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Stochastic seismic response analysis for soil layers with random dynamic parameters

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ABSTRACT: In this paper, the random features of input earthquake and soil dynamic parameters are simultaneously considered. The probabilistic finite element method is introduced into the random vibration analysis. An effective calculating procedure is established for evaluating the stochastic seismic responses of the random soil layers. The horizontal soil layers are taken as an example and the effects of soil parameters' variability on the dynamic responses are investigated. The results show that the variability of the dynamic responses will also reach relative high level when considering the variability of soil parameters. It is very necessary to include these variability in the dynamic reliability analysis of earth structures.

1 INTRODUCTION

As a kind of natural material, the physical and mechanical behaviour of soil shows remarkable variability. The earthquake ground motions also possess intense randomness. The seismic response analysis of soil layers should be the stochastic analysis for mechanical system with random dynamic parameters. In the past investigations, the dynamic analyses of soil layers are mainly concentrated on the deterministic calculations of seismic responses (Seed 1967 & Martin 1982). With the developments of seismic risk analysis and structural reliability evaluation, the theory of random vibration and dynamic reliability is more and more used in the seismic deformation and dynamic stability analyses of soil and foundations. The emphasis, however, is put on the randomness of input earthquake motion (Faccioli 1976 & Gazetas 1981). Recently, some researchers began to introduce the random finite element method to handle the variability of soil parameters (Vanmarcke 1986 & Wu 1991). Using the perturbation theory and Fourier tranform technique, the recurrence equations of finite element are established and solved in frequency domain. The variability of dynamic response caused by the variability of soil paremeters is investigated. But in general, the input ground motion is assumed as the deterministic earthquake wave. In this paper, the random finite element method is combined with the procedure of stochastic response of soil layers. The variability of soil parameters, such as dynamic shear modulus and damping ratio, and the randomness of input seismic motion are taken into consideration by simulating the soil parameters as normal random variables and the earthquake motion as a stationary Gaussian process. Through the second – order perturbation, the equations of mean and variance of power spectrum of dynamic response are established, and then, the spectral curves of mean and variance are obtained from numerical evaluation. Furthermore, the spectral curve of the variation coefficient of power spectrum is given through dividing mean by variance. A horizontal soil layer is taken as the example and the dynamic response analysis is performed. The results show that the variability of dynamic response caused simultaneously by the randomness of soil parameters and earthquake motion is much larger than that caused only by individual randomness of either soil parameters or earthquake motion. It is necessary that a due consideration for the variability of soil parameters and the randomness of ground motion should be made for practical engineering probiems.

2 ANALYSING PROCEDURE

2.1 Soil section model and governing equation

For simplicity, a horizontal soil foundation is selected as the example, and a vertical trip of unit area of soil is taken and divided into N layers shown in Fig. 1 by using one-dimensional lumped-mass model. Under the excitation of the horizontal earthquake motion, $\ddot{x}_g(t)$, the soil layers take place the horizontal shear vibration, and the governing equation in time domain is expressed as

$$M\ddot{X}+D\dot{X}+KX=-MJ_x\ddot{x}_g(t) \quad (1)$$

in which M is a diagonal mass matrix, K and D are the tri-diagonal stiffness and damping matrixes respectively, X(t) is the horizontal displacement vector relative to the base, and J_x is a horizontal load marking vector. By means of Fourier transform technique, the governing equation in frequency domain can be obtained

$$HY(\omega)=-MJ_x\ddot{y}_g(\omega) \quad (2)$$

in which $H = K - \omega^2M + i\omega D$ is a frequency response function, $Y(\omega)$ and $X(t)$ is a Fourier transform pair, so is $\ddot{y}_g(\omega)$ and $\ddot{x}_g(t)$. Obviously, Eq. (2) is formally equivalent to a static equilibrium equation. It is very convenience to apply the random finite element method to slove Eq. (2) if the matrix H is a stochastic matrix.

