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Dynamic response of soil structure interaction – A simple method for calculating the dynamic compliance function of soils

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ABSTRACT: The simple and practical method for calculating the compliance function of soils is developed in this paper. The process of solving the integral equation which is insoluble theoretically is proposed by means of dispersing into the linear simultaneous equations concerned with contact pressure. The coefficients of unknow quantities in the linear simultaneous equations are evaluated by using the Green's function which are the solutions of the points of exciting forces in soils. In this paper, we have proposed to substitute the simple third power spline function instead of Grenn's function, for solving the linear simultaneous equations. The compliance function of soils is finally represnted. In the case of compared with other metohds of caluculating the compliance function of soils, this method has been proposed is more reliable, less calculating time, appliable and especially for more complicated cases.

1 INTRODUCTION

Dynamic analysis of the soil-structure is very important in the design of structures and fundations. Then, this problem has been researched by many researchers, such as T.Odaka[1], H.Tazimi[2] In these researches, one of the research method, substitues spring constants, dampers and attached mass which is concentrated mass system instead of the structure 'is considered to be more economical, simple and more efficient. However, how to evaluate the constants of the springs and the damping efficients of the damper or compliance function of soils is the main problem. Up until now, many kinds calculating methods concerning this problem have been suggested[3][4][5].

If we compared with the advantages of these calculating methods, we have found that the calculating method which is recommended by H.L.Wong has more advantages. This method has less calculating cost, suitable for any sharp functions and easier to applicate. The main process of the method is shown as follows. (1)The contact surface between soil and function is divided into a few small elements. (2)Disperse the integral equation into the linear simultaneous equations conserned with contact pressure. (3)Using Green's function to evaluate these coefficients of every unknow quantities in the linear simultaneaus equations. (4)Solving the linear simultaneaus equations.

However, Green's function is not infinite integral pole, but its integrated function is very complicated. Therefore, when the integral equation is dispersed into the linear simultaneous equations, there are some problems, such as long calculating time, special calculating technique, big calculating error and difficult to applicate. Even though some methods have been recommended to solve these problems, up-to-date be reliable and practicle method has been proposed to show a relatively prefect efficiency.

In this paper, a simple method for calculating the compliance function of soils is proposed. That is, by substituting spline functions instead of Grenn's function, the influence coefficients are thoretically evaluated. Because the third spline function is the sum of many third power polynomial expressions. It is possible to present the influence coefficients theoretically. And all evaluated work becomes easy and accurate.

In order to compare this method with other advanced calculating methods, the compliance function of elastic half-space under the rigid rectangle plate which is placed on its surface has been represented by the above mentioned method.

2 GREEN'S FUNCTION

2.1 The solution of the point of exciting force on the surface of the elastic half-space

Green's function is the solution of the

point of exciting force in soils.

In this paper, in order to easily represent, the method of substitute the 3rd spline function instead of Grenn's function, the solution of the point of exciting force on the surface of the elastic half-space is only discussed. First, we write down the solution.

According to the coordinates system which is shown in Fig.1, the displacements on the surface of the elastic half-space caused by the point force that excits on the surface of the elastic half-space, which is represented by the following equations:

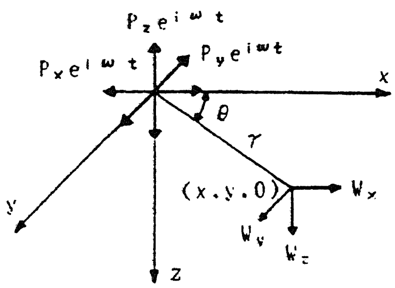

Fig.1 Cylindrical coordinate

$$u_{z,z} = -\frac{Pe^{iwt}}{2\pi G}\, j\int_0^\infty \frac{\xi\sqrt{\xi^2-\gamma^2}}{F(\xi)}\, J_1(\xi a)d\xi \quad \cdots\cdots\cdots(1\text{-}1)$$

