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Dynamic amplification functions of the surface layer considering the variation of soil parameters

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Abstaract: This paper deals with the estimation of the dynamic ground characteristics of a simple soil layer model, employing a stochastic approach to dealing with variations in dynamic soil parameters. The analytical expression of the amplification function is expanded in a Taylor series about the mean value; and the relation between the mean and the standard deviation of the amplification function is presented. Analytical results thus obtained are compared with simulated ones, and the validity of the method of estimation presented here is also examined. The dynamic soil parameters costituted by the effect of soil damping on the amplification function is discussed in detail.

1 INTRODUCTION

It has come to be of importance to clear dynamic ground characteristics related to earthquake damage in recent years. To estimate the characteristics, one- and multi-dimensional soil structural models have generally been analyzed. The two models were used, corresponding respectively to a uniform layer and to a complex layer with an irregular interface, which often makes for complexity of analysis. One problem, however, is that the dynamic ground parameters used in the analysis can be neither perfectly determined nor made to match the *in situ* ground characteristics.

Given these problematic limitations concerning dynamic ground characteristics, researches by Hibino and Kawamura (1988) and Ishida and Ozutani (1988) have already been reported in which variations in the ground parameters are considered from a deterministic view point. So far, however, and with the exception of the papers by Chávez-García and Bard (1989) and Tsujihara (1991), little attention has been paid to the possibility of a stochastic approach to this problem.

In this paper, the analytical model of soil structure that is assumed is a horizontally uniform layer, and the incident model a stationary plane SH wave. Taking into consideration stochastic variation in dynamic ground parameters, we present the dynamic amplification function. The analytical amplification functions including a complex soil structural model are compared with simulated ones.

We consider the thickness, the shear wave velocity, and the density of the surface layer as stochastic dynamic parameters, present the preliminary characteristics of the dynamic amplification function, and discuss the effect of soil damping on this dynamic amplification function and the standard deviation of the amplification factor.

2 ANALYTICAL METHOD

2.1 A technique based on Taylor expansion

We simplify the problem by assuming that the soil structural model is in its depth direction a horizontally uniform layer, and that the incident model is a vertical plane SH wave. The dynamic amplification function $H(\omega)$ analyzed here is represented by site soil parameters such as thickness z, shear wave velocity v, and density ρ. Expanding $H(\omega)$ in a Taylor series about the mean values of the parameters, the resulting expression of $H(\omega)$ is;

$$H(\omega;\mathbf{X}) = H(\omega;\overline{\mathbf{X}}) + \sum_{i=1}^{\infty}\left\{\sum_{\kappa=1}^{n}\frac{(X_\kappa - \overline{X_\kappa})^i}{i!}\left|\frac{\partial^i H}{\partial X_\kappa^i}\right|_{\overline{\mathbf{X}}}\right\} \quad (1)$$

where $\mathbf{X}=(z,v,\rho)$, $\overline{\mathbf{X}}$ representing the mean value of $\mathbf{X}$, and n and ω being respectively the total number of the parameters and the angular frequency. The mean and the approximate mean square of $H(\omega)$ in Eq.(1) can respectively be obtained as follows;

$$E\left[H(\omega;\mathbf{X})\right] = H(\omega;\overline{\mathbf{X}}) = \overline{H} \quad (2)$$

$$E\left[(H-\overline{H})^2\right] = \sum_{\kappa=1}^{n}\sum_{\kappa'=1}^{n} \left|\frac{\partial H}{\partial X_\kappa}\right|_{\overline{\mathbf{X}}} \left|\frac{\partial H}{\partial X_{\kappa'}}\right|_{\overline{\mathbf{X}}} r_{x_\kappa x'_\kappa}\sigma_{x_\kappa}\sigma_{x'_\kappa} \tag{3}$$

In Eqs.(2) and (3), $E[\ ]$ and σ are respectively the ensemble averaging operator and the standard deviation, and r is the correlation coefficient between two of the dynamic soil parameters.

Ignoring the correlation between the two dynamic parameters of soil, and assuming the distribution of the variation of the parameters to be Gaussian, we can obtain the final standard deviation representation σ_H of the amplification function as follows;

$$\sigma_H = \sqrt{\sum_{\kappa=1}^{n}\left|\frac{\partial H}{\partial X_\kappa}\right|^2_{\overline{\mathbf{X}}}\sigma^2_{X_\kappa}} \tag{4}$$

2.2 Simulation

Utilizing a computer, we simulate pseudo random numbers (N) based on a Gaussian probabilistic density function. Based on the amplification function H_s and the mean $\overline{H_s}$ simulated by the random numbers, we can calculate the standard deviation σ_{H_s} of the amplification function using the expression,

$$\sigma_{H_s} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(H_{S_i} - \overline{H_S}\right)^2} \tag{5}$$

where the suffix s means the simulation.