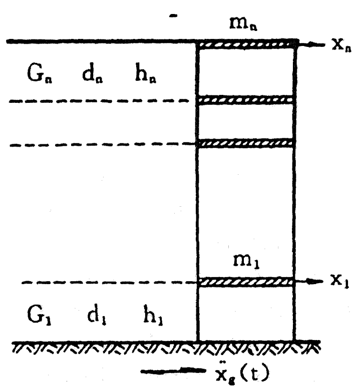

Fig. 1 Simplified Model of Soil Layers

2.2 Ground motion and soil parameter model

A standard stationary filtered white noise Gaussian process, which is widely used in earthquake engineering, is introduced to simulate the stochastic seismic ground motion. The stationary power spectrum can be expressed by Kanai-Tajimi spectrum as

$$S_{\ddot{x}_g}(\omega)=\frac{[1+4\xi_g^2(\omega/\omega_g)^2]S_0}{\{[1-(\omega/\omega_g)^2]^2+4\xi_g^2(\omega/\omega_g)^2\}} \quad (3)$$

where ω_g and ξ_g are the shape parameters and S_0 is a strength parameter. The duration of stationary motion is $[0, \tau]$.

In order to simulate the randomness of soil parameters, for arbitrary j th soil layer, let the mass density, dynamic shear modulus and damping factor be equal to

$$\left.\begin{aligned} m_j&=m_{oj}(1+\Omega_{mj}\alpha_j)\\ G_j&=G_{oj}(1+\Omega_{Gj}\alpha_j)\\ d_j&=d_{oj}(1+\Omega_{dj}\alpha_j)\end{aligned}\right\} \quad (4)$$

where m_{0j}, G_{0j}, d_{0j} are the mean values of the mass, stiffness and damping factor of j th soil layer, Ω_{mj}, Ω_{Gj}, Ω_{dj} express the coefficients of variation of mass, stiffness and damping, and α_j is a random variate following standard normal distribution, that is to say, the mean of α_j is zero and variance equals 1.0. Since the elements of matrixes M, K and D are the random variates, M, K and D are the stochastic matrixes, so is matrix H.

2.3 Solving the random finite element equation

According to the probabilistic finite element procedure, the stochastic matrixes in Eq. (1) are expanded about the mean values of α_j via Taylor series, up to second-order

$$\left.\begin{aligned} M&=M_0+\sum_{i=1}^{n}M'_i\alpha_i+\frac{1}{2}\sum_{i,j=1}^{n}M''_{ij}\alpha_i\alpha_j\\ K&=K_0+\sum_{i=1}^{n}K'_i\alpha_i+\frac{1}{2}\sum_{i,j=1}^{n}K''_{ij}\alpha_i\alpha_j\\ D&=D_0+\sum_{i=1}^{n}D'_i\alpha_i+\frac{1}{2}\sum_{i,j=1}^{n}D''_{ij}\alpha_i\alpha_j\end{aligned}\right\} \quad (5)$$

$$Y=Y_0+\sum_{i=1}^{n}Y'_i\alpha_i+\frac{1}{2}\sum_{i,j=1}^{n}Y''_{ij}\alpha_i\alpha_j \quad (6)$$

It can be noted that since the mixed partial derivatives of M, K, D with respect to α_i and α_j are equal to zero, the calculations will be greatly simplified. Substituting Eqs. (5) and (6) into Eq. (1) and equating equal order terms, the zeroth—, first— and second—order equations corresponding Eq. (2) can be written as

$$H_0Y_0=-M_0J_x\ddot{y}_g(\omega) \quad (7)$$

$$H_0Y'_i+H'_iY_0=-M'_iJ_x\ddot{y}_g(\omega) \quad (8)$$

$$H_0Y''_{ij}+2H'_iY'_j+2H'_jY'_i(1-\delta_{ij})=0 \quad (9)$$

in which δ_{ij} is Krnecker δ operator. It is also interesting to observe that the solutions for Eqs. (7)—(9) only need to inverse matrix H_0 besides the multiplications of the matrixes.

