Earthquake induced displacements of soil-structures systems

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ABSTRACT: An analysis procedure is presented for predicting the earthquake induced displacements of earth dams. The procedure extends the simple Newmark method from a single-degree-of-freedom rigid plastic to a multi-degree-of-freedom flexible system using energy concepts. The method is applied to the San Fernando dams. The lower dam suffered a flow slide on its upstream side while the crest of the upper dam moved 2 meters downstream during the 1971 San Fernando earthquake. The predicted and observed displacements are in good agreement in terms of both the magnitude of displacements as well as their pattern.

1 INTRODUCTION

A number of earth dams have failed or undergone large displacements due to seismically induced liquefaction, for example, two Chilean mine tailings dams (Doby and Alvarez (1976)), and the Mochikoshi tailings dam in Japan (Marcuson (1978); Ishihara (1984)). A number of other dams have undergone large deformations but have not actually failed in as much as the reservoir was not released. The classical example of this was the near failure of the lower San Fernando dam in which a liquefaction induced flow slide occurred on the upstream side removing the crest of the dam. Of perhaps more interest from the analysis point of view was the behaviour of the Upper San Fernando dam. The crest of this dam moved about 2 m downstream during the San Fernando earthquake of 1971. Both of these dams have been extensively studied in the past and their movements and soil properties have been established. These dams have therefore been used to verify the proposed deformation analysis procedure.

The dynamic response of soil-structure systems involving soils whose properties change markedly with cyclic loading is a difficult problem. The difficulty mainly arises from the complexity of the stress-strain relations of the soil - particularly when pore pressure rise and liquefaction occurs. The strains to trigger liquefaction are generally small (<1%). However, once liquefaction is triggered large but limited deformations occur as the soil strain hardens and regains stiffness and strength under the gravity loads and inertia forces. For many structures such as dams, these deformations may be acceptable. It is important, therefore, to develop simple reliable methods for predicting such displacements, and this is the object of this paper.

The deformation analysis proposed here is a pseudo-dynamic finite element procedure which allows both the inertia forces from the earthquake as well as the softening effect of the liquefied soil to be considered. The method is essentially an extension of Newmark’s procedure from a rigid-plastic single-degree-of-freedom system to a flexible multi-degree-of-freedom model. The procedure together with the results are presented in this paper.

2 ANALYSES PROCEDURE

Prediction of earthquake induced movements of earth structures is a difficult problem. Complex effective stress dynamic analyses procedures have been proposed (Finn et al. (1986); Prevost (1981)) but are essentially research tools and not generally appropriate for analysis of most dam structures.

The simplest analysis procedure is that proposed by Newmark (1965) in which a potential slide block is modelled as a single-degree-of-freedom rigid plastic system. Any prescribed time history of acceleration can then be applied at the base and the resulting displacements computed by numerical integration. Newmark also found that the maximum displacement at the end of the
shaking period could be estimated from simple formulae by considering the earthquake to be approximated by a number of pulses.

There are two concerns when applying Newmark's simple procedure to an earth structure such as a dam: (1) the soil, particularly in zones where liquefaction is triggered is not rigid plastic; and (2) the single-degree-of-freedom model does not allow the pattern of displacements to be computed. Byrne (1990) and Byrne et al. (1991) discusses this and show a way of allowing for a general stress-strain relation as well as extending Newmark's approach to a multi-degree-of-freedom system. Basically a pseudo-dynamic finite element procedure is used in which earthquake induced displacements which satisfy energy considerations are achieved by use of a horizontal seismic coefficient. The appropriate seismic coefficient is the one which satisfies the work-energy equation and is found by trial-and-error as described by Byrne et al. (1991). This approach is described here. It is first applied to the Newmark problem and then extended to a general stress-strain and multi-degree-of-freedom system.

Newmark's simplified model is that of a block of mass M resting on an inclined plane of slope $\alpha$, and subjected to a velocity pulse, $V$, relative to the base (Fig. 1a). The resulting displacement is given by

$$D = \frac{6V^2}{2gN}$$

where $D$ = maximum displacement, $V$ = the velocity pulse which Newmark took as the maximum ground velocity, $N$ = the yield acceleration, $g$ = the acceleration as a fraction of "g" required to initiate yield and sliding, and $g$ = the acceleration of gravity. The number 6 in his formula comes from considering 6 pulses of velocity $V$ which Newmark found gave agreement with the integrated records when the ratio $N/A<0.13$ as it usually is for practical cases of concern.

