Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Importance of soil damping idealization in soil-structure interaction computations

R.Zinn
Stangenberg, Schnellenbach und Partner, Bochum, Germany

F.Stangenberg
Ruhr-University Bochum, Germany

ABSTRACT: The importance of properly accounting for the soil damping in earthquake response computations of buildings is discussed. The often used and well-known formulae for the frequency-independent radiation damping coefficients are only valid in case of a homogeneous halfspace. For layered soil sites, however, the soil damping can be dramatically reduced depending on the specific layering. E.g. for one soil layer over bedrock there is nearly no radiation damping below the "cut-off-frequency", i.e. the first frequency of the layer. In the paper it is demonstrated that even for slightly disturbed homogeneous soils by a stiff layer, the radiation damping for certain frequency ranges is much smaller than for the undisturbed homogeneous halfspace.

1 INTRODUCTION

In modal analysis computations, the commonly used equivalent frequency-independent radiation damping coefficients often lead to a wrong prediction of building responses, especially for vibration modes of rigid body translation, where high radiation damping exists. Frequency domain computations using realistic dynamic-stiffness coefficients of the layered soil and the actual foundation geometry are in principle well-suited to properly account for the soil damping effects, see Wong & Luco (1990). But they have certain disadvantages: nonlinear effects are difficult to be introduced, response spectrum methods are not possible and most structural engineers are not familiar with complex response computations and prefer modal analysis methods.

In the paper, stiffness and damping coefficients are presented for different layered soil conditions. In the case of an elastic homogeneous halfspace, simple expressions for the damping coefficients can be utilized in time-domain computations which are based on appropriate functions for the frequency-dependent damping coefficients. In case of a soil stratum on bedrock, the use of these damping coefficients can extremely underestimate the seismic response of buildings, because building frequencies below the so-called "cut-off-frequency" of the layer are very low damped and lead to high earthquake responses. This phenomenon has been discussed by Kausel & Roesset (1975), Kausel et al. (1978), Gazetas(1983), Liou (1989) and Meek & Wolf (1991). For other layered soil conditions, there are very few results published in the literature. Wong & Luco (1985) published results for a square foundation resting on a uniform layer overlying a uniform halfspace. Their results will be reviewed and compared with results of the authors.

As further examples, soil damping coefficients are presented for slightly disturbed homogeneous soils: a square foundation of length 2B is assumed to rest on a homogeneous soil, where a very hard layer of thickness 0.5B is situated at depths 2B, 3B and 4B. The wave reflection effects due to this layer reduce the radiation damping in certain frequency ranges, and the (halfspace) damping coefficients would lead to too low seismic responses.

2 DYNAMIC-STIFFNESS COEFFICIENTS OF ARBITRARILY SHAPED FOUNDATIONS

The equilibrium equations for harmonic motions of a rigid massless foundation with circular frequency ω is conveniently formulated as

$$(K_{ij} (\omega)) \; u_0 (\omega) = P(\omega) \qquad (1)$$

With the 6x6 impedance matrix $(K_{ij}(\omega))$ and the 6-component vectors $u_0(\omega)$ of the displacements and

$P(\omega)$ of the applied load amplitude, respectively. The impedance matrix ($K_{ij}(\omega)$) contains the frequency-dependent impedance functions $K(\omega)$ of the six rigid-body degrees of freedom, which can be written in the form

$$K(\omega) = K^0 \ (k(a_0) + i\ a_0\ c(a_0)) \qquad (2)$$

where $a_0 = \omega\ B/\beta$ is the dimensionless frequency with B = characteristic length of the building (e.g. foundation radius, here half building width) and β = shear wave velocity. $k(a_0)$ and $c(a_0)$ are normalized frequency-dependent stiffness and damping coefficients. K^0 is a static reference value for the normalization: for cirular foundations, K^0 is the static stiffness; in the present study, the values of K^0 are: $K^0 = B\,\mu$ for translational and $K^0 = B^3\,\mu$ for rotational motions with μ = shear modulus of first layer. Values of $k(a_0)$ and $c(a_0)$ for circular foundations on a homogeneous halfspace can be approximated by simple expressions, see Veletsos & Wei (1971) and Kausel et al. (1978). For horizontal (H), vertical (V) and rocking (R) the values $c(a_0)$ are:

$$\begin{aligned} c_H(a_0) &= 0.576 \\ c_V(a_0) &= 0.85 \\ c_R(a_0) &= 0.3\ a_0^2\ /(1 + a_0^2) \end{aligned} \qquad (3)$$