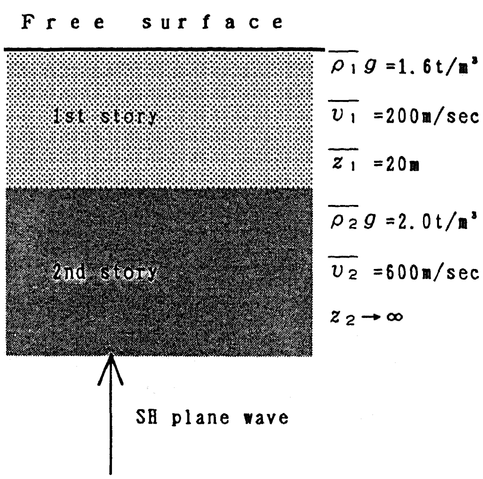


Fig.1 Analytical ground structural model and dynamic soil parameters.

2.3 Parameters for analysis

The analytical model is very simple: a sedimentary layer resting horizontally on a half space developed by Shibata (1981) as shown in Fig.1. For the upper and lower layers, $\overline{z_1}$=20m, $z_2 \rightarrow \infty$, $\overline{v_1}$=200m/sec, $\overline{v_2}$=600m/sec, $\overline{\rho_1}g$=1.6t/m^3, and $\overline{\rho_2}g$=2.0 t/m^3, where g is the gravitational acceleration. The variation of dynamic parameters is chosen as 0~ 15%. The total number of random numbers for simulation N is equal to 100.

Since a viscous type model is here assumed for the soil damping, the following Q value proposed by Kobayashi, Abe and Amaike (1989) is used.

$$Q = vf^n/a \tag{6}$$

In Eq.(6), v and f are respectively the shear wave velocity and the frequency. n and a are the constants and herein we choose a=20~ 60 and n=0.