$$u_{x,z} = \frac{Pe^{iwt}}{2\pi G}\, j\cos\theta \int_0^\infty \frac{\xi^2\{(2\xi^2-1)-2\sqrt{\xi^2-\gamma^2}\sqrt{\xi^2-1}\}}{F(\xi)} \cdot J_1(\xi a)d\xi \quad \cdots\cdots\cdots(1\text{-}2)$$

$$u_{y,z} = \frac{Pe^{iwt}}{2\pi G}\, j\sin\theta \int_0^\infty \frac{\xi^2\{(2\xi^2-1)-2\sqrt{\xi^2-\gamma^2}\sqrt{\xi^2-1}\}}{F(\xi)} \cdot J_1(\xi a)d\xi \quad \cdots\cdots\cdots(1\text{-}3)$$

Where,

Pe^{iwt}: a concentrated load which excites vertically on the origin of the coordinate,

U_{zz}: the vertical displacement on the surface of the elastic half-space when Pe^{iwt} excites vertically on te origin of coordinate,

U_{xz}, U_{yz}: the displacement of x-direction and y-direction for the horizontal displacement respectively,

G: the shear modulus of soils,

ω: frequency of exciting force,

$J_0(\xi a)$, $J_1(\xi a)$: zero power and the 1st power of 1st kind bessel function respectively,

$i = \sqrt{-1}$,

$j = \omega/V_t$: the number of wave,

$F(\xi) = (2\xi^2-1)^2-4\xi^2\sqrt{(\xi^2-\nu^2)(\xi^2-1)}$,

$r = \sqrt{(1-2\nu)/\{2(1-\nu)\}}$,

ν: poisson ratio of soil,

t: time,

$a = \omega r/V_t$: non-dimensional frequency of the exciting force,

V_t: the velocity of shear-wave.

In the other case, when the concentrated load whose direction has θ angle with x-axis excites on the point(0,0,0), the displacements on any point(X,Y,0) is given by following equations.

$$u_{x,x} = \frac{Q_xe^{iwt}}{4\pi G}\, j\int_0^\infty \{(-\frac{\xi\sqrt{\xi^2-1}}{F(\xi)} + \frac{\xi}{\sqrt{\xi^2-1}})J_0(\xi a) + \cos2\theta(\frac{\xi\sqrt{\xi^2-1}}{F(\xi)} + \frac{\xi}{\sqrt{\xi^2-1}})J_2(\xi a)\}d\xi \quad \cdots\cdots(1\text{-}4)$$

$$u_{y,x} = \frac{Q_xe^{iwt}}{2\pi G}\, j\sin\theta\cos\theta \int_0^\infty (\frac{\xi\sqrt{\xi^2-1}}{F(\xi)} + \frac{\xi}{\sqrt{\xi^2-1}}) \cdot J_2(\xi a)d\xi \quad \cdots\cdots\cdots(1\text{-}5)$$

$$u_{z,x} = -\frac{Q_xe^{iwt}}{2\pi G}\, j\cos\theta \int_0^\infty \frac{\xi^2\{(2\xi^2-)-2\sqrt{\xi^2-r^2}\sqrt{\xi^2-1}\}}{F(\xi)} \cdot J_1(\xi)d\xi \quad \cdots\cdots\cdots(1\text{-}6)$$

Where, U_{xx}, U_{yx}, and U_{zx} are the displacements of x-direction, y-direction and z-direction respectively. $J_2(\xi a)$ is the second power of first kind of Bessel function.

In the case of horizontal exciting force in y-direction, the displacements can be given by switching $U_{yx} \to U_{xy}$, $U_{xx} \to U_{yy}$, $U_{zx} \to U_{zy}$ and $U \to (\pi/2, \theta)$ in Eq.(1-3)～Eq.(1-6).