Other, more detailed approximations have been discussed by Veletsos & Verbic (1974) and Roesset (1980). Using the value $D = \mathrm{Im}(K(\omega))/2K^0$ as approximation for the frequency-independent radiation damping of a 1-dof-system it follows:

$$\begin{aligned} D_H &= 0.288\ a_0 \\ D_V &= 0.425\ a_0 \\ D_R &= 0.15\ a_0^3/(1 + a_0^2) \end{aligned} \qquad (4)$$

The damping coefficients (4) coincide with the proposals of Richart et al. (1970) for 1-dof-systems. If ω is inserted as building mode with the dominant displacement in the respective translational or rotational component of motion for multi-dof-systems, (4) leads to the best fit of frequency domain computations with the realistic impedance functions of soil, see Stangenberg et al. (1988).

As an alternative to (2), the impedance functions can be normalized with respect to the complex shear modulus:

$$K(\omega) = K^0\ (\ 1 + 2i\xi\ )\ (k(a_0) + ia_0\ c(a_0)) \qquad (5)$$

where ξ is the hysteretic damping of the soil. Equ. (5) should not be used when damping values of different soil layers are different.

For the computation of the impedance functions (2), the foundation area is divided into n rectangular subregions S_i. Assuming that the traction in subregion S_j can be considered constant, the average displacement in subregion S_i can be expressed according to Wong and Luco (1985) as

$$u_i\ = (\ G_{ij}\ )\ \ P_j \qquad (6)$$

in which (G_{ij}) is the 3 x 3-matrix of Green's functions

$$G_{ij} = \int_{Si} \int_{Sj} G(x\text{-}\xi)\ dS_i(x)\ dS_j(\xi) \qquad (7)$$

The elements of the Green's functions matrix G_{ij} are evaluated by Gaussian integration of solutions for point loads on the halfspace using the algorithms given by Apsel (1979) and Luco & Apsel (1983). The matrices (G_{ij}) for all subregions are assembled to the 3n x 3n-matrix G, and together with the 3n displacement and load vectors u_i and P_j, respectively, the equation of motion is

$$u_i\ =\ G\ \ P_j \qquad (8)$$

For a rigid foundation, the generalized displacement vector u_0 is defined by

$$u_i\ =\ \alpha\ u_0 \qquad (9)$$

where α is the 3n x 6-matrix connecting the average motion of each subregion with the rigid-body degrees of freedom of the foundation. After inserting the vector u_i in eq. (8), the load vector P_j can be determined by solving the system of equations (8). The impedance matrix $(K_{ij}\ (\omega))$ then follows from equation (1) where $P(\omega)$ is the summation of the actual P_j. Obviously, by this procedure any arbitrarily shaped foundation can be treated.

3 FOUNDATION ON LAYER OVER RIGID BASE

Results for the dynamic impedance functions of a rigid circular disk at the surface of a stratum on rigid base have been discussed by Kausel & Roesset (1975), Gazetas (1983) and Meek & Wolf (1991). Chow (1987) studied the case of vertical impedances of rectangular foundations, Liou (1989) presented results for vertical and torsional impedances.

Fig. 1 shows the impedance functions for a rigid square foundation resting on a layer of thickness H = 2B, 3B and 4B over bedrock. Poisson's ratio is $\nu = 1/3$, the hysteretic damping ratio is $\xi = 0.05$.

