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Theoretical assessment of soil-structure interaction effect at bridge structure

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ABSTRACT: In order to develop a simple practical means for assessing the effects of dynamic soil-structure interaction, especially on bridge structures, this paper describes a theory of single-degree-of-freedom system which is equivalent to the single-support bridge model and the multiple-support bridge model where each foundation is modeled as a rigid embedded foundation with a mass and a mass moment of inertia, having two-degree-of-freedom with swaying and rocking motions. To illustrate characteristics and applications in the seismic design of the theory described in this paper, the natural period and the damping ratio of a three-span-continuous steel girder bridge with different foundation size are evaluated.

1 INTRODUCTION

In the tentative seismic design provisions for buildings recommended by the Applied Technology Council in USA (ATC-3,1978), the dynamic soil-structure interaction effects are expressed by an increase in the fixed-base natural period of the buildings, and by a change (generally an increase) in the associated damping ratio, on the basis of primarily the analysis of a simple linear structure of mass and height which is supported through a massless foundation at the surface of a homogeneous elastic half space. This simple structural model may be adequate for buildings since it may be viewed either as the direct model of a single-story building frame or , more generally as the model of a multistory, a multimode building. However, this simple model can not cover the bridge model because the bridge has usually various types of substructure and support conditions with different stiffness of piers and foundations such as a multiple-support bridge, and in some cases has huge foundations deeply embedded into soil. Therefore, for bridges, there is still a need to improve our understandings of the dynamic soil-structure interaction effects at the abutments, the piers, the foundations and the superstructures.

This paper describes a theory of single degree of freedom system which is equivalent to the single-support bridge model and the multiple-support bridge model where each foundation is modeled as a rigid embedded foundation with a mass and a mass moment of inertia, having two-degree-of-freedom with swaying and rocking motions. To illustrate characteristics and applications in the seismic design of the theory described in this paper, the natural period and the damping ratio of a three-span-continuous steel girder bridge with different foundation size are evaluated.

2 EQUIVALENT SINGLE DEGREE OF FREEDOM SYSTEM FOR SINGLE SUPPORT BRIDGE

A three-degree-of-freedom lumped mass-spring model as shown in Fig.1 is examined. The structure is modeled with a mass m_S. The vertical column is massless and inextensible in vertical direction, and the dynamic stiffness of the column is represented by a complex spring K_s^* ($= k_s + i\omega c_s$) with a spring coefficient k_s, a damping coefficient c_s, and frequency ω. The foudation is embedded into soil ,and modeled with a mass M and a mass moment of inertia J_G with respect to centroid. The foundation thus has two-degree-of-freedom with swaying and rocking motions. The effect of soil on the rigid embedded foundation response is modeled with the soil's dynamic stiffness and the effective input motions, on the basis of the sub-structure method where the total soil-structure interaction is given by the sum of the inertial interaction and the kinematic interaction

(Kausel, et al. 1978, Harada, et al. 1981). The soil's dynamic stiffnesses with respect to the top of the foundation are represented by the swaying component K^*_{hh}, the rocking component K^*_{rr}, and the coupled component $K^*_{hr} = K^*_{rh}$. The effective input motions with respect to the top of the foundation are also represented by the swaying component u_{CT} and the rocking component θ_{CT}.

Referring the notations of motion shown in Fig.1, the equations of motions can be obtained in frequency domain such that

$$\begin{bmatrix} -m_S\omega^2 + K^*_S(\omega) & -m_S\omega^2 & -m_S L\omega^2 \\ -m_S\omega^2 & -(m_S + M)\omega^2 + K^*_{hh}(\omega) & -(m_S L - M L_f)\omega^2 + K^*_{hr}(\omega) \\ -m_S L\omega^2 & -(m_S L - M L_f)\omega^2 + K^*_{rh}(\omega) & -(m_S L^2 + J_G + M L_f^2)\omega^2 + K^*_{rr}(\omega) \end{bmatrix} \begin{Bmatrix} u \\ u_T \\ \theta \end{Bmatrix}$$
$$= \begin{Bmatrix} m_S\omega^2 \\ (m_S + M)\omega^2 \\ (m_S L - M L_f)\omega^2 \end{Bmatrix} u_{CT} + \begin{Bmatrix} m_S L\omega^2 \\ (m_S L - M L_f)\omega^2 \\ (m_S L^2 + J_G + M L_f^2)\omega^2 \end{Bmatrix} \theta_{CT} \quad (1)$$

