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The mathematical model for liquefaction induced slip

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ABSTRACT: Dilatancy due to uplift and microcracing in the adjacent material occurs in the frictional slip. If dilatancy develops more rapidly than pore fluid flowing into the newly created void space, effective normal stress is increased in compression, tending to prevent further slip. In this study a model for liquefaction induced permanent ground displacement is given. According to this one dimensional model it is possible to formulate sliding due to liquefaction during the earthquake.

1 INTRODUCTION

Frictional slip is often accompanied by dilatancy due to uplift in sliding over asperities and microcracing in the adjecent material. Dilatancy, or an increase in void volume, is typically associated with the inelastic deformation of relatively intact rock. According to Rice (1979) there would be three possible effects of dilatancy; first effect is the failure instability may be lees, so that, still quasi-static deformation that may cause creep slippage accumalation, second effect is the regional lowering pore pressure as accelerating deformation in the focal region, third effect is the opening of network of cracks.

Instability corresponding to an unbounded slip rate in the absence of pore fluid occurs when the slope of the shear stress versus slip relation is more than the unloading stiffness of the surrounding material.

This study was mainly prepared according to Rudnicki and Chen's paper (1988). Some other papers of Rice and Rudnicki's on the dilatant hardening are as follows.

Rice (1975) considers a compressed layer of saturated rock deformed in shear. Inelastic-stress-strain relations are formulated, and these relations illustrate dilatant hardening when the layer is sheared without drainage at its boundaries. Palmer and Rice (1973) marked relation between shear stress and strain in consolidated clay. As the strain increases, the stress falls from a peak to a smaller residual stress. Rice and Cleary (1976) studying the formulation of fluid drained porous materials. In their study, they have assumed that fluid and solids are taken consistuent compressibility. Also, numerical values of the controlling porous-media elastic parameters are given for several rocks. Rice and Simons (1976) developed a solution for a shear fault in a fluid-infiltrated (drained) elastic porous material. They have found that, slip speeds vary approximately from 3m/yr to 1m/d. These speeds seem to be in a range consistent with the longer-term progressive slope failures. This mechanism of slope stabilization merits further study in the modeling of landslide phenomena. Rice and Rudnicki (1979) report the analysis of two mechanisms by which pore fluids could partially stabilize the earthquake rupture process in natural rock masses. These mechanisms are based on dialtancy strengthening and the increase of elastic stiffness for undrained and drained conditions. Rice (1980) gave mechanics of earthquake rupture in three areas. The closely interrelated areas are: a) Representation of elastic-field generated by earthquakes, b)Fundementals of the rupture process in geological materials, c)Process on a tectonic scale leading to earthquake instabilities. Dilatancy effects during the slip-weakening process in shear crack propogation are given in this paper.

Rudnicki (1977) investigated the models for the inception of earth faulting based on the deformation of a rock mass containing an embedded weakened zone. Constitutive laws appropriate to dilatant, frictional, inelastic behavior are used to characterize the weakened zone material. Two distinct types of instability corresponding to possible models of seismic mechanisms are identified. These are "localization" instabilities, at which essentially homogeneous deformation giving way to localized shearing and "runaway" instabilities at which no further quasi-static deformation is possible and internal effects dominate. Rudnicki (1984) investigated the effects of dilatant hardening on the shear deformation. The analysis considers the of rock mass containing a weakened layer of thickness h. The presence of the weakened layer causes instability characterized by an unbounded ratio of a strain increment in the weakened layer to that in the far field occuring earlier than it would be predicted from the response of the material surrounding the embedded layer. Rudnicki et al. (1984) used collinear crack solutions to examine the effects of fault slip zone interaction on moment, stress-drop and strain energy realese, is extended by considering the effects of interaction between different size slip zones. Roeloffs and Rudnicki (1984) studied water level changes due to creep events by using a coupled deformation-diffusion solution for the "pore pressure" prodused by a shear dislocation moving at a speed V in a saturated porous medium. Coupled solution predicts a water level change comparable in magnitude to the observed change.

