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The extended quasi-three-dimensional ground model

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ABSTRACT: The extended quasi-three-dimensional ground model (EXQ3D model) is presented in this paper, for the analysis of seismic responses of irregularly bounded 3-D surface ground. The quasi-three-dimensional ground model (Q3D model) proposed by Tamura and Suzuki is extended to EXQ3D model so that the new model can take fundamental through arbitrary n-th mode of shear vibrations into consideration. The formulation of the model is described first. Then, earthquake response analyses of a 3-D model ground are carried out using both 3-D finite element method and the proposed model. The validity of the model is verified through the comparison of the results of analyses obtained using both models. It is pointed out, finally, that EXQ3D model is effective in evaluating the effect of surface geology on seismic motions and seismic zonation.

1 INTRODUCTION

Recent large earthquakes, for example, the Mexico Earthquake of 1985 (Suzuki 1986), the Loma Prieta Earthquake of 1989 (Seed et al. 1990) and the Luzon Earthquake of 1990 (Tamura et al. 1990), posed us important lessons suggesting that there exists a close relationship between severe damages and surface geology. There are a lot of attenuation equations and methods to estimate response spectra to evaluate the effects of local soil and geological condition (Trifunac 1989). Based on these measures, much efforts have been taken for seismic zonation. In case that a specific superstructure or an underground tunnel is planned, the prediction of the seismic ground motions due to large earthquakes is necessary to evaluate the earthquake resistance of the structure. In this aspect, much attention has been concentrated on how to evaluate the effect of surface geology on seismic motions and vaious approaches have been proposed to model surface ground.

The quasi-three-dimensional ground model, Q3D model, was proposed (Tamura & Suzuki 1987) to analyze horizontal ground displacement responses of 3-D surface ground as the input data for the earthquake response analysis of an underground tunnel. Q3D model only takes fundamental shear vibration of surface ground into consideration, which is considered to be valid on the ground where the fundamental shear vibration mode of the surface layer is predominant, because the vibration higher than fundamental one does not affect ground displacement response so much especially at the shallow part of the layer. This model was applied to simulate the earthquake response of a shield-driven tunnel in soft ground at the site where the earthquake observation was conducted (Suzuki 1988).

Since Q3D model treats the effect of irregular boundaries of 3-D surface ground, its extention to the model, which can deal with fundamental through arbitrary order of shear vibration modes which are predominant at the site, enables us to simulate not only ground displacement response but also ground acceleration and velocity response of 3-D ground during earthquakes. Thus, the extended model, EXQ3D model, will play an important role in seismic zonation or microzonation. This paper describes the formulation of EXQ3D model first. Then, the validity of the proposed model is shown based on the comparison of the results of earthquake response analyses using this model and 3-D finite element approach.

2 FORMULATION OF EXQ3D MODEL

The formulation for a linear analysis using EXQ3D model is described in this chapter. The surface ground is divided into a number of soil columns in the first step. Each column is replaced by a set of N coupled spring-mass systems or two degree of freedom systems, each of which corresponds with each of the shear vibration mode of the column. Fig.1 illustrates concretely how to model the soil column, replacing it by spring-mass-damper system. The mass of the n-th mode of vibration system $M^{(n)}$ is given by the following equation:

$$M_i^{(n)} = \frac{\left(\int_0^{H_i} m_i(z) f_{i,n}(z) dz\right)^2}{\int_0^{H_i} m_i(z) f_{i,n}^2(z) dz} \quad (1)$$

in which: $M_i^{(n)}$ denotes the effective mass of soil column i on n-th shear vibration mode; $m_i(z)$ mass of soil column i at depth z; $f_{i,n}(z)$ normalized modal vector on n-th shear vibration mode; H_i thickness of soil column i; and z depth below ground surface.

