Seismic earth response considering various mechanical models

Gh. Mărmureanu, S. F. Bălan & I. Vasile
Center of Earth Physics, Romania

ABSTRACT: The paper summarizes the results obtained by authors regarding the influence of the soil mechanical modelling on seismic earth massive response. There were used constitutive models (nonlinear viscoelastic, linear viscoelastic, nonlinear elastic and linear elastic model) and finally it was computed the response values, that is, maximum accelerations, fundamental periods, amplification spectrum, acceleration and velocity response spectra.

1 INTRODUCTION

The characteristics of the soil at the site play a significant role in defining both the free field ground motion and the soil behaviour during soil structure interaction. Soils are seldom homogenous and they seldom lie in clearly defined horizontal layers. Selecting a mathematical model (constitutive models) to represent the stress-strain behaviour of the soil is a difficult problem. In general, the stress-strain behaviour of the soil is strongly nonlinear, anisotropic, elasto-plastic, loading-path dependent and also depends on the water constant and possible flow conditions in the soil. The role of the constitutive models is to compress the data into a form which is efficient for computations.

2 NONLINEAR VISCOELASTIC MODEL FOR SOILS

The analytic forms of the constitutive equations of soils can be evaluated by means of experimental data obtained from creep triaxial tests. (Bratocin 1986). From this data by polynomial regression method at each time \( t \), we can obtain \( m \) constitutive relationships in the form (Bratocin 1983):

\[
\tau_j(t) = g_j(t, \gamma) = \sum_{k} g_{kj} (-\gamma)^k
\]

or in integral form:

\[
\tau(t) = \int_{0}^{t} g(\gamma, t - \lambda) \dot{\gamma}(\lambda) d\lambda
\]

where \( \tau \) and \( \gamma \) are the tensor invariants of the stress \( \sigma \) and strain \( \varepsilon \):

\[
\tau = \left\{ \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 \right]^{1/2} + (\sigma_3 - \sigma_1)^2 \right\}^{1/2}
\]

\[
\dot{\gamma} = \left\{ \left[ (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 \right]^{1/2} + (\varepsilon_3 - \varepsilon_1)^2 \right\}^{1/2}
\]

and \( g(\gamma, t) \) is the nonlinear relaxation function:

\[
g(\gamma, t) = \sum_{j} \left[ g_{j^*}^0 + (g_{j^*}^\infty - g_{j^*}^0) \exp(-\beta_j t) \right] (-\gamma)^k
\]

\( g_{j^*}^0, g_{j^*}^\infty \) and \( \beta_j \) are material constants. Function (5) is in compliance with three-relaxation model. This model has sufficient accuracy for practical purposes and satisfies the reduction condition to the nonlinear form (Bratocin 1986). The values of the constants \( g_{j^*} \) (\( i = 0, 1, 2; j = 1, 2, \ldots, m \)) from equation (5) are the values obtained at time \( t_j \) for the corresponding exponential \( G_i(t) \) from eq. (1). For each of these "exponential" \( G_i(t) \) (i=0, 1, 2) we have \( m \) pairs \( (g_{j^*}, t_j) \) formed by the values of the argument \( t = t_j \) and the
values of this function at time $t_1$; $G_i(t_1) \propto g_i$. Statistical analyses of these pairs (logarithmic transformation and linear regression) can give the parameters $g_i, g_i'$ and $\beta_i$ of the non-linear relaxation function $G(\gamma, t)$. If the strain history is in the form:

$$\gamma(t) = \gamma_0 \exp(i\omega t)$$  \hspace{1cm} (6)

where $\gamma_0$ is the strain amplitude and $\omega$ is the excitation frequency and from eq. (2), (5) and (6) and after integration we obtain the dynamic shear constitutive equation in the form (Bratosin 1986):

$$\tau(\omega, t) = \sigma(\gamma_0, i\omega) \gamma(t)$$  \hspace{1cm} (7)

where $\sigma(\gamma', i\omega)$ is the complex modulus function:

$$\sigma(\gamma_0, i\omega) = G_2(\gamma_0, \omega) + i G_1(\gamma_0, \omega)$$  \hspace{1cm} (8)