His model will now be developed in terms of work-energy and this will allow its extension to a general formulation.

The work-energy theorem states that the work done by the internal forces or stresses minus the work done by the external forces must equal the change in kinetic energy of the system, namely,

$$W_{\text{INT}} - W_{\text{EXT}} = \frac{1}{2} M V^2$$

The work done by the internal forces depends on the stress-strain relations of the material and since Newmark assumed the material to be rigid plastic, the internal force or resistance is constant with displacement as shown in Figure 1b. The work done is the area beneath the resistance line. The external force is the gravity driving force, $Mg \sin \alpha$, and in this case is constant with displacement as shown in Figure 1b. $W_{\text{EXT}}$ is therefore the area beneath the driving force line. The net work done is the difference between the two areas, namely the shaded area and this must equal $1/2 MV^2$.

Now $W_{\text{INT}} = (sL)D$, where $s$ = the shear strength of the soil and $L$ is the length of the slide block, and $W_{\text{EXT}} = (Mg \sin \alpha)D$. Thus equation 2 reduces to

$$D(sL - Mg \sin \alpha) = \frac{1}{2} MV^2$$

or

$$D = \frac{1}{2} MV^2 / (sL - Mg \sin \alpha)$$

$$= \frac{1}{2} V^2 / gN$$

where the yield acceleration, $N$, is given by

$$N = (sL - Mg \sin \alpha) / Mg$$

Equation 3a is for a single velocity pulse and when 6 pulses are considered the result is identical to Newmark’s equation 1.

Soil when triggered to liquefy will not behave in a rigid plastic manner and this is examined herein. The triggering of liquefaction of loose saturated sandy soils by earthquake loading is a small strain phenomenon (Byrne (1990)). Upon liquefaction the stress in the soil drops from A to B. Its resistance then increases with strain to a residual value, $s_r$, shown in Fig. 1c. The driving force from the ground slope remains constant, however, so that the system accelerates and deforms. When the strain reaches point C the material has hardened so that the stress developed is now sufficient to balance the driving stress as shown in Fig. 1d. However, the system has a velocity at this point and the stress continues to increase until point D is reached where the net energy ($W_{\text{INT}} - W_{\text{EXT}}$) is zero. If the system also had an initial velocity at the time liquefaction was triggered it would carry on to point E. If the driving stress exceeds the residual strength, a flow slide will occur.

Comparing the rigid plastic Newmark approach with the proposed extension to a general stress-strain relation (Figs. 1b and 1d) it may be seen that Newmark is missing the displacement from A to D.
\[ V = \text{velocity} \]
\[ M = \text{Mass of the block} \]
\[ D = \text{displacement} \]

Fig. 1: (a) Block on inclined plane subjected to velocity pulse \( V \); (b) Work-energy, Newmark; (c) Characteristic of pre- and post-liquefaction monotonic stress-strain curves; and (d) Work-energy, extended Newmark.

This could be a very considerable displacement since strains of 20 to 50% are commonly required to mobilize the residual strength, \( s_r \). However, in carrying out analyses where liquefaction is triggered only one pulse is considered appropriate, whereas Newmark considered a range of pulses up to 6 depending on the ratio N/A. So there may be compensating factors here.

For a single-degree-of-freedom system, the displacement can be computed directly from the energy equation 2 and this is described in detail in (7).

For a multi-degree-of-freedom system a finite element approach can be used. The displacements are computed from the solution of

\[
[K](\Delta) = (F + \Delta F)
\]

where \([K]\) is the global stiffness matrix of the system, \((\Delta)\) is the vector of nodal displacements, \(F\) is the static load vector acting on the system (gravity plus boundary loads), and \(\Delta F\) is an additional load applied to satisfy the energy balance of equation 2. If \(\Delta F = 0\), then for the single-degree-of-freedom, a displacement corresponding to \(C\) would be predicted. An additional force is required to balance the energy and predict points \(D\) or \(E\). This additional force can be considered as a seismic coefficient. However, its value is not related to the peak ground acceleration but is selected by trial and error so as to balance the energy in accordance with equation 2.