The spring constants and the constants of viscous dampers of n-th shear vibration system are, then, determined using the circular frequency of n-th mode of the soil column as:

$$K_{3x,i}^{(n)} = K_{3y,i}^{(n)} = \omega_{i,n}^2 M_i^{(n)} \quad (2)$$

$$C_{3x,i}^{(n)} = C_{3y,i}^{(n)} = \frac{2D_{i,n}}{\omega_{i,n}} K_{3x,i}^{(n)} = \frac{2D_{i,n}}{\omega_{i,n}} K_{3y,i}^{(n)} \quad (3)$$

in which: $K_{3x,i}^{(n)}$ and $K_{3y,i}^{(n)}$ denote shear spring constants connecting mass $M_i^{(n)}$ with the base ground in the horizontal x and y direction, respectively; $C_{3x,i}^{(n)}$ and $C_{3y,i}^{(n)}$ constants of viscous dampers connecting mass $M_i^{(n)}$ with the base ground in the horizontal x and y direction, respectively; $\omega_{i,n}$ circular frequency of n-th mode; and $D_{i,n}$ damping factor of soil colum i on n-th mode of shear vibration.

$D_{i,n}$ is the weighted mean calculated by the damping factor of soil column i, $h_{i,n}(z)$, and the displacement function, $F_{i,n}(z)$, given as follows:

$$D_{i,n} = \frac{\int_0^{H_i} h_{i,n}(z) \cdot |F_{i,n}(z)| dz}{\int_0^{H_i} |F_{i,n}(z)| dz} \quad (4)$$

$$F_{i,n}(z) = \beta_{i,n} f_{i,n}(z) \quad (5)$$

in which, $\beta_{i,n}$ denotes the participation factor of soil column i on n-th mode of shear vibration.

Each two degree of freedom system composed of a mass, 2 springs and 2 dampers is connected with each other by finite plate elements with unit thickness as illustrated in Fig.2. Fig.2(b) is the schematic representation of the n-th mode of shear vibration system when the 3-D surface ground illustrated as Fig.2(a) is replaced by EXQ3D model. The elastic modulus of plate element J, $E_{J,n}$, is given by the following equation, when the element connects 4 masses with a rectangular shape.

$$E_{J,n} = \frac{1}{4} \sum_{i=1}^{4} R_{i,n} \quad (6)$$

$$R_{i,n} = \int_0^{H_i} E_i(z) \cdot |F_{i,n}(z)| dz \quad (7)$$

In which: $E_i(z)$ denotes the elastic modulus of soil column i at depth z below surface; and $R_{i,n}$ denotes the equivalent rigidity of the nodal point i in consideration of n-th shear vibration mode, and can be defined by integrating the product of $E_i(z)$ and the absolute form of $F_{i,n}(z)$ over the thickness of the layer at nodal point i. Another parameter, which is necessary to form two dimensional elasticity matrix represented as D, is Poisson's ratio of plate element J, $\nu_{J,n}$, defined as follows:

$$\nu_{J,n} = \frac{1}{4} \sum_{i=1}^{4} \nu_{i,n} \quad (8)$$

$$\nu_{i,n} = \frac{\int_0^{H_i} \nu_i(z) \cdot |F_{i,n}(z)| dz}{\int_0^{H_i} |F_{i,n}(z)| dz} \quad (9)$$

in which: $\nu_i(z)$ represents the Poisson's ratio of the layer at nodal point i; and $V_{i,n}$ is Poisson's ratio at nodal point i in n-th mode of shear vibration system, and is given as the weighted mean obtained using the same manner as that used in equation (4). Thus, the element stiffness matrix of plate element J, $[K_2]_J^{(n)}$, can be given, using finite element procedure for a plate element with unit thickness under plane-stress condition:

$$[K_2]_J^{(n)} = \int_{vol} B^T D B dv \quad (10)$$

in which, B and D denote strain matrix and two-dimensional elasticity matirix respectively under plane-stress condition. The damping matrix of plate element J in n-th mode of shear vibration system is, therefore, given by the following equation:

$$[C_2]_J^{(n)} = \frac{2D_{J,n}}{\omega_{J,n}} [K_2]_J^{(n)} \quad (11)$$

$$\omega_{J,n} = \frac{1}{4} \sum_{i=1}^{4} \omega_{i,n} \quad (12)$$

$$D_{J,n} = \frac{1}{4} \sum_{i=1}^{4} D_{i,n} \quad (13)$$

Then, the total element stiffness matrix, $[K_2]^{(n)}$, the total damping matrix, $[C_2]^{(n)}$, for plate elements in n-th mode of shear vibration system is given as follows:

$$[K_2]^{(n)} = \sum_{J=1}^{N_e} [K_2]_J^{(n)} \quad (14)$$

$$[C_2]^{(n)} = \sum_{J=1}^{N_e} [C_2]_J^{(n)} \quad (15)$$

in which, N_e denotes the total number of plate elements used in the model. Defining the stiffness matrix for the spring-mass systems of the model constituted of $K_{3x}^{(n)}$ and $K_{3y}^{(n)}$ as $[K_3]^{(n)}$, and the damping matrix constituted of $C_{3x}^{(n)}$ and $C_{3y}^{(n)}$ as $[C_3]^{(n)}$, the total stiffness matrix, $[K]^{(n)}$, and the total damping matrix, $[C]^{(n)}$, of n-th mode of shear vibration system of EXQ3D model can be written as:

$$[K]^{(n)} = [K_2]^{(n)} + [K_3]^{(n)} \quad (16)$$

$$[C]^{(n)} = [C_2]^{(n)} + [C_3]^{(n)} \quad (17)$$

Thus, the equation of motion for n-th mode of shear vibration system of EXQ3D model illustrated in Fig.2(b) is written as:

$$[M]^{(n)}\begin{bmatrix}\ddot{X}\\ \ddot{Y}\end{bmatrix}^{(n)} + [C]^{(n)}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}^{(n)} + [K]^{(n)}\begin{bmatrix}X\\ Y\end{bmatrix}^{(n)} = -[M]^{(n)}\begin{bmatrix}\ddot{u}\\ \ddot{w}\end{bmatrix}^{(n)} \quad (18)$$

in which: $[M]^{(n)}$ denotes the effective mass matrix of n-th mode of shear vibration system; $[C]^{(n)}$ damping matrix of n-th mode of shear vibration system; $[K]^{(n)}$ stiffness matrix of n-th mode of shear vibration system; X and Y horizontal displacement in x and y direction, respectively; and $\ddot{u}$, $\ddot{w}$ input horizontal acceleration in x and y direction, respectively

A set of N coupled above equations are induced when fundamental through N-th mode of shear vibrations are taken into account in EXQ3D model. The earthquake responses of nodal point i or soil column i at arbitrary depth z and time t are given by the incremental mode-superposition technique adopted in some earthquake response analyses of multi degree of freedom system regarding one-dimensional shear vibration problems (Elgamal 1991, Mohraz 1991).

$$\ddot{x}_i(z,t) = \sum_{n=1}^{N} \ddot{X}_{i,n}(t)\, F_{i,n}(z) + \ddot{u}_i(t) \quad (19)$$

$$\dot{x}_i(z,t) = \sum_{n=1}^{N} \dot{X}_{i,n}(t)\, F_{i,n}(z) \quad (20)$$

$$x_i(z,t) = \sum_{n=1}^{N} X_{i,n}(t)\, F_{i,n}(z) \quad (21)$$

$$\ddot{y}_i(z,t) = \sum_{n=1}^{N} \ddot{Y}_{i,n}(t)\, F_{i,n}(z) + \ddot{w}_i(t) \quad (22)$$