It is, now, quite advantageous in assessing the dynamic soil-foundation effects on structural responses to rewrite Eq.(1) by eliminating the foundation motions, u_T and θ, from Eq.(1), such that

$$[-m_S\omega^2 + K^*_e]U_e = m_S\omega^2 U_{ge} \quad (2)$$

where

$$K^*_e = \frac{K^*_S A}{A + K^*_S B}, \; U_{ge} = \frac{C}{A}u_{CT} + \frac{D}{A}L\theta_{CT} \quad (3-a)$$

$$U_e = \frac{K^*_S}{K^*_e}u \quad (3-b)$$

In Eq.(3), A, B, C, D are given by,

$$\begin{aligned} A &= M J_G\omega^4 - [J_G K^*_{hh} + M L_f(L_f K^*_{hh} + 2K^*_{hr}) \\ &+ M K^*_{rr}]\omega^2 + (K^*_{hh}K^*_{rr} - K^{*2}_{hr}) \end{aligned} \quad (4-a)$$

$$\begin{aligned} B &= -[J_G + M(L + L_f)^2]\omega^2 \\ &+ (K^*_{rr} + K^*_{hh}L^2 - 2K^*_{hr}L) \end{aligned} \quad (4-b)$$

$$\begin{aligned} C &= -[J_G K^*_{hh} + M(L + L_f)(L_f K^*_{hh} + K^*_{hr})]\omega^2 \\ &+ (K^*_{hh}K^*_{rr} - K^{*2}_{hr}) \end{aligned} \quad (4-c)$$

$$\begin{aligned} D &= -\Big[\frac{J_G K^*_{hr}}{L} + \frac{M(L + L_f)}{L}(L_f K^*_{hr} + K^*_{rr})\Big]\omega^2 \\ &+ (K^*_{hh}K^*_{rr} - K^{*2}_{hr}) \end{aligned} \quad (4-d)$$

The quantities, A, B, C, and D are only functions of soil-foundation properties and column's height, but do not contain the structural mass m_S and the column's dynamic stiffness K^*_S. Equation (2) thus can be interpreted as a equation of motion of a single-degree-of-freedom lumped mass-spring model as shown in Fig.2(c). All the effects of dynamic soil-foundation interaction on structural responses are included in the equivalent complex spring K^*_e and the equivalent input motion displacement U_{ge} defined by Eqs.(3-a). Equation (2) can also be derived by applying the substructure method at the interface between the structural mass and the column.

From Eq.(2), the natural period T and the damping ratio h of the structure, can be obtained such as

$$T = 2\pi\sqrt{\frac{m_S}{Re(K^*_e)}}, \quad h = \frac{Im(K^*_e)}{2\omega\sqrt{m_S Re(K^*_e)}} \quad (5)$$

where $Re(x)$ and $Im(x)$ represent the real and imaginary part of x, respectively. The complex spring K^*_e in Eq.5 has to be evaluated for the required natural frequency of the equivalent system shown in Fig.2(c). As this natural frequency is unknown at first, it has to be determined by iteration until the period given by Eq.(5) is satisfied. The damping ratio can then be evaluated at the natural frequency. From experience, satisfactory accuracy can be obtained in two or three iterations.

The examination of Eqs.(3-a), and(4), in conjunction with the numerical studies, can yield the approximate formulae for T and h given by Eq.(5), and $|C/A|$ and $|D/A|$ of input motion U_{ge} given by Eq.(3-a)(Harada, et al. 1991).

3 EQUIVALENT SINGLE DEGREE OF FREEDOM SYSTEM OF MULTIPLE SUPPORT BRIDGE

The theory descibed in Section 2 can be extended to obtain an equivalent single-degree-of-freedom system of a multiple-support bridge as shown in Fig.3, by employing the principle of virtual displacement method; (for example,Clough and Penzien, 1975). The procedure is as follows:

Step 1. For a specified frequency, calculate the complex spring K^*_{ej} of the jth support by using Eq.3 , and then construct a structural model with fixed base condition as shown in Fig.4 where the bridge superstructure is modeled as a distributed weight continuum $w(x)$ supported by the complex spring K^*_{ej}.

Step 2. Apply the distributed weight $w(x)$ in transverse direction , and then evaluate the shape function $\psi(x)$. Similar procedure is valid for the longitudinal direction.