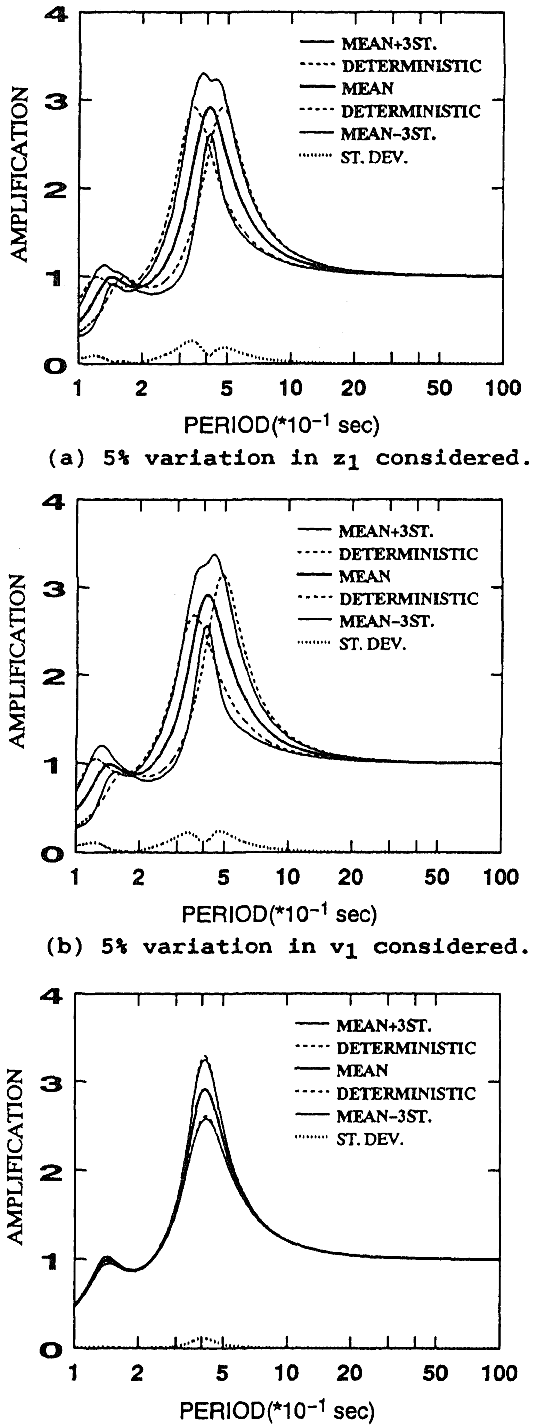
3 RESULTS

3.1 Results based on Taylor expansion

Fig.2.1 (a)~ (c) shows the relation between the amplification function H and the period T, where a 5% variation in the respective mean values of z_1, v_1 , and ρ_1 is considered in case of a=20. The two broken lines in the figure represent the amplification functions in case of the positive and the negative variations given by the deterministic method, and the fine solid line represents the mean amplification function. These functions are hereafter referred to as the deterministic amplifications.

The two thick solid lines approximately enveloping the maximum and minimum values of the deterministic amplification represent the amplification functions gained when the positive and the negative variations are given according to the stochastic method. These functions are represented considering the mean and three times as large as the standard deviation σ_H of Eq.(4), and hereafter referred to as the stochastic amplifications. The σ_H is shown at the bottom of the respective figures with a dotted line.

From Figure (a), where the variation of z_1 is considered, it is seen that the period of the deterministic amplification peak point (about 0.4sec) shifts by 0.06sec toward a longer period in the case of the positive variation. On the other hand, it shifts by 0.06sec toward a shorter period in the case of the negative one. The deterministic amplifications in these two cases have identical peak values, these agreeing quite well with the peak value of the mean amplification function $\overline{H}$. The deterministic amplifications are approximately enveloped by

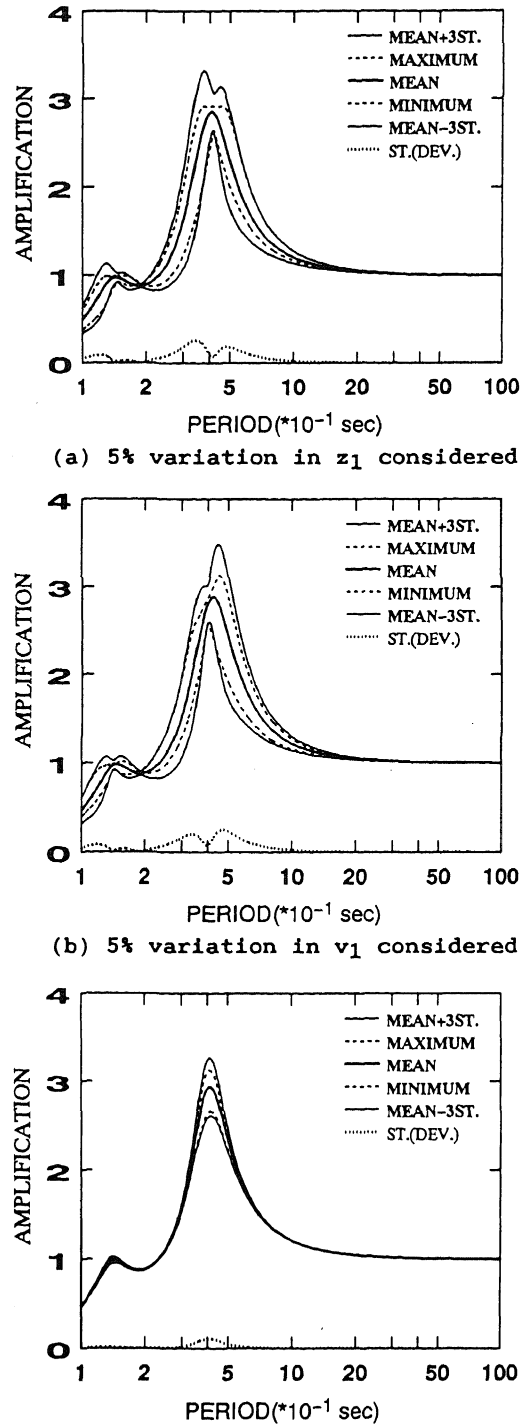


(a) 5% variation in z_1 considered.

(b) 5% variation in v_1 considered.

(c) 5% variation in ρ_1 considered.

Fig.2.1 Amplification function based on the Taylor expansion (a=20, n=0).



(a) 5% variation in z_1 considered

(b) 5% variation in v_1 considered

(c) 5% variation in ρ_1 considered

Fig.2.2 Amplification function based on the simulation (a=20, n=0).

the stochastic ones. It is interesting to note that σ_H has a valley at its fundamental period.