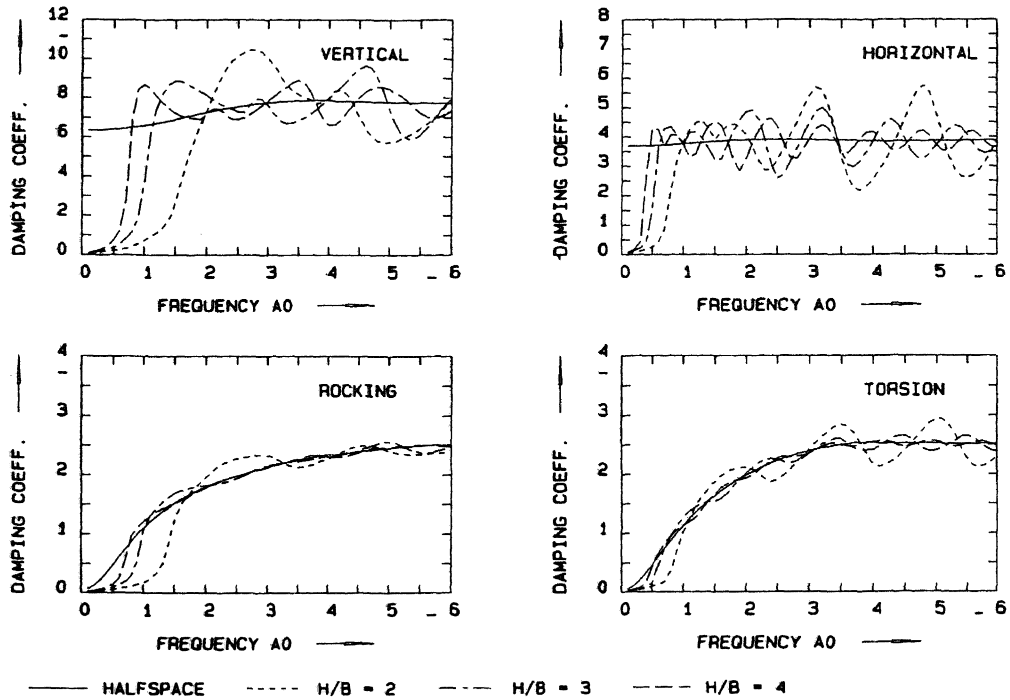

Fig. 1 Damping coefficients for a square foundation on a stratum over bedrock, $\nu = 1/3$

The oscillations of the functions in Fig. 1 are very similar to those of a rigid circular footing, c.f. Gazetas (1983). The fundamental frequencies of the stratum of thickness H are:

$$f_{S,P} = \beta_{S,P} / 4H \qquad (10)$$

with the velocities of S and P waves β_S and β_P, respectively. For the present case $\nu = 1/3$, the ratio between these velocities is $\beta_P/\beta_S = 2$. As an example, the frequencies $f_{S,P}$ for the case H/B = 2 are $a_{0,S} = \pi/4$ and $a_{0,P} = \pi/2$. It can be clearly observed in Fig.1 , that for horizontal and torsional motions, the damping nearly vanishes below $\pi/4$, whereas for vertical and rocking motions, which induce mainly P waves in the stratum, the "cut-off-frequency" is $\pi/2$.

4 FOUNDATION ON LAYER OVER HALFSPACE

As a further example, the normalized impedance functions for a square foundation resting on a uniform layer overlying a uniform halfspace are plotted in Fig. 2 versus results of Wong & Luco (1985) for ratios H/B = 2, $\beta_1 / \beta_2 = 0.6$, $\rho_2 / \rho_1 = 1.13$ and $\nu = 0.45$, $D_1 = 0.05$, $D_2 = 0.03$. The results of the procedure acc. to chapter 2 are in good agreement with the results of Wong & Luco, which is no surprise since the authors use in principle the same approach as Wong & Luco.

5 FOUNDATION ON DISTURBED HALFSPACE

A slightly disturbed homogeneous halfspace is analysed: a hard rock layer of thickness 0.5B is situated at depths 2B, 3B and 4B. Below this rock layer, the halfspace has the same properties as the first soil stratum. The hysteretic damping ratio is $\xi = 0.05$. Figures 3 and 4 show the resulting dynamic stiffness coefficients for the Poisson ratios $\nu=1/3$ and $\nu=0.45$. Although the deviation from the halfspace values is smaller than for a layer over rigid base, the influence cannot be neglected: for certain frequencies, the damping is 30 - 50 % lower than for an elastic halfspace.