Eq.(1-1)～Eq.(1-6) are infinite integral with singular pole. They can be calculated by changing them into the finite integral and residual parts. Therefore, Eq.(1-1)～Eq.(1-6) can be written as follows(6):

$$u_{z,z} = -\frac{Q_{z,z}e^{iwt}}{2\pi G} \cdot \frac{1-\nu}{\gamma}\{f_1(a) + i\, f_2(a)\} \quad \cdots\cdots\cdots(1\text{-}7)$$

$$u_{x,z} = -\frac{Q_ze^{iwt}}{4\pi G} \cdot \frac{(1-2\nu)x}{\gamma^2}\{e_1(a) + i\, e_2(a)\} \quad \cdots\cdots\cdots(1\text{-}8)$$

$$u_{y,z} = -\frac{Q_ze^{iwt}}{4\pi G} \cdot \frac{(1-2\nu)y}{\gamma^2}\{e_1(a) + i\, e_2(a)\} \quad \cdots\cdots\cdots(1\text{-}9)$$

$$u_{x,x} = \frac{Q_xe^{iwt}}{2\pi G}\left[\frac{1-\nu}{\gamma}\{g_1(a) + i\, g_2(a)\} + \frac{\nu x^2}{\gamma^3}\{h_1(a) + i\, h_2(a)\}\right] \quad \cdots\cdots\cdots(1\text{-}10)$$

$$u_{y,x}=\frac{Q_x e^{iwt}}{2\pi G}\cdot\frac{\nu xy}{\gamma}\{h_1(a)+i\,h_2(a)\} \quad \cdots\cdots(1\text{-}11)$$

$$u_{z,x}=\frac{Q_x e^{iwt}}{4\pi G}\cdot\frac{(1-2\nu)x}{\gamma}\{e_1(a)+i\,e_2(a)\} \quad \cdots\cdots(1\text{-}12)$$

Where, $f_1(a), f_2(a)$ are the real number part and imaginary number part respectively.

2.2 Substitution of the 3rd spline function for Green's function

When the dynamic compliance function of soils is calculated by divided method, it is necessary to integrate with respect to Eq.(1-7)～Eq.(1-12) for calculating the influence coefficients. However, because the integrated functions in Eq.(1-7)～Eq.(1-12) are very complicated, the calculation is very hard. The method that we have mentioned above is that Green's function is substituted by the third power spline function which is the sum of many third power polynomial expression. Compared with other approximate calculating methods, the method has many advantages such as spline, practical and accurate. The substitution process is shown as following:

(1) Several values of real number part and imaginary number part at a region we need in Eq.(1-7)～Eq.(1-12) respectively.

(2) Calculate out the 3rd spline functions which substituted real number part and imaginary number part of Green's function in Eq.(1-7)～(1-12) respectively.

The comparison of the accurate value of the real number part $f_1(a)$, the imaginary number part $f_2(a)$ and value of spline function coresponding $f_1(a)$ and $f_2(a)$ is shown in Fig.2. It can be seen that there is almost no error came out by using spline function instesd of Green's function. And in the range of $[-\infty, \infty]$, this effect still exists. Hence, the Eqs.(1-2) and (1-12) can be written as the following forms.

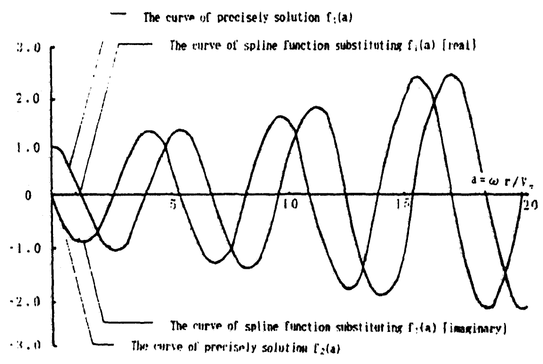

Fig.2 Substitution of spline function for Green's function (ν=1/3)