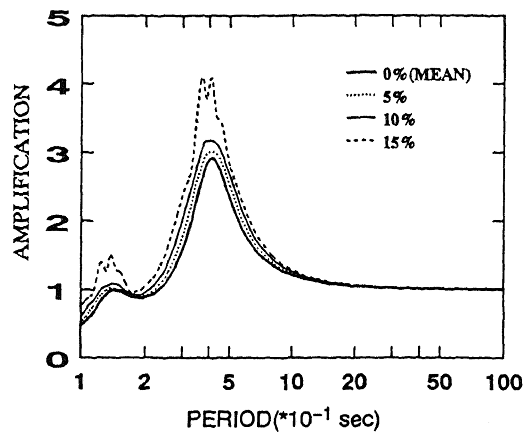
From Figure (b), it is seen that the positive variation of v_1 shortens the period of the deterministic amplification peak by 0.5sec with decrease in the peak value, and that the negative variation of v_1 lengthens the period by 0.5sec with increase in the peak value. The σ_H shows the same characteristics of the standard deviation as in the case of the variation in z_1.

From Figure (c), it is also seen that slight variation in ρ_1 produces variation in the peak amplification value, and that in this case the stochastic amplifications approximately envelope the deterministic ones. Furthermore, σ_H has characteristics similar to the mean amplification function, and it is obvious from consideration of the variations of z_1, v_1, and ρ_1 that σ_H in case of the variation in ρ_1 is smaller than the other standard deviations.

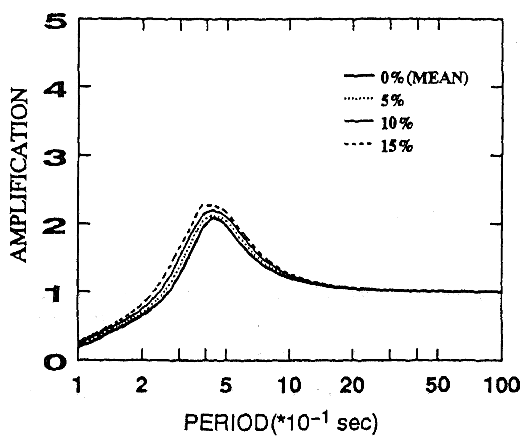
3.2 Comparison

In Fig. 2.2(a)~ (c), the amplification function H_s gained by a simulation method is plotted as a function of period, where the two broken lines with symbols are respectively the maximum and the minimum values of H_s, the fine solid line is the mean $\overline{H_s}$ of H_s, and the two thick solid lines are $\overline{H_s} \pm 3\sigma_{H_s}$. The standard deviation of H_s, σ_{H_s}, is shown at the bottom of the figure with the dotted line. The Figures (a), (b) and (c) correspond to the results gained in respect to the variations in z_1, v_1 and ρ_1.

Comparing the maximum value of H_s with $\overline{H_s} + 3\sigma_{H_s}$ in Figure (a), it is found that the two sets of values nearly agree with each other except at peak value, a pattern reproducing that seen in Figure (b); in Figure (c), however, it is seen that these two sets of values coincide almost unanimously over all periods. Thus, $\overline{H_s} \pm 3\sigma_{H_s}$ in almost all cases covers the maximum and the minimum values

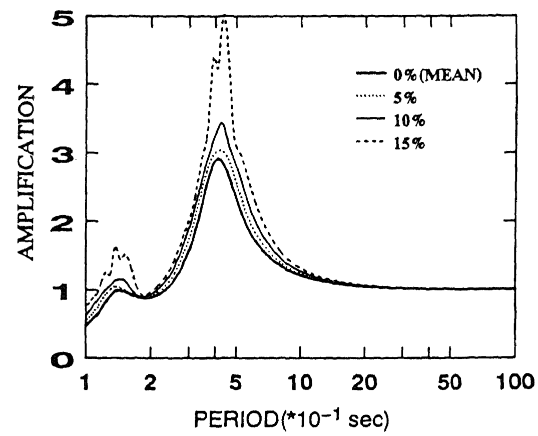


(a) Case of a=20 and n=0.