6 SUMMARY

The radiation damping of rigid foundations over layered soils can be significantly smaller than for the case of a homogeneous halfspace. Even for an only slightly disturbed homogeneous halfspace, the damping can be reduced by some 30 - 50 %. This has

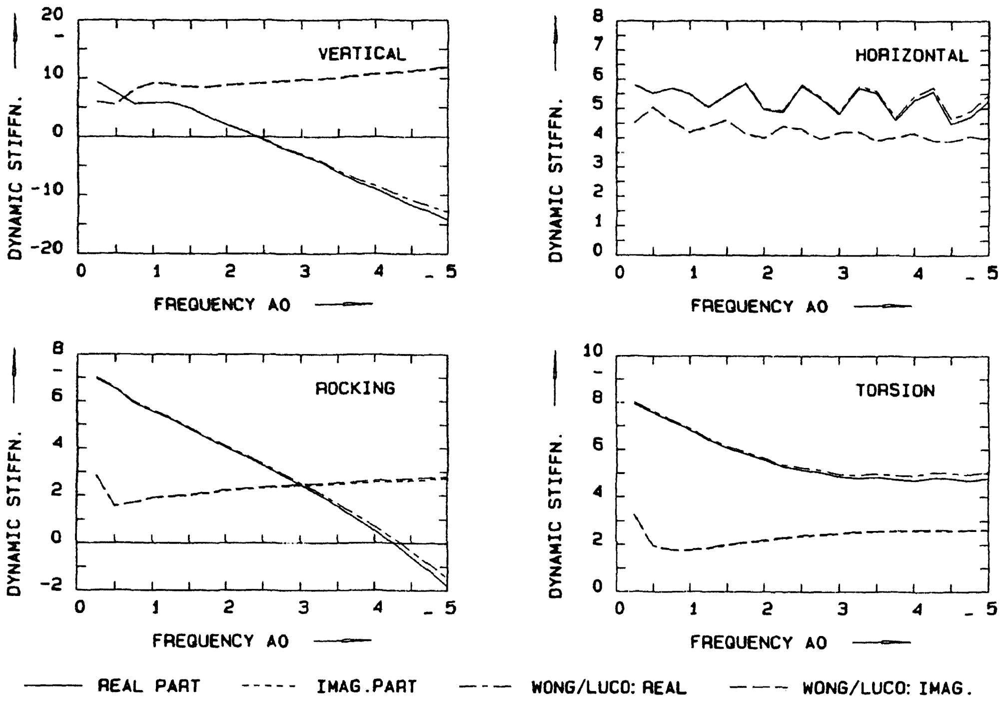

Fig.2 Impedance functions for a square foundation on a layer over halfspace, H/B=2, β_1/β_2=0.6, ν=0.45

been demonstrated for a square foundation on a nearly homogeneous halfspace disturbed by a stiff layer at different depths below the foundation. In modal analysis computations, the damping coefficients should be determined by use of the impedance functions of the specific layering and not by use of the formulae (4).

REFERENCES

Apsel, R.J. 1979. Dynamic Green's functions for layered media and applications to boundary-value problems. *Ph.D. Thesis*, Univ. of Cal.San Diego.

Chow,Y.K. 1987. Vertical vibration of three-dimensional rigid foundations on layered media. *Earthqu. Eng. Struct Dyn.* 15: 585-594.

Gazetas, G. 1983. Analysis of machine foundation vibrations: state of the art. *Soil dynamics and Earthqu. Eng.* Vol.2, No.1: 1-42.

Kausel, E.& Roesset,J.M. 1975. Dynamic stiffness of circular foundations. *J. Eng.Mech. Div.*: 771-785.

Kausel,E., Whitman, R.V., Morray,J.P. & Elsabee,F. 1978. The spring method for embedded foundations. *Nucl. Eng.& Design*: 377-392.

Liou,G. 1989. Analyt. solutions for ssi in layered media. *Earthqu.Eng.Struct.Dyn 18: 667-686.*

Luco, J.E.1976. Vibrations of a rigid disk on a layered viscoel. medium. *Nucl.Eng.&Des.*36: 325-340.

Luco, J.E. & Apsel, R.J. 1983. On the Green's functions for a layered half-space, parts I and II,*Bull. Seism. Soc. Am.* 73: 909-951.

Meek,J.W. & Wolf,J.P. 1991. Insights of cut-off-frequency for foundations on soil layer. *Earthqu. Eng. Struct. Dyn.* 20: 651-665.

Richart, F.E., Hall, J.R. & Woods, R.D. 1970. *Vibrations of soils and foundtns.* New Jersey:Prent.-Hall.

Roesset,J.M. 1980. Stiffness and damping coefficients of foundtns. *Dyn. Resp. Pile Fdns.*, ASCE: 1-30.

Stangenberg, F., Zinn, R. & Danisch, R. 1988. Soil damping effects on vibrations of noncircular foundations, *8th World Conf. on Earthqu. Eng.*,Tokio.

