

# Influence of random mechanical parameter on earthquake response analysis of site

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**ABSTRACT:** In this paper, a Monte Carlo simulation technique is introduced to compute the site earthquake response, in which the mechanical parameters of layered soil are taken as random variables. A method to estimate the statistical data of the soil shear wave velocity and a convergence criterion are given for imitation computation. According to a number of computations, it can be affirmed that the influence of the random parameter variability of soil layers is another important reason causing the earthquake response spectra being serious variance. The influence of random seismic inputs is almost as large as that of the variance of shear wave velocity.

## 1 INTRODUCTION

There are many kinds of random uncertainties caused by subjective or objective reasons in the analysis of site earthquake response. The random uncertain factors which are included in the computation of site earthquake response can be generally classified into three major categories: (1) The uncertainty of earthquake input which is caused by earthquake source mechanism, transmission path and others, is usually expressed in seismic parameters. (2) The uncertainty in computing model, for example, the computation of an engineering site, the linear elastic model or viscoelastoplastic model can be used, and the computing method can be the imitation of lumped mass or the solution of vibration equations, and so forth. (3) The uncertainty of site itself, which includes two parts: the distribution of soil layers and the mechanical parameters of soil layers.

At present, the earthquake inputting parameters can be assessed in probability by means of the seismic risk analysis (1990). And on the difference of variant computation methods a lot of works have been done and the appropriate computing model can be essentially chosen in the main. About the uncertainty of site itself, however, has not made many exploration. Recent years, a new method—probabilistic finite element method is introduced (1986). It is suitable for that the coefficient of variant (COV) of parameters is less than 20 percent. The soil parameters, in practical engineering site, disperse in a great range. Figure 1 shows the shear wave velocity (SWV) at different depth a kind of soil measured in china, which the COV is about 36 percent. And it is frequent that the COV of the SWV is about 30 to 40 percent. On the condition of this COV, the site earthquake response is quite discrete (fig. 2).

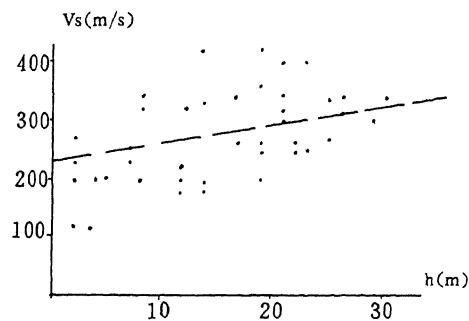


Figure 1. Shear wave velocity at different depth

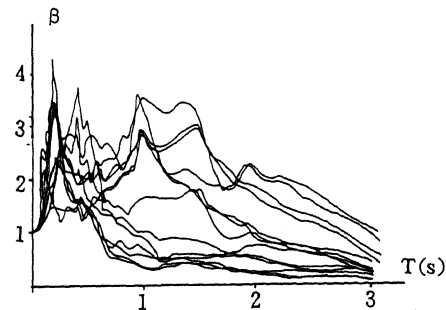


Figure 2. Ground response spectra with variant soil parameters

Based on the discussion above, the objective of this paper is concentrated on to examine the effect of random soil parameters variability on the earthquake response of site. A Monte Carlo simulation technique is introduced to imitate the random parameters of soil. A Bayes method is

put forward to estimate the SWV where the data measured are in short. A convergence criterion of mean response spectra is given. Some interesting conclusions on the mean response spectra computed from a lot of samples are discussed.

## 2 COMPUTING METHOD

In practical engineering, the site is usually layered and nearly horizontal, so that the layered soil is assumed as level and spread and the bedrock is assumed as half-space (fig. 3). The ground motion of this kind of site, general say, is made by shear waves transmitted from bedrock. The one dimension shear wave model

No.	Thickness	Coodinates	Parameters
1	$h_1$	$z_1 \quad u_1$	$V_{s1}, \rho_1, \eta_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	$h_j$	$z_j \quad u_j$	$V_{sj}, \rho_j, \eta_j$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$h_n$	$z_n \quad u_n$	$V_{sn}, \rho_n, \eta_n$
	$\infty$		

Figure 3. Computing model of soil

can be used. Each soil layer is assumed as homogeneous and isotropic viscoelasticity. Linearisation method is introduced to consider the nonlinearity of soil. In this case, the layer j with the shear stiffness  $G_j$ , damping ratio  $\eta_j$  and mass  $\rho_j$  is satisfied with the vibration equation:

$$\rho_j \frac{\partial^2 u_j}{\partial t^2} = G_j \frac{\partial^2 u_j}{\partial z_j^2} + \eta_j \frac{\partial^3 u_j}{\partial z_j^2 \partial t} \quad (1)$$

The shear stiffness  $G_j$  is obtained from SWV of soil layer j,  $G_j = (V_{s_j})^2 \rho_j$ . From the solution of equation (1), the displacement of layer j is expressed as follows:

$$u_j(z_j, t) = E_j e^{i(k_j z_j + \omega t)} + F_j e^{-i(k_j z_j - \omega t)} \quad (2)$$

Here  $\omega$  is radian frequency.