$$u_{z,z}=\frac{Q_z e^{iwt}}{2\pi G}\cdot\frac{1-\nu}{\gamma}\sum_{i=1}^{n}[\{y^f_i+b^f_i(a-a_i)+c^f_i(a-a_i)^2+d^f_i(a-a_i)^3\}+\sqrt{-1}\{y'^f_i+b'^f_i(a-a_i)+c'^f_i(a-a_i)^2+d'^f_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}1)$$

$$u_{x,z}=-\frac{Q_z e^{iwt}}{4\pi G}\cdot\frac{(1-2\nu)x}{\gamma^2}\sum_{i=1}^{n}[\{y^e_i+b^e_i(a-a_i)+c^e_i(a-a_i)^2+d^e_i(a-a_i)^3\}+\sqrt{-1}\{y'^e_i+b'^e_i(a-a_i)+c'^e_i(a-a_i)^2+d'^e_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}2)$$

$$u_{y,z}=-\frac{Q_z e^{iwt}}{4\pi G}\cdot\frac{(1-2\nu)y}{\gamma^2}\sum_{i=1}^{n}[\{y^e_i+b^e_i(a-a_i)+c^e_i(a-a_i)^2+d^e_i(a-a_i)^3\}+\sqrt{-1}\{y'^e_i+b'^e_i(a-a_i)+c'^e_i(a-a_i)^2+d'^e_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}3)$$

$$u_{x,x}=\frac{Q_x e^{iwt}}{2\pi G}\cdot\sum_{i=1}^{n}[\frac{1-\nu}{\gamma}[\{yg_i+bg_i(a-a_i)+cg_i(a-a_i)^2+dg_i(a-a_i)^3\}+\sqrt{-1}\{y'g_i+b'g_i(a-a_i)+c'g_i(a-a_i)^2+d'g_i(a-a_i)^3\}]+\frac{\nu x^2}{\gamma^3}[\{y^h_i+b^h_i(a-a_i)+c^h_i(a-a_i)^2+d^h_i(a-a_i)^3\}+\sqrt{-1}\{y'^h_i+b'^h_i(a-a_i)+c'^h_i(a-a_i)^2+d'^h_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}4)$$

$$u_{y,x}=-\frac{Q_x e^{iwt}}{2\pi G}\cdot\frac{\nu xy}{\gamma^3}\sum_{i=1}^{n}[\{y^h_i+b^h_i(a-a_i)+c^h_i(a-a_i)^2+d^h_i(a-a_i)^3\}+\sqrt{-1}\{y'^h_i+b'^h_i(a-a_i)+c'^h_i(a-a_i)^2+d'^h_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}5)$$

$$u_{z,x}=-\frac{Q_x e^{iwt}}{4\pi G}\cdot\frac{(1-2\nu)y}{\gamma^2}\sum_{i=1}^{n}[\{y^e_i+b^e_i(a-a_i)+c^e_i(a-a_i)^2+d^e_i(a-a_i)^3\}+\sqrt{-1}\{y'^e_i+b'^e_i(a-a_i)+c'^e_i(a-a_i)^2+d'^e_i(a-a_i)^3\}] \quad (a_i<a<a_{i+1}) \quad \cdots\cdots\cdots\cdots(2\text{-}6)$$

Here, when $a=a_i$ in Eq.(1-7), y^f_i, y'^f_i are the values of the real number part $f_1(a)$ and the imaginary number part $f_2(a)$ respectively. y^h_i, y'^h_i are the real number part $h_1(a)$ and the imaginary part $h_2(a)$ respectively in Eq.(1-11). b^h_i, c^h_i, d^h_i are coefficients of the 3rd spline function in the imaginary number part of Eq.(1-11). b'^h_i, c'^h_i, d'^h_i are coefficients of the 3rd spline function in the imaginary number part of Eq.(1-11). The rest of symbols are similar to that mentioned above.