The real part $G_2(\gamma_0, \omega)$ is the storage modulus function and the imaginary part $G_1(\gamma_0, \omega)$ is the loss modulus function and they have the following forms:

$$G_2(\gamma_0, \omega) = \sum_{i=1}^{Q^2} e_i f(\gamma)^i + \sum_{i}^{Q_1} (e_i - e_i') f(\gamma)^i (9)$$

$$G_1(\gamma_0, \omega) = \sum_{i}^{Q_2} (e_i - e_i') f(\gamma)^i \beta_i$$  \hspace{1cm} (10)

The complex modulus function can have the form:

$$\sigma(\gamma_0, i\omega) = G(\gamma_0, \omega) \exp[i \tan^{-1}G(\gamma_0, \omega)]$$  \hspace{1cm} (11)

where:

$$G(\gamma_0, \omega) = [G(\gamma_0, i\omega)]^{-1} = G_2(\gamma_0, \omega) + G_1(\gamma_0, \omega)$$  \hspace{1cm} (12)

$$D(\gamma_0, \omega) = G_1(\gamma_0, \omega)/G_2(\gamma_0, \omega)$$  \hspace{1cm} (13)

The first function (eq.12) is called dynamic torsional modulus and the second function (13) is called torsional damping function, from equations 9,10,11,12 and 13 we can obtain another forms of the dynamic torsional modulus function and damping functions:

$$G(\gamma_0, \omega) = \sum_{i=1}^{Q_2} g_i(\gamma_0)^i - (14)$$

$$D(\gamma_0, \omega) = \sum_{i}^{Q_1} d_i(\gamma_0)^i - (15)$$

where $g_i$ and $d_i$, $i=0,1,2$, are functions only of frequency $\omega$ and contain only material parameters: $e_i, e_i', \omega$, $i=0,1,2$, which are found from resonant column data by polinomial regression method. On the other hand, the analysis of several resonant column tests shows a major weight of the strain level on modulus and damping values and a minor influence of the frequency values above 1 Hz on frequency values $G_i(\omega)$ and $D_i(\omega), i=0,1,2$. Then from practical point of view, we can consider $G$, and $D$, as constants over the frequency range which is of main interest in the analysis and in the case, the complex modulus function becomes:

$$\sigma(\gamma_0) = G(\gamma_0) \exp[i \tan^{-1}D(\gamma_0)]$$  \hspace{1cm} (16)

or

$$\sigma(\gamma_0) = \sum_{i}^{Q_2} g_i(-\gamma)^i$$  \hspace{1cm} (17)

where $G_i$ are complex constants of form:

$$G = G_1 + i G_1', \hspace{1cm} i=0,1,2$$  \hspace{1cm} (18)

where $G_1$ and $G_1'$ are combinations of the constants $G_1$ and $D_1$, $i=0,1,2$, obtained from resonant column data, (Bratosin 1986):

$$G_o = G_o (1 - 0.5D_1^2) ; \hspace{1cm} G_0'' = G_o D_0$$

$$G_1 = G_1 (1 - 0.5D_1^2) + G_0 D_0 D_1$$

$$G_1' = G_1 D_0 = G_0 D_1$$

$$G_2 = G_2 (1 - 0.5D_1^2) + G_0 D_0 D_1 - G_o (0.5D_1^2 + D_0 D_2)$$

$$G_2' = G_2 D_0 - G_1 D_1 + G_0 D_2$$

The data from resonant column tests are very useful for checking the behaviour of the nonlinear viscoelastic model under dynamic conditions. As it is shown in Fig. 1 a good agreement is obtained between dynamic shear modulus (14) and damping
functions and resonant Hardin and Drnevich column data from our laboratory for diluvial clay from layer number two (Fig. 2). The same data are obtained for layers number 1,3 and 4.

3 THE INFLUENCE OF MODELLING

The theory considers the responses associated with vertical propagation of shear waves through the linear viscoelastic ($G_i=$constant and $D_i=$constant) or linear elastic ($G_i=$constant and $D_i=0$) system.