2 MATHEMATICAL MODEL AND FORMULATION

A recent study on dilatant hardening is done by Rudnicki and Chen (1988). Their study is an outgrow of previous work of Rudnicki (1984) which examined the effects of dilatant hardening in a narrow zone slightly weaker than the surrounding material and he found dilatant hardening did delay the onset of an instability compared with its occurance in the absence of pore fluid. Rudnicki and Chen (1988) considered one-dimensional model consisting of a long slab (Fig.1) loaded at its top surface by a shear displacement U and by a normal stress σ.

A reservoir of pore fluid maintaned at a constant pressure p is also connected to the slab at y=h. The slab is bounded at y=0 by a fault or frictional surface assumed to be formed at peak shear stress. For constant normal stress and pore pressure, the shear stress on the frictional surface τ is related to the relative slip δ by the following expression.

$$\tau=\tau_p-(\tau_p-\tau_r)g(\delta/\delta_o) \qquad (1)$$

In which τ_p= the peak stress, τ_r= the residual shear stress and $g(\delta/\delta_o)$= a function that describes the decrease of τ from τ_p to τ_r (Fig.2a). The simplest form of g(x), from the experiments done by Rice (1980), Wong (1982) and Barton (1972, 1976) is as follows

$$g(x)=-2x^3+3x^2 \qquad 0\leq x\leq 1 \qquad (2)$$

If the normal stress and pore pressure are not constant, on additional term must be added to (1) (Fig.2b). For frictional slip, effective stress is the difference between the total compressive stress and pore pressure.

$$\tau=\tau_p-(\tau_p-\tau_r)g(\delta/\delta_o)+m_o[\sigma-(p_o-p_1)] \qquad (3)$$

Where σ= the normal stress measured at peak shear stress, p_o= the value of pore pressure at y=0 and p_1= its initial value of p.

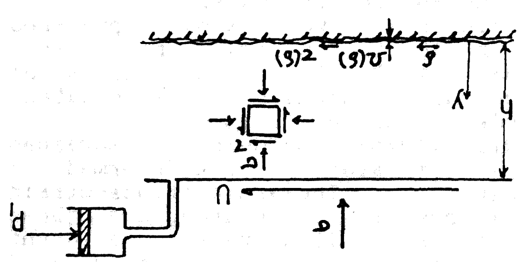

Fig.1. Geometry of mathematical model

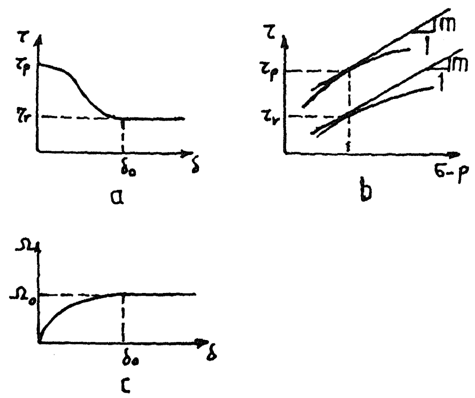

Fig.2.Constitutive relation for the frictional surface. a)The shear stress τ decreases from a peak value τ_p to a residual value τ_r as the slip δ increases from zero to δ_o. b)The level of shear stress with the effective stress σ-p. c)The uplift Ω increases to a maximum of Ω_o when $\delta=\delta_o$.