From Eqs. (7)—(9), $Y_0(\omega)$, $Y'_i(\omega)$ and $Y''_{ij}(\omega)$ are derived respectively

$$Y_0(\omega)=-H_0^{-1}MJ_x\ddot{y}_g(\omega)=A_0\ddot{y}_g(\omega) \quad (10)$$

$$Y'_i(\omega)=-H_0^{-1}[H'_iA_0+M'_iJ_x]\ddot{y}_g(\omega) =B_i\ddot{y}_g(\omega) \quad i=1,2,\cdots,n \quad (11)$$

$$Y''_{ij}(\omega)=-H_0^{-1}[2H'_iB_j+2H'_jB_i(1-\delta_{ij})]\ddot{y}_g(\omega) =2C_{ij}\ddot{y}_g(\omega) \quad i,j=1,2,\cdots,n \quad (12)$$

And then, substituting Eqs. (10), (11) and (12) into Eq. (6), $Y(\omega)$ is

$$Y(\omega)=[A_0+\sum_{i=1}^{n}B_i\alpha_i+\sum_{i,j=1}^{n}C_{ij}\alpha_i\alpha_j]\ddot{y}_g(\omega) \quad (13)$$

2.4 Analysis of random vibration

Considering the high damping property of the soil layers excited by strong earthquake, the responses of soil layers will also reach stationary period in a short duration when the input motion is stationary. According to the definition of power spectrum, the stationary power spectrum matrix of response $X(t)$ can be written directly

$$S_x(\omega)=[A_0+\sum_{i=1}^{n}B_i\alpha_i+\sum_{i,j=1}^{n}C_{ij}\alpha_i\alpha_j] \cdot [A_0^*+\sum_{k=1}^{n}B_k^*+\sum_{k,l=1}^{n}C_{kl}^*\alpha_k\alpha_l]^T S_{\ddot{x}_g}(\omega) \quad (14)$$

in which A_0^* is the conjugate matrix of A_0 and A_0^T is the transformation matrix of A_0.

Expanding Eq. (14), $S_x(\omega)$ becomes

$$S_x(\omega)=A_0A_0^{*T}+\sum_{i=1}^{n}(A_0B_i^{*T}+B_iA_0^{*T})\alpha_i +\sum_{i,j=1}^{n}(B_iB_j^{*T}+A_0C_{ij}^{*T}+C_{ij}A_0^{*T})\alpha_i\alpha_j +\sum_{i,j=1}^{n}\sum_{k=1}^{n}(B_kC_{ij}^{*T}+C_{ij}B_k^{*T})\alpha_i\alpha_j\alpha_k +\sum_{i,j=1}^{n}\sum_{k,l=1}^{n}C_{ij}C_{kl}^{*T}\alpha_i\alpha_j\alpha_k\alpha_l \quad (15)$$

It can be noted that the response power spectrum matrix is also a random matrix due to the variability of soil parameters. Taking the mathematical expection for Eq. (15) and considering the property of N(0, 1) distribution of α_j, the mean value of power spectrum is

$$E[S_x(\omega)]=AA_0^{*T}+\sum_{i,j=1}^{n}(B_iB_j^{*T}+A_0C_{ij}^{*T}+C_{ij}A_0^{*T})E[\alpha_i\alpha_j] +\sum_{i,j=1}^{n}\sum_{k,l=1}^{n}C_{ij}C_{kl}^{*T}E[\alpha_i\alpha_j\alpha_k\alpha_l] \quad (16)$$

Similarly, the variance of power spectrum is

$$V[S_x(\omega)]=E[S_x(\omega)-E[S_x(\omega)]] \cdot [S_x^*(\omega)-E[S_x^*(\omega)]]^T \quad (17)$$

Obviously, the calculations in Eqs. (16) and (17) will be involved with the correlation information of random variates α_i, α_j, α_k, α_l. Up to now, it is a very difficult task to determine the correlation coefficients among the different dynamic parameters in soil layers. From initial statistical analysis, it has been found that the correlations among the parameters are not strong when the distances among the soil elements go beyond 3 or 5 meters. Therefore, for simplicity, the correlations of dynamic parameters are neglected in this paper. That is to say, $E[\alpha_i\alpha_j]=0$ when $i\neq j$. Then, Eq. (16) is simplified as

$$E[S_x(\omega)]=AA_0^{*T}+\sum_{i=1}^{n}(B_iB_i^{*T}+A_0C_{ii}^{*T}+C_{ii}A_0^{*T}) +3\sum_{i=1}^{n}C_{ii}C_{ii}^{*T} \quad (18)$$

And correspondingly, Eq. (17) is also simplified greatly.