The system (Fig. 2) consists of 4 horizontal layers extended to infinity in the horizontal direction and has an elastic half-space as the bottom layer. Each layer is homogeneous and isotropic and is characterized by the thickness $h$, mass density $\rho$, dynamic torsional modulus function $G_i=\gamma(\gamma)$ and torsional damping function $D_i=D_0(\omega)$. If $G_i=G_0(\gamma)$ and $D_i=0$ then we have a nonlinear elastic model. The nonlinearity of the shear modulus and damping ratio is accounted for by the use of equivalent linear soil properties using an iterative procedure to obtain values for modulus and damping compatible with the effective strains in each layer.

The input base motion is the same in all cases and the maximum acceleration at the base rock is 0.041 g. The response values, that is, maximum accelerations, fundamental periods, amplification spectrum, acceleration and velocity spectra (0% and 5% damping values) are given in Tables 1 to 4 and Figures 3 and 4.

All values were obtained by using AMPLOC (St. Bâlân 1987) and SHAKE (Schmabel 1972) programs which are based on the continuous solution to the wave equation adapted for use with transient motions through the Fast Fourier Transform algorithms.

4 CONCLUSIONS

The analytic forms of the constitutive equations of soils can be evaluated by means of experimental data. The resonant column sample is subject to harmonic longitudinal and torsional vibrations and for a given excitation level it can give
one value of the dynamic shear modulus function (\(G\)) and one value of the damping function (\(D\)). The dynamic torsional modulus function and damping function become polynomials in terms of strain and can be entirely determined from resonant column data by polynomial regression method. An example is shown in Fig. 1.

Vertically propagating shear waves show that different constitutive models for the same site and the same seismic input lead to different results. The difference between a nonlinear viscoelastic model and a linear elastic one is often as great as a factor of 1.32 in peak surface acceleration (Table 1), of 1.7 in maximum acceleration spectra (Table 2), of 2.7 and 1.13 in maximum absolute acceleration spectra of damping 0% respectively, 5%. There are some shifts in the fundamental periods of the site due to softening of soil layers (Fig. 2 and 3, Table 2).

While these effects may be relatively unimportant in the special case of vertically propagating shear waves in the free field, they may be significant when more ground shaking is considered and are definitely important to the complex stress states associated with seismic soil structure interaction.

REFERENCES


Bratosin D., 1986, Mechanical models for dynamic behaviour of soils.
Table 1

<table>
<thead>
<tr>
<th>Level (m)</th>
<th>Nonlinear viscoelastic model</th>
<th>Linear viscoelastic model</th>
<th>Nonlinear elastic model</th>
<th>Linear elastic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 m</td>
<td>0.20695 g</td>
<td>0.2310 g</td>
<td>0.2525 g</td>
<td>0.2710 g</td>
</tr>
<tr>
<td>-14.0 m</td>
<td>0.08904 g</td>
<td>0.1191 g</td>
<td>0.1150 g</td>
<td>0.1402 g</td>
</tr>
<tr>
<td>-38.0 m</td>
<td>0.08761 g</td>
<td>0.0742 g</td>
<td>0.1025 g</td>
<td>0.0905 g</td>
</tr>
<tr>
<td>-48.0 m</td>
<td>0.08284 g</td>
<td>0.0673 g</td>
<td>0.0941 g</td>
<td>0.0772 g</td>
</tr>
</tbody>
</table>

Amplification Spectrum, $|H(i\omega)|/10$

| $|H(i\omega)|/10$ | 3.55 | 4.72 | 5.40 | 6.93 |

Period, $T$

| $T$ | 0.206 s | 0.201 s | 0.252 s | 0.201 s |

Table 2

Maximum Absolute Acceleration Spectra (in g) at $T=0.2$ s: TABLE 3

| D=0% | 4.295 g  | 7.880 g  | 4.550 g  | 11.770 g |
| D=5% | 0.980 g  | 0.905 g  | 1.289 g  | 1.107 g  |

Maximum Relative Velocity Spectra (m/s) at $T=0.2$ s: TABLE 4

| D=0% | 1.330 m/s | 2.45 m/s | 1.440 m/s | 3.650 m/s |
| D=5% | 0.306 m/s | 0.298 m/s | 0.374 m/s | 0.359 m/s |

Fig. 3
Fig. 4

Preprint EP-30 ICEPIZ, Buch. R.
Schnabel Per B., J. Lysmer, H. Bolton Seed, SHAKE a computer program for earthquake response analysis of horizontally layered sites. Report No. EERC 72-12, University of California.