$$\dot{y}_i(z,t) = \sum_{n=1}^{N} \dot{Y}_{i,n}(t)\, F_{i,n}(z) \quad (23)$$

$$y_i(z,t) = \sum_{n=1}^{N} Y_{i,n}(t)\, F_{i,n}(z) \quad (24)$$

in which: $\ddot{x}_i(z,t)$, $\dot{x}_i(z,t)$ and $x_i(z,t)$ are absolute acceleration, relative velocity and displacement, respectively, of nodal point i in the horizontal x direction at depth z and at time t; $\ddot{y}_i(z,t)$, $\dot{y}_i(z,t)$ and $y_i(z,t)$ absolute acceleration, relative velocity and displacement, respectively, of nodal point i in the horizontal y direction at depth z and time t; $\ddot{X}_{i,n}(t)$, $\dot{X}_{i,n}(t)$ and $X_{i,n}(t)$ relative acceleration, velocity and displacement, respectively, of mass i in x direction at time t in n-th mode of shear vibration system; $\ddot{Y}_{i,n}(t)$, $\dot{Y}_{i,n}(t)$ and $Y_{i,n}(t)$ relative acceleration, velocity and displacement of mass i in y direction at time t in n-th mode of shear vibration system; and $\ddot{u}_i(t)$ and $\ddot{w}_i(t)$ input ground motions acting on soil column i in x and y direction, respectively, at time t.

3.VERIFICATION OF EXQ3D MODEL

3.1 Model ground and conditions of analyses

In order to validate EXQ3D model, earthquake response analyses of 3-D surface ground were carried out, using EXQ3D model and finite element model (FEM3). The model ground used is illustrated in Fig.3. The model ground is a 3-D valley bounded by sharp slopes of base ground. The stratigraphy of surface layer of the model is shown in Fig.4. The surface layer is consisted of 4 different types of soil with different properties. As shown in the figure, lower three modes of shear vibration are predominant in such a layer. In the analyses using EXQ3D model, therefore, fundamental through third modes of shear vibration are taken into account.

Fig.5 shows the discritization of the model ground for EXQ3D model. The number of nodes used is 315, while that of freeedom is 487. Fig.6 shows a schematic representation of the discritization for FEM3. The horizontal mesh for FEM3 includes 121 nodes, which is considerably rough compared with that for EXQ3D. Suface layer is divided into 4 sublayers vertically corresponding to 4 soil types. Consequenltly, the number of nodes used for FEM3 is 605, while that of freedom is 972, which is twice as much as that used for EXQ3D.

Earthquake response analyses were conducted using NS and EW components of Hachinohe waves, Tokachi-oki Earthquake of 1968 as input ground motions in the horizontal y and x direction, respectively. The damping factor of surface layer was fixed to 0.1 in both models of analysis.

3.2 Comparison of results

The results of analyses obtained by EXQ3D and FEM3 are compared with respect to both acceleration and displacement responses. Table 1 summarizes the values of maximum acceleration and maximum displacement obtained by the two models. In terms of maximum responses, as shown in the table, results obtained by EXQ3D give good approximations of finite element solutions except for cases of No.137, GL.-5.0 m. In consideration of the rough discritization in FEM3, it can be concluded that the results in Table 1 validate EXQ3D sufficiently.

Table 1 Comparison of maximum responses

Location	Max. Accel.(gal)		Max. Disp. (cm)	
	EXQ3D	FEM3	EXQ3D	FEM3
137-X-0	390.6	403.3	2.756	2.896
137-Y-0	326.3	329.3	1.895	1.533
137-X-5	245.4	268.5	1.569	1.813
137-Y-5	310.3	190.4	1.377	0.923
137-X-10	182.0	177.4	0.385	0.386
137-Y-10	197.0	225.4	0.408	0.333
137-X-15	169.3	163.2	0.160	0.165
137-Y-15	167.2	186.7	0.169	0.153
141-X-0	366.8	355.4	2.587	2.324
141-Y-0	315.0	328.7	1.841	1.642
200-X-0	392.1	420.5	2.766	2.990
200-Y-0	323.7	311.5	1.894	1.473
204-X-0	365.7	393.5	2.593	2.494
204-Y-0	308.7	309.9	1.777	1.750