Step 3. Apply the motion displacement U_{gej} through the complex springK^*_{ej} at the jth support to the massless (weightless) bridge ($w(x) = 0$), and then evaluate the response displacement of massless bridge $U_k(x)$.

Step 4. Calculate the generalized mass m^*, the generalized spring and damping coefficients k^*, c^*, and the generalized input motion dispacement U^*_g;

$$m^* = \frac{1}{g}\int w(x)\psi^2(x)dx \quad , \; k^* = \int w(x)\psi(x)dx \qquad (6-a)$$

$$c^* = \frac{1}{\omega}\sum_j Im(K^*_{ej})\psi^2(x_j) \; , \; U^*_g = \frac{\int w(x)\psi(x)U_k(x)dx}{\int w(x)\psi^2(x)dx} \qquad (6-b)$$

Step 5. Calculate the natural period and the damping ratio;

$$T = 2\pi\sqrt{\frac{m^*}{k^*}}, \qquad h = \frac{c^*}{2\sqrt{m^*k^*}} \qquad (7)$$

Similarly to the natural period and the damping ratio in Eq.(5), the natural period and the damping ratio given by Eq.(7) has to be determined by iteration until Eq.(7) is satisfied because the complex spring K^*_{ej} is a function of frequency. Finally, the frequency equation of motion of the equivalent system of the multiple-support bridge is obtained such that

$$[-m^*\omega^2 + k^*]U^* = m^*\omega^2U^*_g \qquad (8)$$

4 NUMERICAL EXAMPLES

The transverse vibration characteristics are presented in this numerical examples of a highway bridge model of three-span-continuous steel girder bridge with an equal span length of 50 m (Fig.3). The properties of the bridge model are as follows;
$w(x) = 13t/m$; $EI(x) = 95tm^2$
$k_{s1} = k_{s4} = 8.9\times10^5t/m$; $k_{s2} = k_{s3} = 5.4\times10^4t/m$
$L = 15m$; $H_s = 30m$; $a = 2, 6, 10m$
$V_{ss} = 100m/s$; $w_s = 1.5t/m^3$; $\nu_s = 0.45$; $D_s = 0$
$V_s = 500m/s$; $w = 1.8t/m^3$; $\nu = 0.3$; $D = 0.05$
where V_{ss}, w_s, ν_s, and D_s are S-wave speed, soil weight per unit volume, Poisson ratio, and soil damping ratio for surface layer, respectively. The soil properties for base layer are represented by V, w, ν, and D. The complex spring of the column is defined such as $K^*_{sj} = k_{sj}(1 + i2SD)$ with structural damping ratio $SD = 0.02$. The soil's dynamics stiffnesses are evaluated by the approximate formulae (Harada,et al. 1981). In order to illustrate the effects of the foundation size on the bridge response characteristics, the radius of the inner two foundations are changed as a =2 m,6m,10m.

The frequency variation of the equivalent spring and damping coefficients k_{ej} (= $Re(K^*_{ej})$) and c_{ej}(= $Im(K^*_{ej}/\omega)$) of the inner support j=2 or 3 with a=6m are shown in Fig.5. Fig.5(a) indicates that k_{ej} takes almost constant value at $\omega = 0$ up to the first natural frequency of the jth foundation ω_{fj}, while it converges to the value of the jth pier spring coefficient with fixed-base condition above ω_{fj}. It is observed from Fig.5(b) that c_{ej} has a single peak at the frequency nearly equal to ω_{fj}. Although the frequency range shown in Fig.5 does not cover the second natural frequency of the jth foundation, the observations from Fig.5 are still valid for the high frquency region (Harada,et al. 1991).

The period T and damping ratio h of the three-span bridge model are shown in Fig.6 with the shape function $\psi(x)$. The shape function in fixed-base condition is indicated by dashed curve. The foundation size clearly affects T and h , resulting in an increase in the fixed-base natural period of bridge as the foundation size decreases. The damping ratio has a maximum at the foundation size a = 6m where the natural frequency of the bridge coincides approximately with ω_{fj} (j=2 or 3). This observation on damping ratio is important because it is usually recognized that the radiation damping increases as the foundation size increases.