For the multi-degree-of-freedom system \(W_{INT}\) equals the work done by the element stresses and strains, and \(W_{EXT}\) equals the work done by the static load vector \(= (F)(\Delta)^T\). The additional force \(\Delta F\) is not included as it is merely an artifact to obtain the appropriate displacements.

The procedure has been incorporated into the finite element computer code SOILSTRESS (Byrne and Janzen (1981)) and found to give an exact agreement with Newmark when the assumptions made correspond to a single-degree-of-freedom rigid plastic system. It gives good agreement with liquefaction induced field observations reported in Hamada et al. (1987).

The procedure predicts a flow slide failure of the Lower San Fernando dam on its upstream side as was observed. This occurred because the residual strength after liquefaction was triggered was insufficient to resist the gravity driving stresses, consequently the upstream slope slumped coming to rest when its final geometry was consistent with residual strength together with energy considerations. Due to space limitation the lower San Fernando dam predictions are not shown. This paper concentrates on predicting the seismic displacements of the upper San Fernando dam.

3 UPPER SAN FERNANDO DAM

The San Fernando dams were located approximately 14 km from the epicentre of the M6.6 Richter Magnitude earthquake which
occurred on February 9, 1971. The Lower dam suffered a major flow slide on its upstream side, while the crest of the upper dam moved approximately 2 m downstream.

The Upper San Fernando dam was constructed commencing in 1921 to provide reservoir storage capacity for the Los Angeles Aqueduct. The main body of the dam was constructed of hydraulic fill resting upon an alluvium foundation soil. The top dyke section was constructed of rolled fill. A cross-section of the dam showing the various material types, the water table, and the predicted zones of liquefaction is shown in Fig. 2.

For the proposed analysis procedure, the zones in which liquefaction is triggered must first be identified. The assumed zones of liquefaction were based on the analysis carried out by Seed et al. (1973). Their study involved a comparison of the cyclic resistance of the soil and the dynamic stresses caused by the earthquake. Liquefaction was assumed to be triggered in the zones where the dynamic stresses exceeded the cyclic resistance. The cyclic resistance was based on laboratory testing of samples and the dynamic stresses were evaluated from a dynamic analysis using a modified form of the Pacoima record scale to a peak acceleration of 0.6g.

The triggering of liquefaction is a small strain phenomenon, with strains generally less than 1%. Such strains are generally not of concern for earth dams and are not considered herein. It is the much larger strains that arise when the gravity forces together with inertia forces act on the very much softer post-liquefaction stress-strain curves that are of concern. These strains can be in the range 10-50%. It is these strains and the resulting displacements that are computed in the analysis proposed herein.

The soil properties required for the analysis are the pre- and post-quake values and these are listed in Table 1. The general procedure for obtaining these is described in Byrne (1990) and Byrne et al. (1991). The pre-quake values for all 5 soil types were based on data reported by Seed et al. (1973).

The post-quake values for liquefied zones 1, 2 and 4 were kept the same as the pre-quake values. For the liquefied sandy zone 5, the modulus number, $K_G$, was drastically reduced to model the very large reduction in stiffness that occurs upon liquefaction, and the strength was dropped to its residual value. These reduced values were based on the average normalized standard penetration value, $(N_1)^{50} = 15$ together with post-liquefaction strengths and strains proposed by Seed and Harder (1990), Byrne (1990), and Seed et al. (1984).

The clayey core, material 3, although not considered to liquefy was modelled as undergoing a marked reduction in stiffness. Based on data presented by Byrne et al. (1984), the initial modulus, and hence $K_G$, was reduced by a factor of 3.3 as shown in Table 1 due to pore pressure rise. A large pore pressure rise in the core was recorded. The residual strength of the clay was based on Torvane shear tests reported by Seed et al. (1973). The authors suggested an $s_t/s_{vo} = 0.24$ and based on an average $\sigma_{vo} = 110$ kPa, this leads to $s_t = 26$ kPa as shown on the table.

The zones of liquefaction would drain sometime after the earthquake, causing additional volumetric strains and settlements. Potential volumetric strains of 2% were assumed based on Tokimatsu and Seed (1987) and an average $(N_1)^{50} = 15$, and were accounted for in the analysis by applying appropriate loads to each liquefied elements in a manner similar to that commonly used in thermal elasticity.