137-X-0: No.137,Direction=X,Depth=0.0m

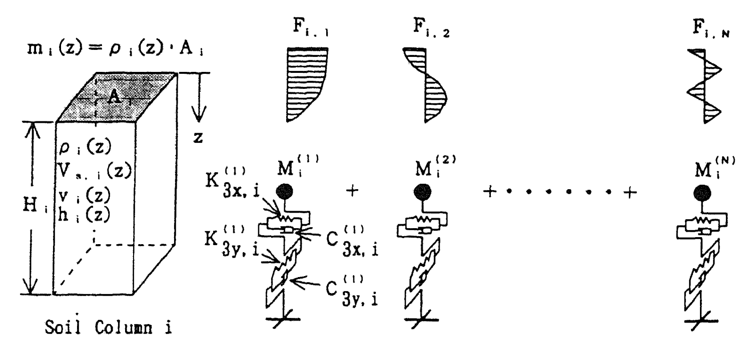
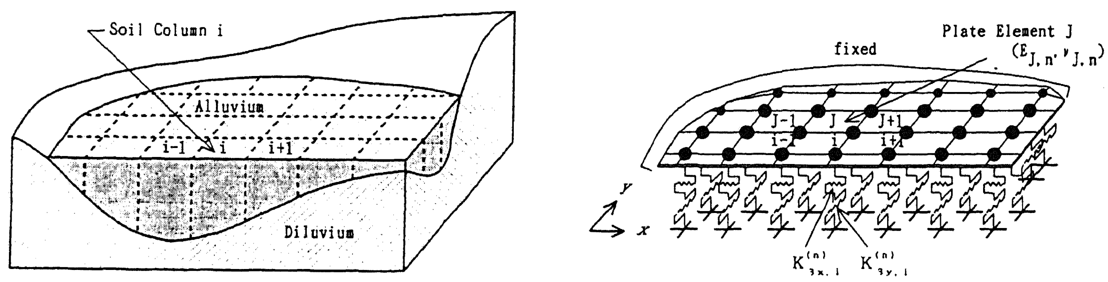


Fig.1 A schematic representation of a set of N coupled spring-mass-damper systems



(a) irregularly bounded 3-D surface ground (b) n-th mode of shear vibration system

Fig.2 A schematic representation of EXQ3D model

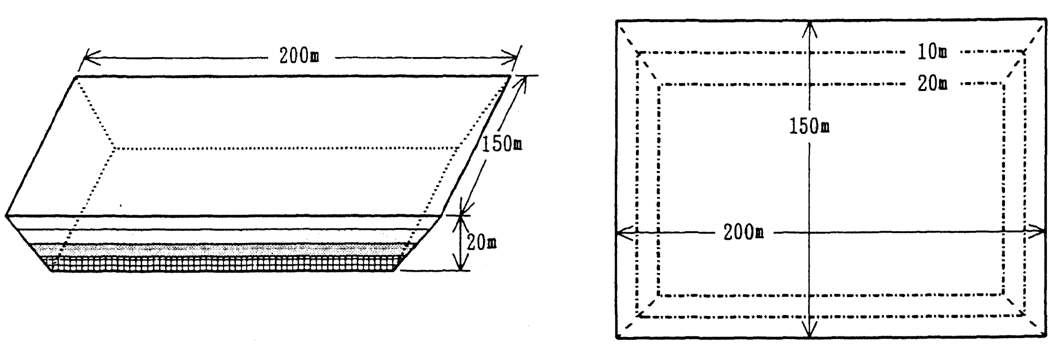
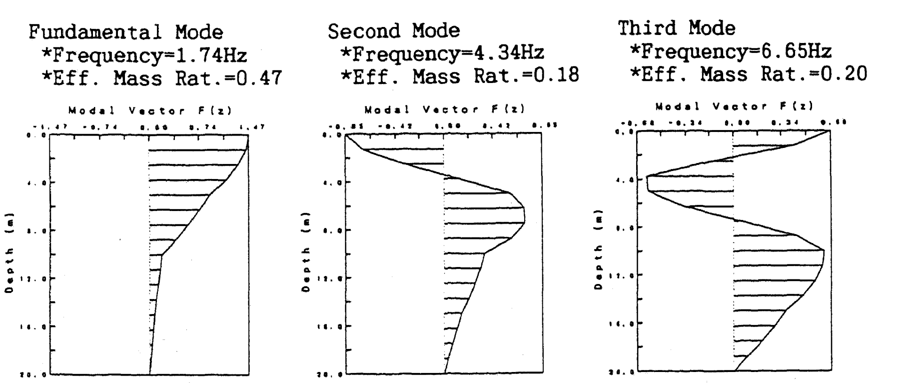