According to the boundary conditions, the relations of amplitude layer j to ground and accelerate, or velocity of layer j to ground or layer k can be obtained as well. Therefore, the earthquake response of every soil layer can be computed.

For computing the earthquake response of site, a Monte Carlo simulation technique is used to imitate the random variability of SWV. The procedures of computation are as follows:

1. According to the geological age, feature and distribution of soil layer, seismic engineering geological unites are zoned. The statistic parameters of the SWV are esti-

ated by the data collected and measured. Based on these parameters, a series of samples of SWV are generated randomly by computer.

2. By means of one dimension shear wave model and linearisation method, the earthquake response spectra are computed iteratively with the samples. And hence the mean response spectra are computed.

3. The results obtained are checked, if they are converged then outputting, otherwise giving another set of samples and competing again until reaching the convergence criterion.

From the computing steps mentioned above, two problems have to be solved before the computation.

### 2.1 Estimating the statistic parameters of soil

It will be very expensive to get the SWV by means of a lot of drilling holes and measuring in site. It is usual, we all know, that there are much data about site which can be get from the engineering geologic investigation measured before, though the data may not give the value of SWV directly. The data give us a lot of the information about SWV, from which the experience and knowledge about SWV can be got. By means of Bayes method, the information combined with some other data measured in site can give more accurate and overall feature of site. The process of computation are given below.

1. Based on the available data of soil layers, the priori distribution  $h(\theta)$  can be got (Here  $\theta$  is the parameter of SWV. It may be, in general, the mean or mean square deviation of the parameter distribution).

2. According to the available data and the situation of site, supplementing drilling and measuring are made. The values of shear wave velocity measured  $X = (X_1, X_2, \dots, X_n)$  are described in a kind of distribution  $P(X|\theta)$  or  $P(X_1, X_2, \dots, X_n|\theta)$ , with the parameter  $\theta$ .

3. In this way, the posterior distribution of SWV which combines the priori distribution  $h(\theta)$  from empirical estimate with the values measured or observed. The renew estimation to the parameter  $\theta$ —the posterior distribution  $h(\theta|X)$  is given in equation:

$$\begin{aligned} h(\theta|X) &= \frac{P(X_1, X_2, \dots, X_n|\theta)h(\theta)}{\int_{-\infty}^{\infty} P(X_1, X_2, \dots, X_n|\theta)h(\theta)d\theta} \\ &= KP(x|\theta)h(\theta) \end{aligned} \quad (3)$$

Here  $K$  is a coefficient which is unrelated to the parameter  $\theta$ ;

$$K = \left[ \int_{-\infty}^{\infty} P(X_1, X_2, \dots, X_n|\theta)h(\theta)d\theta \right]^{-1} \quad (4)$$

This approach makes best use of the existed information about SWV as well as human experience. It gives more accurate statistic parameters of SWV.

## 2.2 The determination of convergence criterion

In general, Monte Carlo simulation takes much time in digit computations. The purpose of earthquake response computation is to give average response spectra of site. With the same goal, the digit simulation must take the average response spectra as convergence criterion. Through the comparison of some convergence criteria, it can be sure that the convergence ought to be the values of average response spectrum at every period point (1991). It is meant that the residual values at every period points are not greater than the prescribed error  $\epsilon$ , comparing the average response spectrum computed  $i$ th time  $\beta_i(t)$  with  $i+1$ th time  $\beta_{i+1}(t)$ , i. e.

$$\frac{|\beta_{i+1}(t) - \beta_i(t)|}{\beta_i(t)} \leq \epsilon \quad (5)$$

At the same time, the criterion (5) has to be satisfied strictly several times before stopping computation, as the character of simulation is considered.

## 3 ANALYSIS OF UNCERTAINTY FACTORS

For the purpose of comparing and analysing the influence of various uncertainty factors variability on site earthquake response, three engineering geological sections with different stiffness of soil are chosen, in which the soil layers of every section are eight (Table 1). The SWV of every sections are selected. And two another sections are given (Table 2) for comparison, in which the relation of variable SWV is same as section 2 but different in the distribution of layers.