The following procedures are employed in practice computations. First, $S_{\ddot{x}_g}(\omega)$ is scattered at a series of frequency points. For each discrete point ω_s, complete the calculations of Eqs. (17), (18) and obtain the mean value and variance of power spectrum. Then, the variance of relative displacement of i th soil layer can be expressed as

$$\sigma_{xi}^2=\int_0^\infty S_x^{(i)}(\omega)d\omega \approx \sum_{s=1}^{p}[S_x^{(i)}(\omega_s)+S_x^{(i)}(\omega_{s+1})]\Delta\omega_s/2 \quad (19)$$

in which $S_x^{(i)}(\omega)$ is the i th diagonal element of matrix $S_x(\omega)$ and p is the total number of discrete frequency points. Again, taking the mathematucal expection and variance for above equation, the mean value and variance of response variance are as follows

$$E[\sigma_{xi}^2]=\sum_{s=1}^{p}\{E[S_x^{(i)}(\omega_s)] +E[S_x^{(i)}(\omega_{s+1})]\}\Delta\omega_s/2 \quad (20)$$

$$V[\sigma_{xi}^2]=E[\sigma_{xi}^2-E[\sigma_{xi}^2]]^2 \quad (21)$$

Furthermore, in order to evaluate the variability degree of dynamic response, the coefficient of variation of the response variance of relative displacementis also introduced as

$$\Omega_{\sigma_{xi}^2}=\sqrt{V[\sigma_{xi}^2]}\Big/E[\sigma_{xi}^2] \tag{22}$$

Other responses, such as shear strain, shear stress, absolute acceleration, etc, can also be transformed conveniently from the X(t).

2.5 Equivalent linearization iteration

The computations presented above are only the random vibration analyses for linear system with random dynamic parameters. Pactically, the dynamic shear modulus and damping ratio of soils are intensively dependent on the cyclic shear strain amplitude, γ_e. In this paper, the hyperbolic model presented by Hardin and Drnevich (1972) is utilized to describle the dynamic nonlinear property of soils, i. e. , the relationships among shear modulus G, damping ratio ξ, cyclic shear strain amplitude γ_e are expressed as

$$\left.\begin{aligned} G=G(\gamma_e)=\frac{G_m}{1+|\gamma_e/\gamma_R|} \\ \xi=\xi(\gamma_e)=\frac{|\gamma_e/\gamma_R|\xi_m}{1+|\gamma_e/\gamma_R|} \end{aligned}\right\} \tag{23}$$

in which, γ_R denotes the reference shear strain, ξ_m is

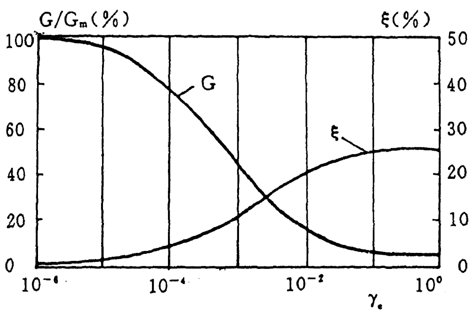

Fig. 2(a) Mean Curves of $G\sim\gamma_e$ and $\xi\sim\gamma_e$

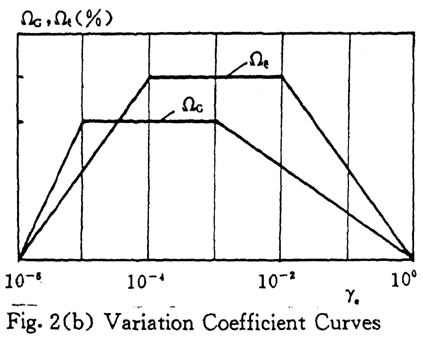

Fig. 2(b) Variation Coefficient Curves

the maximum damping ratio, G_m represents the maximum shear modulus which is related to the average effective surrounding pressure σ'_m. The relationship between G_m and σ'_m is selected as follow

$$G_m=69.9k_m(\sigma'_m)^{1/2} \quad (\text{ton/m}^2) \tag{24}$$

where k_m is known as the maximum shear modulus coefficient.