2.3 Evaluation of the Dynamic function in elastic half-space

In order to explain the advantages of using the above substitution method to replace Green's function to calculate compliance function in soils, an example for the calculation of the compliance function is given.

Considering a rigid structure of area S where exsit on the surface of elastic half-space. In this situation, the distribution of the contact pressure can be evaluated by solving the following Fredholm's integral equation.

$$W_I(x,y)=\sum_{J=x,y,z}\int\int_{S_{i,j}} G_{IJ}(x,y\mid\xi,\eta)\cdot q_J(\xi,\eta)\,d\xi d\eta \quad \cdots\cdots\cdots\cdots(3\text{-}1)$$

Here, I, J = x,y,z. The footnotes I, J represent the directions of displacement. Σ represents the sum of the displacements cause by contact pressures of the three directions. W_I is the surface displacement inside of the contact surface S. q_J is the unknown contact pressure inside of contact surface S. $G_{IJ}(x,y\mid\xi,\eta)$ is Green's function in elastic half-space. Namely, it is the displacement at (x,y) which is caused by a unit point excited force efforted on the surface (ξ,η).

A numerically analytical method is used to divid integral equation. As shown in Fig.3, divided contact surface into a few finite elements. Then, divided integral equations into linear simultaneous equations. Finary, solved these equations. Eq.(3-1) can be changed to the following form.

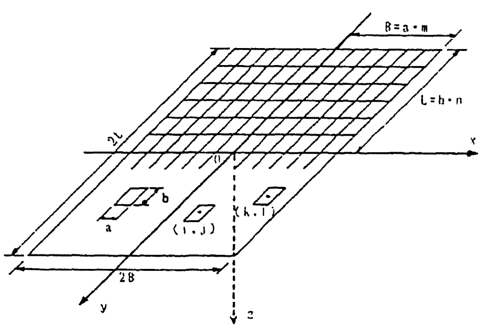

Fig.3 Divisional madel on the contact surface

$$W_I(k,l)_E=\Big[\sum_{J=x,y,z}\sum_{i=-m}^{m}\sum_{j=-n}^{n} q_J(i,j)\int\int_{S_{i,j}} G_{IJ}(x_k,y_l\;\xi,\eta)d\xi d\eta\Big]_E \quad \cdots\cdots\cdots(3\text{-}2)$$

$$(k=-m\sim m,\ l=-n\sim n,\ E=1\sim 4mn)$$

Where,

$W_I(k,l)$: displacement of (k,l) element,

$q_J(i,j)$: equal distributed contact pressure in (i,j) elements,

$S_{i,j}$: area in (i,j) element,

X_k,y_l: the coordinates of the central point in (k,l) element,

$\int\int G_{IJ}(X_k,y_l\mid\xi,\eta)d\xi d\eta$: double integral of Green's function over (k,l) element.

It can be seen from Eq.(3-2) that if the influence coefficients are given and in addition boundary conditions are known, then we can get contact pressure by solving linear simultaneous equations. By using all contact pressures we get above, the dynamic compliance function can be evaluaued.

$$C_{vv}=GB\Delta/\Big(\sum_{i=-m}^{m}\sum_{j=-n}^{n} q_z(i,j)\ S_{i,j}\Big)$$

$$C_{hh}=GB\Delta/\Big(\sum_{i=-m}^{m}\sum_{j=-n}^{n} q_x(i,j)\ S_{i,j}\Big) \quad \cdots\cdots\cdots(3\text{-}3)$$

$$C_{nn}=GB^3\Delta/\Big(\sum_{i=-m}^{m}\sum_{j=-n}^{n} q_z(i,j)\ S_{i,j}\ x_i\Big)$$

C_{vv}, C_{hh}, C_{nn} are compliance function of the vertical vibration, horizontal vibration and rotated vibration along y-axis respectively. Δ, Ω are the displacement and the angle of rotation in the direction of vibration.