5 CONCLUSION

The theory and the concepts described in this paper provide the simple practical means for assessing the effects of dynamic soil-structure interaction in the seismic design of structure, especially of bridges or multiple-support lifeline structures. The theory of the equivalent system presented in this paper is reduced to that adopted in ATC-3 for buildings if the mass and the embedment of rigid foundation are neglected in the single-support bridge model.

6 REFFERENCE

Applied Technology Council,1978.Tentative provisions for the development of seismic regulation for buildings. National Bureau of Standards Special Publication, 510.

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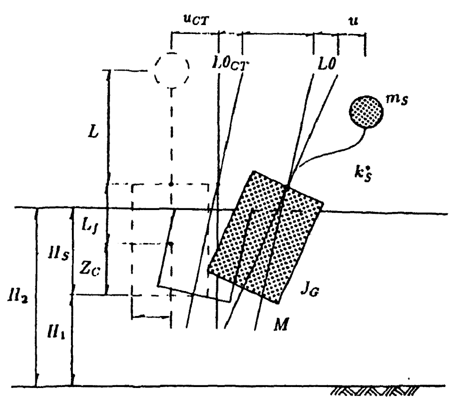


Fig.1 A three-degree-of-freedom lumped mass-spring model and its notations

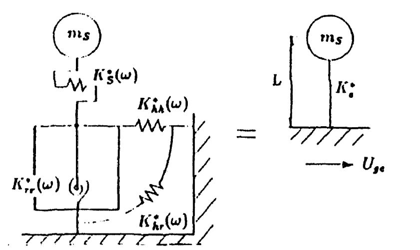


Fig.2 An equivalent single degree of freedom lumped mass-spring model

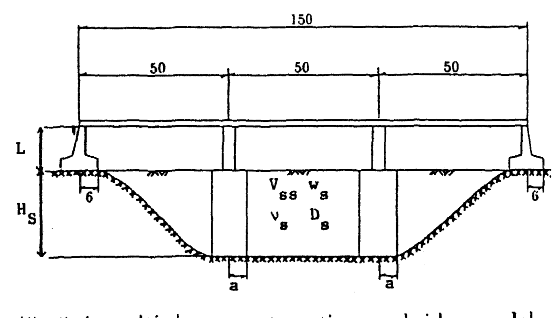


Fig.3 A multiple support continuous bridge model

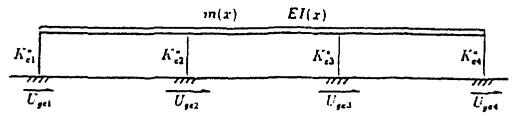
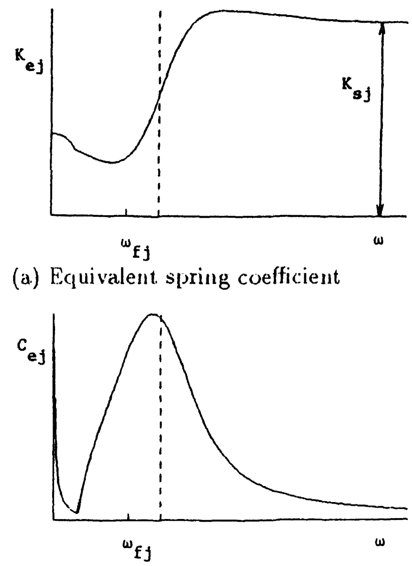


Fig.4 A multiple support continuous bridge model with fixed base conditions



(a) Equivalent spring coefficient

(b) Equivalent damping coefficient

Fig.5 Frequency variations of equivalent spring and damping coefficients of inner support (j=2 or 3)

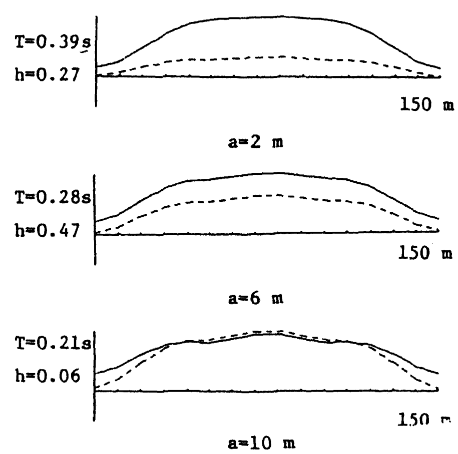


Fig.6 The periods, damping ratios, and shape functions of the three span bridge models with three foundation sizes (a= 2m,6m, and 10m). The dashed curves are for the shape functions in fixed-base conditions.