In this paper, Eq. (23) is used to express the mean curves of $G\sim\gamma_e$ and $\xi\sim\gamma_e$. Furthermore, in order to simulate the different variation levels of G and ξ under the different cyclic shear strain amplitude γ_e, the curves of $\Omega_G\sim\gamma_e$ and $\Omega_\xi\sim\gamma_e$ are introduced in Fig. 2, which are initially assumed as the piece-wise linear functions.

A modified equivalent linearization approach (Wu 1990) is applied to obtain the strain compatible results. In the approach, the equivalent amplitude of the cyclic shear strain, γ_e, is taken as

$$\gamma_e=\sqrt{\pi E[\sigma_\gamma^2]/2} \tag{25}$$

in which $E[\sigma_\gamma^2]$ is the mean value of the variance of shear strain response.

The following steps are employed in iteration procedures. At the begining, a group of initial mean values and variation coefficients of the shear moduli and damping ratioes are assumed for each soil layer. Then, the linear system are formed and the stochastic vibration analyses for the linear system with random parameters are conducted. The mean value of the variance of shear strain response, $E[\sigma_\gamma^2]$, can be calculated easily for each soil layer. Through Eq. (25), Eq. (24) and Fig. 2, the new mean values and variation coefficients of the shear modulus and damping ratio are obtained. The initial values are replaced by the new ones and the random responses of soil layers are computed again. This iteration procedure is carried out repeatly until the strain compatible results, including modulus, damping ratio, acceleration, shear stress, etc, are obtained. These results are taken as the final response values and the computations are completed.

3 APPLICATION

A homogeneous soil foundation with 28 meters in depth is chosen to verity the effectiveness of the method proposed here. The unit weight of soil is 17 kN/m^3. The soil section is divided into 7 layers and the thickness of each layer is 4 meters. According to the statistical investigation for general middle — hard site, the Kanai — Tajimi spectral parameters in Eq. (3) are taken as $\omega_g = 17.9$ (rad/s), $\xi_g = 0.45$ and $S_0 = 0.0023 m^2/s^3$, which repersents the average maximum earthquake acceleration $\bar{a}_m = 1.0 m/s^2$. The nonlinear characteristic parameters of soils in Eq. (23) and Eq. (24) are selected as $\gamma_R = 3 \times 10^4$, $\xi_m = 0.3$ and $k_m = 65$. For each layer the variation coefficient of mass is taken as $\Omega_m = 5\%$, and in Fig. 2 the maximum variation coefficients of shear modulus and damping ratio are chosen as $\Omega_G = 15\%$ and $\Omega_\xi = 30\%$ respectively.

After determining above parameters the random finite element and random vibration analyses are conducted for this site. First, Fig. 3 shows the mean and standard deviation curves of response power spectrum of shear strain at the elevation −16m under the ground surface. By all appearances, if there is not the variability of soil parameters, only one curve can be obtained, that is $E[S_\gamma(\omega)]$. When considering the variability, however, not only the mean curve but also variance or standard deviation curve can be obtained simultaneously. Also it can be seen from Fig. 3 that the variability of dynamic response reaches relative high level and the variation coefficient at the peak goes beyond 80%, that is to say, the response power spectrum is very discrete due to the variability of soil parameters. The spectrum curves are very smooth and the dominant frequency is about 8.2 (rad/s).