When the calculating the influence coefficients, we paid a special attention on it. Because a lot of time is used to calculate the influence coefficients, in addition the accuracy of the influence coefficients directly effects the accuracy of the compliance function of soils, we can easily get the influence coefficients by substituting spline function instead of the complicated Green's function and then directly making double integral of Eq.(1-1) ~(1-6) over the elements instead of either doing over-laborate numerical integral or substituting concentrated loads instead of distributed force to do approximate calculation which causes divided elements increase. The integral results are shown in literature.

3 COMPARISON OF RESULTS

In order to make sure the advantages of the calculation method in this article, the compliance function of elastic half-space under the rigid square plate which is placed on its surface has been represents by the above mentioned. The comparison of the results among this method and other methods are shown. Calculation is based the following conditions. The divided number of the contact surface is 2m=2n=6, 2m=2n=8 respectively; Divided element is square; Vibration is vertical, horizontal and rotated vibration; No binding force is considerd in this paper.

3.1 The comparison of accuracy

H.L.Wong(4) and T.Kitamura(6) also used divided method to calculate compliance function in elastic half-space. The result of the comparison is shown in Fig.4～Fig.9.

H.L.Wong supposes that the contact pressure in all elements are equal distributed. Then double integral of Green's function makes over every elements. Finaly, obtains the influence coefficients accurately. However, T.Kitamura is proposed simplified calculation under the same assumption. That is, substitution of equal distributed contact pressure in each elements for the concentrated loads on each unit. The influence coefficients are obtained as displacements caused by these concentrated loads. Therefore, the accuracy of this method is between H.L.Wong and T.Kitamura's methods. It can be seen in Fig.4～Fig.9 that the accuracy of this method is much more higher than T.Kitamura, almost the same as H.L.Wong's result. In the mean time the error of T.Kitamura increases with the increases of non-dimensional vibration frequency a, but there is no phenomenon in this method.

It can be concluded that the accuracy of this method is not decreased even though this method we used is approximate method in which spline function is substituted instead of Green's function. The result almost had the same accuracy as H.L.Wong's method.

3.2 The comparison of computation time and amount of work

It can be seen from Fig.3～Fig.8 that if comparing 8 elements division in our method with 10 elements division in T.Kitamura's method, our method has higher accuracy. Even though the number of division is not decreased compared with H.L.Wong's method. Making double integral of Green's function over every element is needed in H.L.Wong's method, which makes the amount of calculation work very big. All in all, the amount of computation work decreases to 2/3 of T.Kitamura's method. And personal computer can easily be used to solve the problem. The cpmputation time and amount of work decrease several time compared with H.L.Wong's method.

3.3 Comparison of application

The greater the stiffenss of structures is and the softer the soil is, the grater the effect of dynamic interaction in the soil-structure is. For the colossal rigid structure especially such as nuclear power station and ocean structure, their non-dimensional frequency a are very high. As

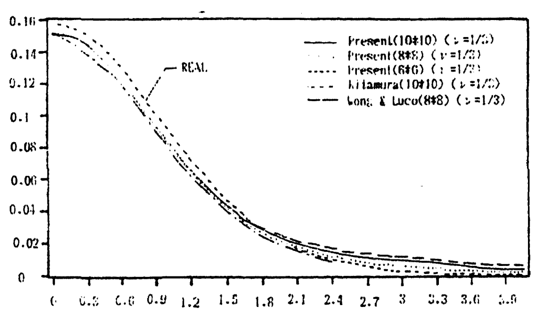

Fig.4 Comparison with other researchs (vertical vibration L/B=1)

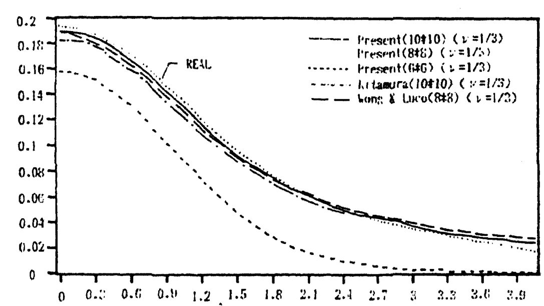

Fig.6 Comparison with other researchs (horizontal vibration L/B=1)