Table 1. Sample of computing sections

No.	Depth (m)	Soil type	SWV (m/s)		
			Sec. 1	Sec. 2	Sec. 3
1	2.5	Silt clay	101	161	231
2	5.0	Clay	112	172	242
3	10.0	Sandy silt	134	194	264
4	15.0	Clay	156	216	286
5	35.0	Medium sand	244	304	374
6	55.0	Sandy silt	332	392	462
7	70.0	Medium sand	398	450	470
8	80.0	Medium sand	500	500	500

### 3.1 Comparison of different soil stiffness

When the sections with different soil stiffness are computed, the curves of response spectra are shown in figure 4. Here sections 1, 2, 3, are computed. The curves are very discrete. It is in common that the value of SWV is greater or the soil is more stiffness, the difference of response spectra is smaller. This indicates that the harder the soil of site, the smaller the effect of variation in the value of SWV. The magnitude of soil stiffness is one of important factors affecting the earthquake response.

Table 2. Sample of comparing sections

No.	Depth (m)	SWV Sec. 4	Depth (m)	SWV Sec. 5	Depth (m)	SWV Sec. 6
2	7.5	183	12.0	203	5.0	172
3	17.5	227	22.0	247	10.0	194
4	32.5	293	32.0	291	15.0	216
5	42.5	337	42.0	335	20.0	238
6	62.5	425	52.0	379	25.0	260
7	72.5	469	62.0	423	30.0	282
8	80.0	500	80.0	500	35.0	304
9					40.0	326
10					50.0	370
11					60.0	410
12					70.0	458
13					80.0	500

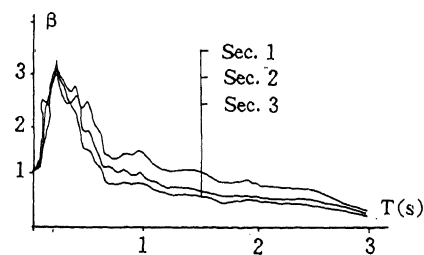


Figure 4. Comparison of different soil stiffness

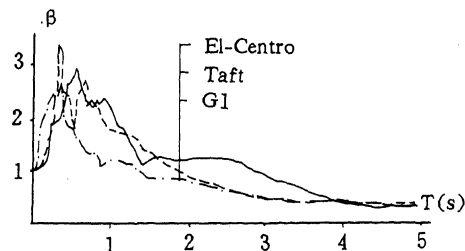


Figure 5. Influence of earthquake inputs variability

### 3.2 The influence of earthquake inputs variability

It is obvious, from figure 5, that the average response spectrum is sensitive to the random variations of earthquake input. Here three typical kinds of earthquake waves are selected: El-Centro, Taft, and G1 (an artificial wave). Figure 5 illustrates that the periods corresponding to the peak point of the spectrum are discrete, and the variations of spectra in middle and long periods are evident too. By contrast, the discrete degree of the later is similar to that of soil with different stiffness in figure 4, but is smaller to that of random variations in figure 2. If the response spectra with random parameter variability shown in figure 2 are taken as the usual spectra computed by the deterministic method. The computations demonstrate that the discreteness of response spectra

caused by the random parameter variability may be more significant comparing with that caused by the variant earthquake input. For a long time, the discreteness of response spectra in same site is only originated from the randomness of the earthquake inputs. This paper demonstrates that another source causing earthquake response spectra discreteness is from the randomness of site parameters.

### 3.3 The influence of discrete range of SWV

Here gives the computations of section 2, in which five COVs 0%, 10%, 20%, 30% and 40% are considered in digit simulations. And the  $\beta$  spectra corresponding to every COV illustrated in figure 6. It can be found obviously that the average response spectra varied with the COV. Within the short period, the variance is not clear. In the middle and long period, however,  $T > 0.6$  for instance, the disparity is greater. Figure 7 gives

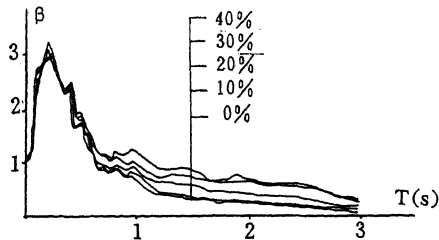


Figure 6. Influence of discrete range of SWV

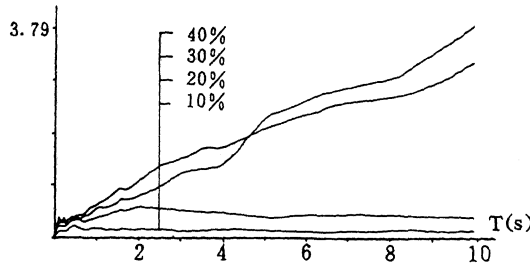


Figure 7. The COVs variation of response spectra

the COV of the response spectra at every period. The COV of response spectra varying with the COV of parameters in regularly, when the COV of the parameters is greater, the most discrete value is about long period. This gives the fact that the long period of response spectrum is sensitive to the parameter variability of soil. It is similar to the computations of rest sections.