Veletsos, A.S. & Wei, Y.T. 1971. Lateral and rocking vibrations of footings. *J. Soil Mech. Found. Div.*

Veletsos, A.S. & Verbic, B. 1974. Basic response functions for elastic foundations. *J. Eng. Mech. Div.ASCE* : 189-202.

Wong, H.L. & Luco, J.E. 1985. Tables of impedance functions for square foundations on layered media, *Soil Dyn. Earthqu. Eng.* 4: 64-81.

Wong, H.L. & Luco, J.E. 1990. Seismic response of Lotung containment model: theory and observations. *Journ. of Eng. Mech.* 116: 1332-1350.

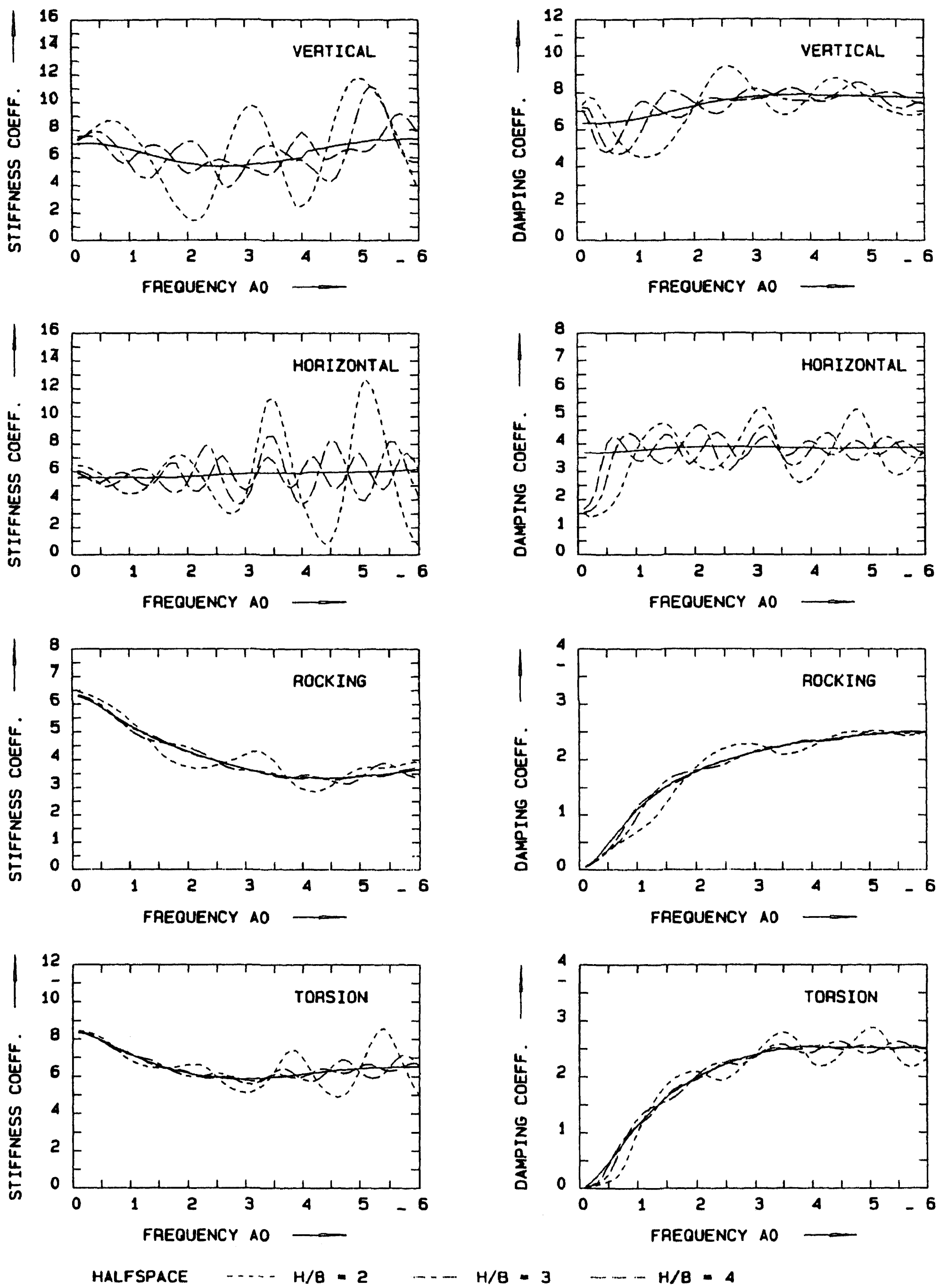

Fig.3 Impedance functions for a square foundation on halfspace with stiff layer, $\nu = 1/3$

