process of uplifting.

2. When uplifting occurs, the higher frequency components of the tank response become more and more abundant. The stiffer the underlying foundation soil, the richer the higher frequency components.

3. Uplifting induces greater radial tensile stresses at the base plate on the uplifting side, which governs the stress condition of the base plate.

4. The influences of foundation soil rigidity on the axial compressive stresses response of the tank wall were very significant during uplifting. Place the tank on softer ground is preferable.

Further refinement of the approach by introducing geometrical nonlinear finite elements is suggested.

REFERENCES

Barton, D. C. & Parker, J. V. 1987. Finite element-analysis of the seismic reponse of anchored and unahchored liquid storage tanks. J. Earthquake Eng. and Structural Dynamics. 15:299-322.

Clough, R. W. Niwa A. & Clough D. P. 1979. Experimental seismic study of cylindrical tanks. J. Struct. Eng. ASCE 105-12: 2565-2590.

Guan Y. P. 1991. Research on multi-variable quasi-conforming degenerated shell finite elements. Ph. D Dissertation. Dalian University of Technology. Dalian. China.