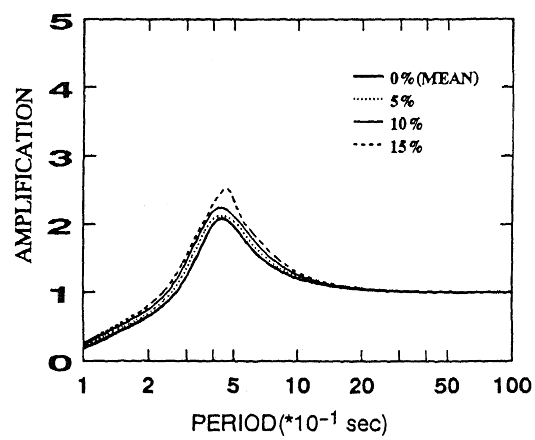


(b) Case of a=20 and n=0.

Fig.3.1 Amplification function considering the variation in z_1.



(a) Case of a=20 and n=0.



(b) Case of a=20 and n=0.

Fig.3.2 Amplification function considering the variation in v_1.

of H_s in the period range presented in this analysis. With the exception of the peak value, each σ_{H_s} in Fig.2.2 agrees quite well with the corresponding σ_H in Fig.2.1.

Consequently, it is concluded that the present analytical method is sufficient for estimating the amplification function taking into account variation in the dynamic soil parameters.

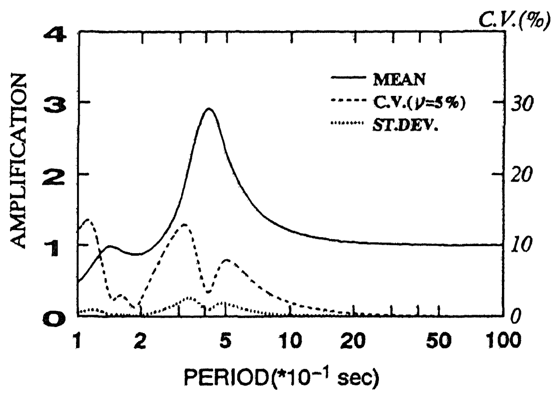
3.3 Effect of soil damping

In this section, the change in the constants is considered in Eq.(6); n=0, a=20, 60. In Figs. 3.1 and 3.2, the amplification function H is plotted as a function of period T by selecting the parameter a as a variable, where the variances of the most largely influential parameters z_1 and v_1 are taken into consideration. The parameter a provides the Q value governing the soil damping. The four kinds of lines in the figure represent the results gained where the coefficient of variation ν is set as 0, 5, 10, and 15%. The ν means the variation in the mean value as follows;

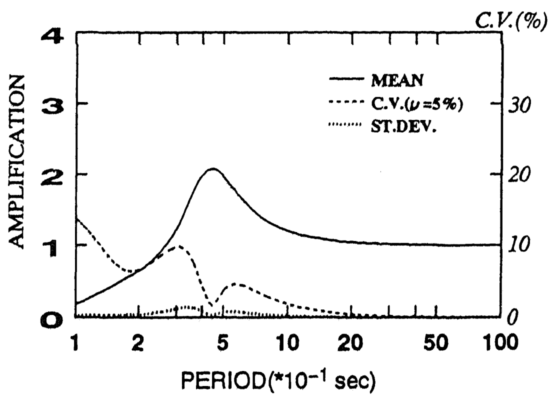
$$\nu = \frac{\sigma_x}{\bar{x}} \tag{7}$$

And Figures (a) and (b) show the results for two cases, in which a=20 and a=60, respectively.

In Fig.3.1(a), it is noticed that the first peak values of the amplification function for the case in which ν=15% and for cases in which ν=0, 5 and 10% are respectively about 4.5 and 3.0, and the peak value where ν=15% is 1.5 times of the peak value for the other cases. Furthermore, for the second peak, the peak values of each case are respectively about 1.5 and 1.0, and the peak value for the case of ν=15% is also 1.5 times the peak value for the other cases. As shown in Figure (b), when the parameter a -*i.e.* soil damping- increases, the amplification functions for the four cases decrease over the whole

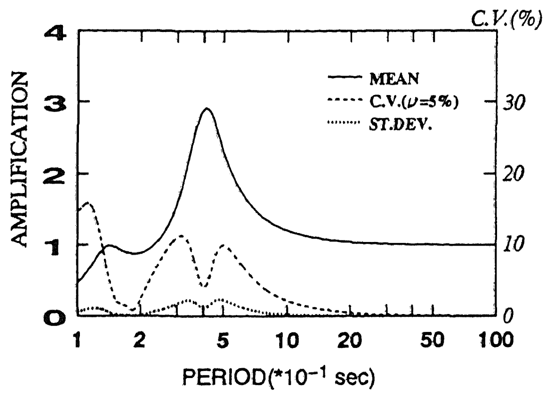


(a) Case of a=60 and n=0.