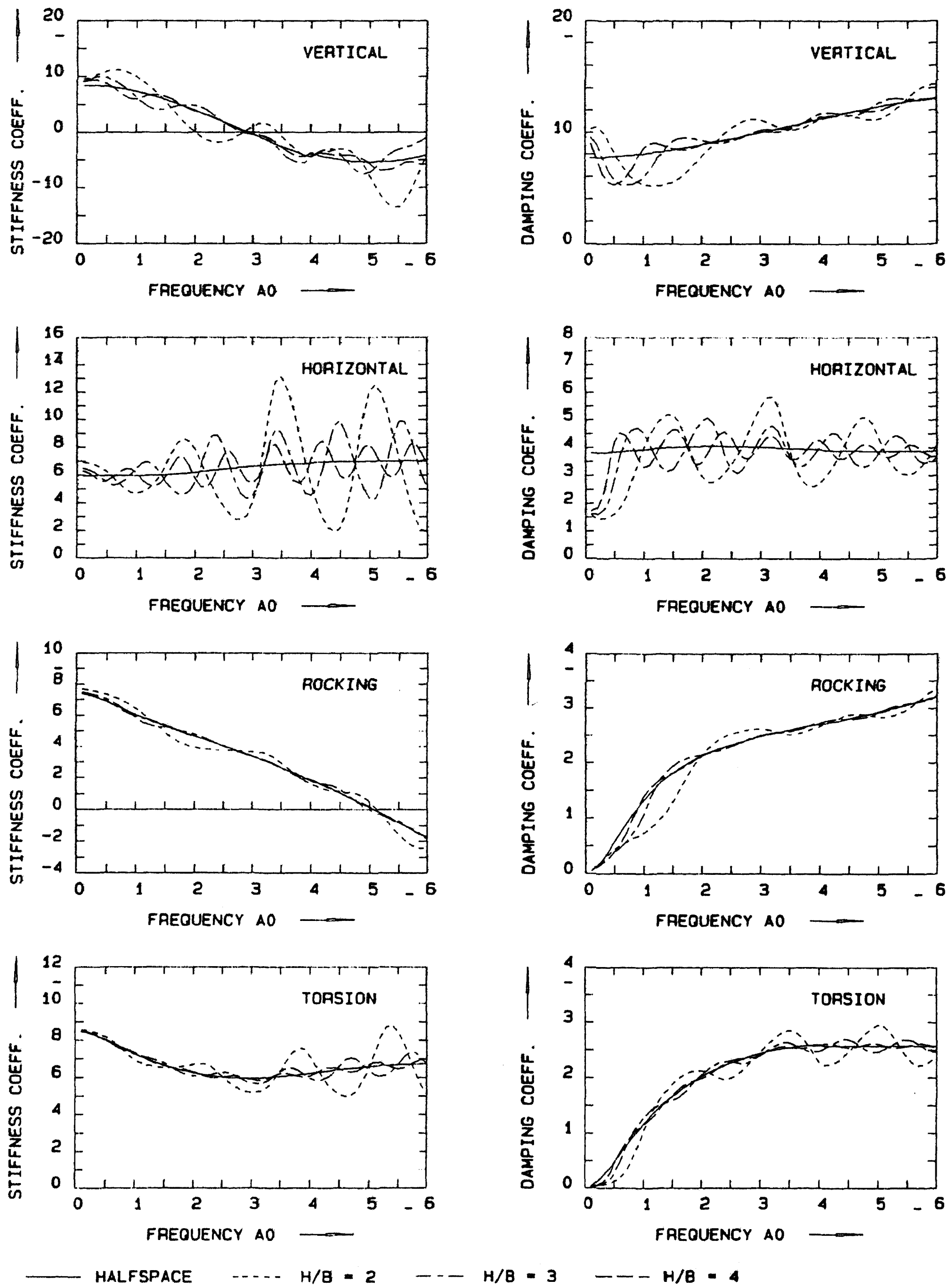

Fig.4 Impedance functions for a square foundation on halfspace with stiff layer, ν = 0.45