### 3.4 The influence of upper and lower soil layers

For comparing the influence of the upper and lower soil

layers, two cases are considered in the computation of section 2. The case 1 is that the random parameter variability only upper soil layers are taken into account and the rest soil layers are considered as determinate parameters. The case 2 is on the other hand, the parameters of upper soil layers (15 meters up) are thought of as determinate, and the rest lower soil layers are considered as random parameters. The computational results demonstrate that the average response spectrum curve of case 1 is close to that of determinate parameters. The curve of case 2, however, is very close to that of all soil are random variable (fig. 8). This means the influence of parameter variability of lower soil on the earthquake response of site is greater than that of upper soil.

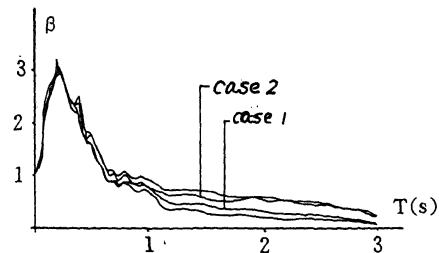


Figure 8. Influence of upper and lower soil layers

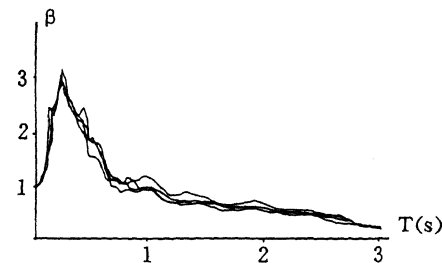


Figure 9. Influence of soil layer distribution variability

### 3.5 The influence of the soil layer distribution variability

The soil layer distribution is usually different in same geological unit, but the statistical relation of SWV is similar. just as the sections given in table 1, the section 2, 4, 5, 6. After computing these four sections, the average response spectra are illustrated in figure 9. It is clear that the curves are not very discrete, though the soil distribution is disparate. Therefore, it can be sure that the effect of layer distribution on the average response spectrum is not significant, especially compared with that of the random parameter variability.

### 3.6 The influence of overburden

In practical engineering site, the bedrock is usually very deep under the ground. It is expensive and unnecessary to compute from the bedrock so depth. It is normal to

take the harder soil layer as the assumed bedrock. For example, the value of SWV greater than 500 m/s is taken as the assumed bedrock in China Cord. This paper five situations are discussed in the computation of section 2. (1) The thickness of overburden is 55m, the SWV of layer 6 is taken 500m/s. (2) The thickness of overburden is 35m, the SWV of layer 5 is taken 500m/s. (3) The upper six layers are computed only, that is the bedrock value of SWV is equal to 450m/s. (4) Upper five layers are computed only, that is the bedrock value of SWV is equal to 392m/s. (5) Eightlayers are computed, that is a layer 15m with SWV 500m/s added to the section 2. The computational results are illustrated in figure 10. Obviously, all of five kinds of situations are not very discrete. Some difference are mainly in middle and long period. In the same times, it can be seen that the effect of the assumed bedrock value of SWV on the average response spectrum is not great. This also indicates primarily that the average response spectra is insensitive to the thickness of overburden and the assumed bedrock value of SWV is not an unchangeable value. However, it is necessary to make further research to this problem.

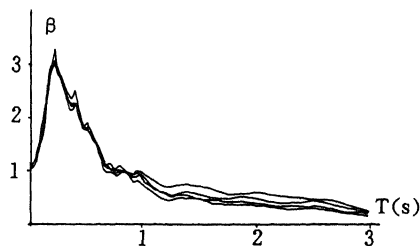


Figure 10. Influence of overburden

#### 4 CONCLUSION

According to various uncertainty factors in engineering site, this paper makes use of Monte Carlo simulation technique to compute earthquake response. Some factors are examined the effect on the average response spectrum. Specific conclusions on individual results presented in preceding sections are given below.

1. The random parameter (SWV) variability in general site have a significant effect on the earthquake response. It is an important reason causing the discreteness of recorded earthquake response spectra.

2. The influence of deeper soil compared with upper soil layer is greater. The earthquake response spectrum is not very sensitive to the variation of the distribution of soil layers.

3. Both the stiffness of soil and the earthquake input have great effect on the output, They are nearly in same magnitude of degree.

4. The determination to assumed bedrock is a problem need to further research. It can be sure that the harder

soil layer taken as assumed bedrock has smaller effect on the computed results.

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