Fig.3 Three-dimensional surface ground used for earthquake response analyses



Soil Properties

γ_t (t/m³)	V_s (m/sec)	ν
1.5	60	0.495
1.6	80	0.495
1.8	210	0.470
1.9	270	0.450

Fig.4 Soil properties and shear vibration modes of surface layer used in analyses

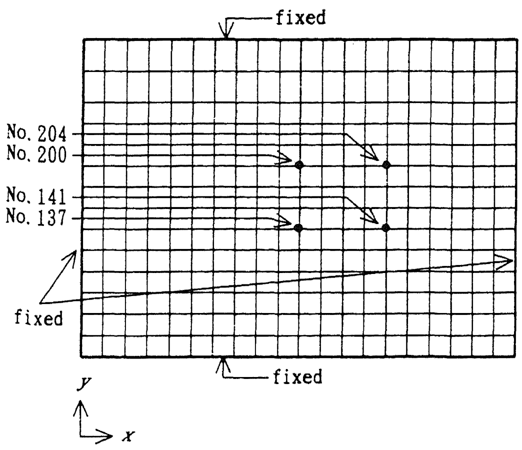


Fig.5 Discritization of plate elemet mesh used for the analysis by EXQ3D

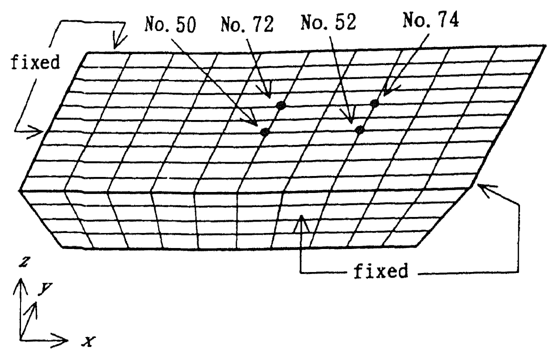


Fig.6 Discritization of finite element mesh used for the analysis by FEM3

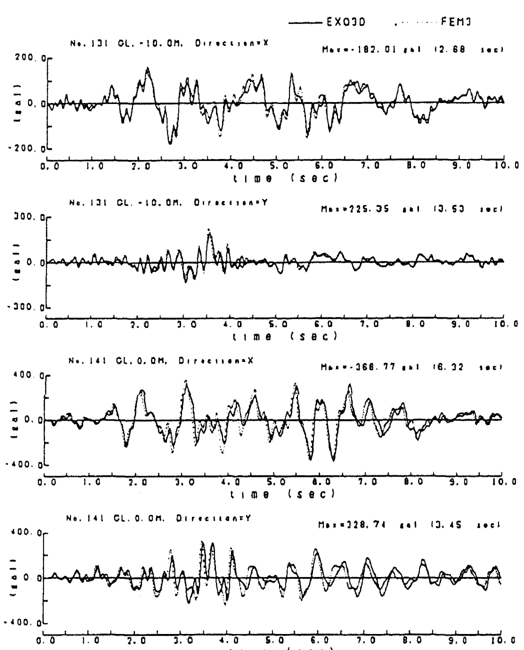


Fig.7 Comparison of time histories of acceleration between the analyses by EXQ3D and FEM3