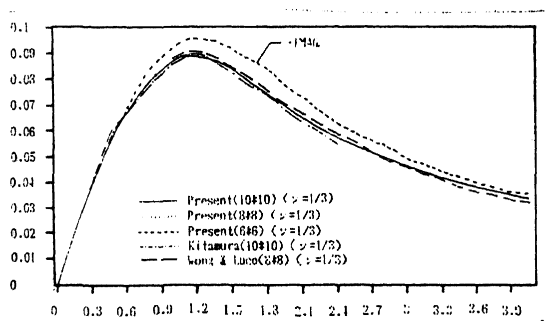

Fig.5 Comparison with other researchs (vertical vibration L/B=1)

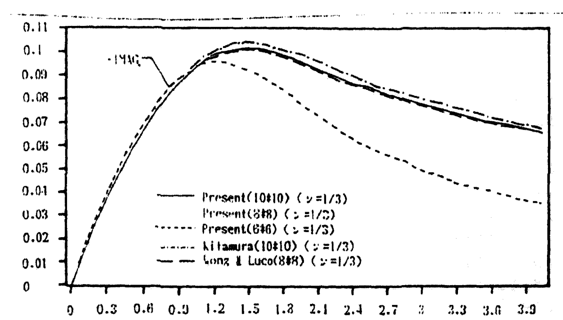

Fig.7 Comparison with other researchs (horizontal vibration L/B=1)

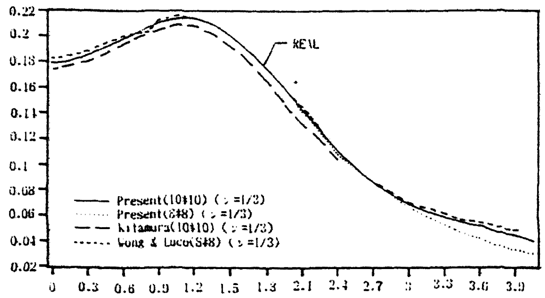

Fig.8 Comparison with other researchs (rotating vibration L/B=1)

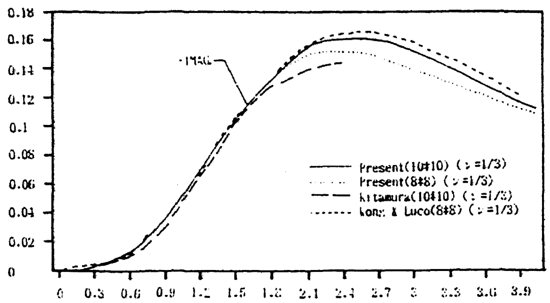

Fig.9 Comparison with other researchs (rotating vibration L/B=1)

what mentioned above, the error of T.Kitamura's method increase in the big non-dimensional frequency field, so that it is unreliable to use T.Kitamura's method in the high non-dimensional frequency field. Although there is no such problem in H.L.Wong's method, it is also difficult to applicate because of huge work of and long time of computation. This method overcomes the shortcomings of the two methods. It also has good future for some complicated situations such as multi-layer soils and buried fundations.

4 CONCLUSIONS

Some advantages in this method have been established in this reseach.

1. Green's function is substituted with the spline function, its error is very small. And computation precision has been improved and the results are more reliable.
2. Green's function is integrated numerically, the computation time and amount of work has decreased more than 50% in comprison with other methods.
3. Even though the method in this paper is an approximate computation method, the evaluate results are also more reliable in the field of high non-dimensional frequency.
4. The method is applicable especially for more complicated cases such as multi-layer soils and fundations which are buried.

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