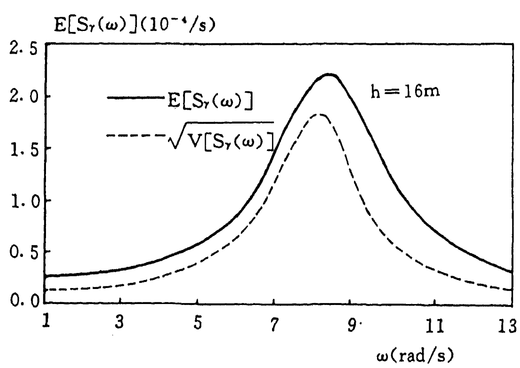

Fig. 3 Mean and Standard Deviation Curves of Power Spectrum

Then, the distribution curves of mean, standard deviation and variation coefficient of shesr strain response are given in Fig. 4, in which RR represents that the input earthquake motion is Random and the soil parameters are also Random, RD represents that the earthquake is Random but the soil parameters are Deterministic, and the shaded part represents the area of mean plus and minus standard deviation. It can be easily found that the influence of the variability of soil parameters on the mean curve is not strong, but the variability of dynamic response caused by the variability of soil parameters is very intensive. The variation coefficient is about 35% at the base and goes beyond 70% at the ground surface, although the maximum variation coefficient of soil parameters is only 30%. The shaded part shows that the discrete band is relative wide.

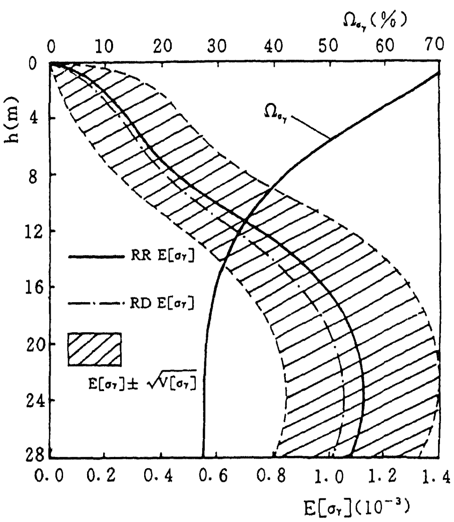

Fig. 4 Distribution of Shear Strain Respouses

Finally, Fig. 5 gives the distribution curves of mean values of dynamic shear stress and modulus. It can be noted that the differences of $E[G]$ between RR and RD is bigger than those of $E[\sigma_\tau]$. Because the $E[\sigma_\gamma]$ obtained from RD is smaller than that from RR, which can be found out from Fig. 4, according to Eq. (23), the $E[G]$ from RD is larger than that from RR. However, the dynamic shear stress is the product of shear strain and modulus. Therefore the differences of $E[\sigma_\tau]$ between RR and RD is not big.

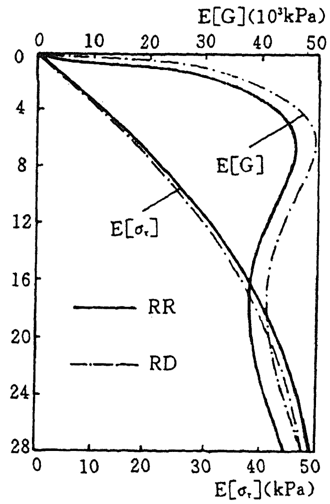

Fig. 5 Distribution of Shear Stress and Modulus

It is not difficult to compute other response statisticses or failure probability. But limlted by the paper length, these results will not be discussed here.

4 CONCLUSIONS

The aseismic design of earth structures is often involved with a series of indeterminate factors. The designers will pay more and more attention to the indeterminate factors with the transformation of structural design from safety coefficient method into reliability method. The physical and mechanical properties of soils, taken as a kind of natural and special construction materials, show intensive randomness. In this paper, the stochastic properties of soil parameters and earthquake ground motion are simultaneously taken into consideration in evaluating the seismic responses of soil horizons, and the influences of the variability of soil parameters on the dynamic responaes are investigated with emphasis. The analysis results for a horizontal soil layers show that the variability of dynamic responses goes remarkably beyond the variability of input soil parameters. It is very necessary to consider this kind of influences for evaluating the seismic responses and dynamic reliability of earth structures.

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