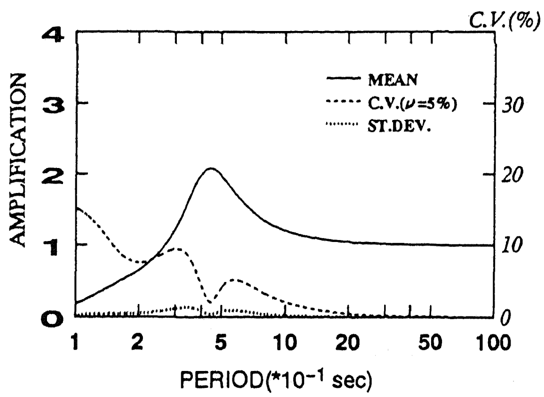


(b) Case of a=60 and n=0.

Fig.4.1 Amplification function of the mean, the standard deviation and the coefficient of variation considering 5% variation in z_1.



(a) Case of a=60 and n=0.



(b) Case of a=60 and n=0.

Fig.4.2 Amplification function of the mean, the standard deviation and the coefficient of variation considering 5% variation in v_1.

period range, the peak value showing a particularly remarkable decrease. As the result, the peak value becomes about 2.0. In the shorter period range below 0.1sec, the amplification functions gradually decrease, and, finally becoming independent of the ν value, have almost identical value.

In Fig.3.2(a), the first peak value of the amplification function is about 5.0 for ν=15%, this value being about 2.0 times larger than the corresponding peak value for the case of ν=0. From Figure (b), it is seen that the rate of decrease in the amplification function over the whole period range is smaller than that relating to z_1 as shown in Fig.3.1. Moreover, in the shorter period range below 0.1sec, all the amplification functions again become independent of the ν value, having virtually identical values. This characteristic of v_1 is the same as those observed in the case of z_1.

Consequently, when the 15% variations in the thickness of the soil layer and the shear wave velocity are taken into consideration, the peak value of the first period of the amplification function is 1.5$\sim$ 2.0 times larger than the corresponding peak value when this variation is ignored; it is the shear wave velocity that has the more significant influence on the amplification function of these two dynamic soil parameters.

3.4 Standard deviation and coefficient of variation

In Figs. 4.1 and 4.2, the coefficient of variation *C.V.*, the mean amplification $\overline{H}$, and the standard deviation σ_H are all plotted as a function of period, where the fine solid, the broken, and the dotted lines correspond to the $\overline{H}$, the *C.V.*, and the σ_H with 5% variation in z_1 and v_1, respectively.

In these figures, in the vicinity of the first peak period, the maximum fine solid line, the minimum dotted line, and the middle broken line represent the mean amplification $\overline{H}$, the standard deviation σ_H, and the coefficient of variation *C.V.* of the amplification function. The *C.V.* shows characteristics similar to those of σ_H over the range except the shorter period. And it is clear that in the period range below one second the *C.V.* reveals a large variation, equal to 10 $\sim$ 25%, which is larger than 5% variation given for z_1 and v_1.

4 CONCLUSIONS

In this paper, we used a horizontally uniform soil layer as a soil structure model and a vertical *SH* plane wave as an incident model, and estimated dynamic amplification functions based on analysis and simulation taking into consideration variation in the soil parameters. The results indicated that the stochastic amplification functions covered the deterministic ones, that except with regard to the mean value the two amplification functions showed satisfactory agreement, and that the agreement between the respective standard deviations of the amplification functions was quite satisfactory. Therefore, the analytical approach presented here is useful when variations in the soil parameters are considered from a stochastic view point.

We discussed the effects of both variation in the dynamic soil parameters and damping on the amplification function. The results indicated that of all the parameters the exerting greatest influence on the function was the shear wave velocity of the upper soil layer. The soil damping caused a decrease in the peak amplification function for the longer period range. However for the shorter period range, no such decrease can be clearly recognized from the results.

The fluctuation rate of the peak value of the amplification function is greater than the one given to the coefficient of variation of the dynamic parameters beforehand. Though the coefficient of variation of the amplification function increases for the shorter period range, this does not have much effect on the seismic response, due to the considerably smaller absolute amplification function of the surface layer.

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