Dilatancy is assumed to accompany frictional slip according to a relation

$$\Omega=\Omega_o f(\delta/\delta_o) \quad (4)$$

The maximum amount of dilatancy Ω_o is achieved when the shear stress has been reduced to its residual value δ_o (Fig.2a-c). Ω includes both the actual opening of the surface due to uplift and the increase in void volume due to microcracking in material adjecent to the fault surface. The value of f is given by Barton (1976)

$$f(x)=2x-x^2 \qquad 0\le x\le 1 \quad (5)$$

It is assumed that the volume of moving pore fluid is equal to that of the newly created void space occuring due to movement of the frictional surface. Density of pore fluid is μ, mass of fluid drawn-in per unit slip surface is $\mu\Omega$. Rate of fluid mass influx q is given by;

$$q=\mu\dot{\Omega}+\mu\Omega\dot{p}_o/K_f \quad (6)$$

Here superposed dot denotes the time derivative and $K_f=\mu dp/d\mu$ is the bulk modulus of pore fluid. This change in fluid mass must be balanced by flow from surrounding material. This flow is simply assumed to be proportional to the difference between the pore pressure p_o on the fault surface and pressure p_1 in the reservoir.

$$q=\mu K(p_1-p_o)/h \quad (7)$$

The permeability K can be expressed as $K=k/\mu$, where k has units of area and μ is the fluid viscosity. The material adjecent to the fault surface is assumed to be elastic. This is true if material unloads from peak stress. Elastic deformation shear stress is given by eq.(8)

$$\tau=\tau_p-G(U-\delta)/h \quad (8)$$

G= the elastic shear modulus governing unloading. Since equilibrium requires the stress given by (8) must be equal to that given in (3) one gets,

$$U=\delta-[(\tau_p-\tau_r)h/G]g(\delta/\delta_o)+(hm_o/G)[\sigma-(p_o-p_1)] \quad (9)$$

Equating (6) and (7) yields;

$$K(p_1-p_o)/h=\dot{\Omega}+\Omega\dot{p}_o/K_f \quad (10)$$

For simplicity, U will be assumed to increase at a constant rate U_o. Eq.(9) can be rewritten in a nondimensional form,

$$\epsilon dY/dT=\{2[T-Y+(2A/3)g(Y)]+\epsilon\alpha f(Y)\}/\{f'(Y)+\alpha f(Y)[1-(2A/3)g'(Y)]\} \quad (11)$$

$Y=\delta/\delta_o$, $T=\dot{U}_o t/\delta_o$, $P=m_o h(p_1-p_o)/G\delta_o$, $A=3(\tau_p-\tau_r)h/(2G\delta_o)$, $\epsilon=U_o t_o/\delta_o$, $t_o=2\Omega_o m_o h^2/(\delta_o KG)$ and $\alpha=G\delta_o/(m_o hK_f)$

For undrained response, the deformation is too rapid to allow time for fluid mass exchange. In this case, left hand side of (10) is zero. Hence,

$$\dot{p}_o=-K_f f'(\delta/\delta_o)\dot{\delta}/[\delta_o f(\delta/\delta_o)] \quad (12)$$

Differentiating (3) and substituting in (12) gives slope of undrained response as,

$$d\tau/d\delta=-[(\tau_p-\tau_r)/\delta_o]g'(\delta/\delta_o)+m_oK_f f'(\delta/\delta_o)/[\delta_o f(\delta/\delta_o)] \quad (13)$$

Hardening effect is the second term of (13) which is proportional to the friction coefficient m_o and pore fluid bulk modulus K_f. Large decrease in pore pressure can reduce K_f by allowing dissolved gases to come out of solution or reaching the liquid-vapur transition. In the limit of K_f falling to zero the hardening effect disappears.

3 CONCLUSION

According to Rice (1980) characteristic slip amount in situ is δ=0.1m. By using this value for slip at the begining of the residual stress level,it is found that δ_o=0.2m. For the 1983 Nihonkai-Chubu earthquake, the permanent ground displacements with a maximum horizantal amplitude of 5m, and for the 1964 Niiagata earthquake, the maximum horizantal displacement of 5.5m were found from Hamada et al. (1986). Comparing this value with the mathematical model results is seen big difference Sener (1987-1987a). The main reason for this difference is due to the analitic model used in analysis is not obtained for liquefaction induced land sliding case. Three or two dimensional models may give better representation of liquefaction induced permanent ground displacements.

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