4 RESULTS

The predicted deformed finite element mesh is shown in Fig. 3, and the predicted and observed displacements at locations A to F on the figure, are listed in Table 2. The observed values were taken from Serff et al. (1976). It may be seen that the predicted and observed horizontal displacements are in excellent agreement at points A to E. At location F the predicted displacement is lower than

![Fig. 2: Upper San Fernando Dam: Soil Types, Zones of Liquefaction and Water Table.](image-url)
Table 1. Soil properties used in the analysis.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>γ (kN/m³)</th>
<th>d₁ (kPa)</th>
<th>c₁</th>
<th>n₁</th>
<th>K₁</th>
<th>n₂</th>
<th>K₂</th>
<th>m₁</th>
<th>R₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alluvium</td>
<td>20</td>
<td>37</td>
<td>0</td>
<td>117</td>
<td>.80</td>
<td>2000</td>
<td>.40</td>
<td>.66</td>
<td></td>
</tr>
<tr>
<td>Hyd. Fill</td>
<td>19</td>
<td>37</td>
<td>0</td>
<td>175</td>
<td>.52</td>
<td>2000</td>
<td>.26</td>
<td>.78</td>
<td></td>
</tr>
<tr>
<td>Clay Core</td>
<td>19</td>
<td>37</td>
<td>(26)</td>
<td>175</td>
<td>.52</td>
<td>2000</td>
<td>.26</td>
<td>.78</td>
<td></td>
</tr>
<tr>
<td>Rolled Fill</td>
<td>22</td>
<td>25</td>
<td>124</td>
<td>125</td>
<td>.76</td>
<td>2000</td>
<td>.38</td>
<td>.90</td>
<td></td>
</tr>
<tr>
<td>Lqf. Soil</td>
<td>19</td>
<td>37</td>
<td>(24)</td>
<td>175</td>
<td>.52</td>
<td>2000</td>
<td>.26</td>
<td>.78</td>
<td></td>
</tr>
</tbody>
</table>

Note: - Numbers in brackets indicate the properties after earthquake.
- Pre- and post-earthquake properties of soil 1, 2 and 4 are assumed identical.

Table 2. The observed and predicted displacements of Upper San Fernando Dam.

<table>
<thead>
<tr>
<th>Location</th>
<th>Observed</th>
<th>Pred.</th>
<th>Observed</th>
<th>Pred.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.49</td>
<td>1.59</td>
<td>-0.76</td>
<td>-0.90</td>
</tr>
<tr>
<td>B</td>
<td>1.49</td>
<td>1.72</td>
<td>-0.58</td>
<td>-0.53</td>
</tr>
<tr>
<td>C</td>
<td>1.95</td>
<td>1.91</td>
<td>-0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>D</td>
<td>2.19</td>
<td>2.07</td>
<td>-0.43</td>
<td>+0.05</td>
</tr>
<tr>
<td>E</td>
<td>1.77</td>
<td>1.79</td>
<td>-0.52</td>
<td>-0.13</td>
</tr>
<tr>
<td>F</td>
<td>1.10</td>
<td>0.34</td>
<td>-0.06</td>
<td>**-0.02</td>
</tr>
</tbody>
</table>

*Refer to Fig. 3.
**Negative value signifies settlement.

5 SUMMARY

An analysis procedure is presented for predicting earthquake induced displacements of dams. The procedure is an extension of the simple Newmark method from a single to a multi-degree-of-freedom system taking into account the softened stress-strain response of liquefied soils.

The method is based on the assumption that the strains to trigger liquefaction are small and can be neglected compared to the subsequent strains caused by the combined gravity and inertia forces acting on the softened post-liquefaction stress-strain curves. It is these curves that control liquefaction induced strains and displacements.

The method was applied to the San Fernando dams. The procedure predicts that the lower San

![Fig. 3: Upper San Fernando Dam Computed Deformed Shape. Displacement Magnification Factor = 2.](image-url)
Fernando dam would suffer a flow slide on its upstream slope in agreement with the field observation. The procedure also predicts that the crest of the upper San Fernando dam will move downstream about 2 m in agreement with the field observations. In addition, the predicted pattern of deformations is generally in good agreement with the field observations.

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7 REFERENCES