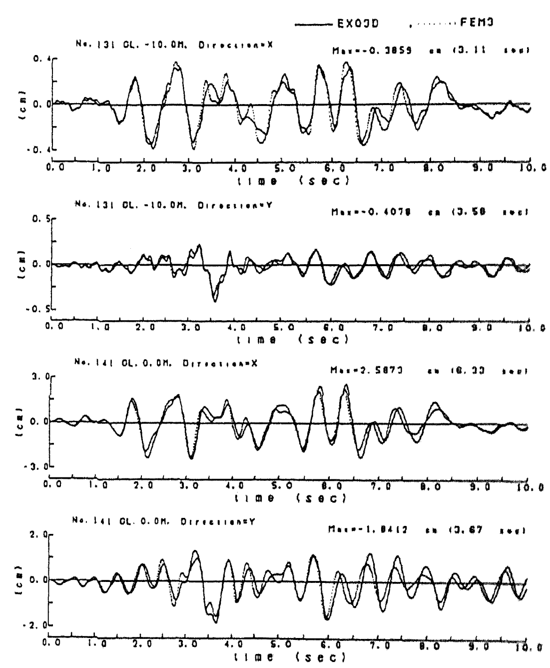


Fig.8 Comparison of time histories of displacement between the analyses by EXQ3D and FEM3

In order to compare time-dependent characteristics of responses obtained by the two models, time histories of acceleration and displacement were plotted at selected locations. Fig.7 shows comparative plotting on time histories of acceleration. The upper two histories are acceleration responses at GL.-10 m of No.131, where the effect of second and third modes of shear vibration is relatively large. The lower two time histories are, on the other hand, acceleration responses on the ground surface of No.141. As shown in the figure, EXQ3D gives good approximation of 3-D finite element solutions in terms of time -dependent characteristics of acceleration.

Fig.8 shows comparative plotting on time histories of displacement. The selected locations for plotting are the same that are used in Fig.7. Displacement responses near ground surface are mostly governed by fundamental mode of shear vibration. However, EXQ3D model also gives good simulation of ground displacement responses in the deep part, as shown in the figure. Therefore, EXQ3D model enables us easily to simulate earthquake response of underground structures deeply buried in surface ground.

4.LIMITATIONS OF EXQ3D MODEL

Since EXQ3D is the simplified model based on the incremental mode-superposition technique, there are some limitations in its application. The model is fully applicable to the surface ground consisted of a layer with uniform thickness and without any local irregularity. However, a large amount of error may generate, if the incremental mode-superposition technique is applied to the ground, where the frequency of fundamental shear vibration mode of a soil column is almost identical to that of the second or third mode of adjoining columns. Therefore, the following limitations should be kept in mind:

1. The discritization of a mesh should not be excessively rough. The difference in thickness of adjoining colums should be kept at the level less than 50 %;
2. The model can not be applied to the surface ground with excessively sharp slope of base rock. The slope angle should not exceed 45 degrees at maximum; and
3. The fifth mode will be the heighest mode desirable to be incrementally superposed. Modes higher than the fifth one do not predominate in natural soil deposits in general. In such cases, the extent of abovementioned error becomes compatible to or larger than the increase in accuracy due to the incremental mode-superposition.

5.CONCLUSIONS

In this paper, the author proposed the extended quasi-three-dimensional ground model (EXQ3D model) for earthquake response simulation of surface ground under three dimensional geological conditions. The validity of the model was verified, based on the comparison between the results of earthquake response analyses using both EXQ3D model and 3-D finite element model. Through the analyses, the followings are summarized as conclusions:

1. EXQ3D model enables us to simulate easily acceleration responses of 3-D surface ground, while Q3D model can not sufficiently simulate because it takes only fundamental mode of shear vibration into consideration.
2. Using EXQ3D model, the accuracy of earthquake response analyses of underground structures is improved, even if the structures are buried deeply in 3-D surface ground.
3. EXQ3D model not only eliminates laborious works for generating 3-D mesh in 3D-FEM, but also reduces computation time due to the reduction of number of freedom needed.
4. In applying EXQ3D model, limitations of the model, based on the assumption that the incremental mode-superposition can be applied, should be kept in mind.

REFERENCES

Elgamal, A.W. 1991. Shear hysteretic elasto-plastic earthquake response of soil system. Earthq. Eng. Struc. Dyn. 20: 371-389

Mohraz, B. et al. 1991. An incremental mode-superposition for non-linear dynamic analysis. Earthq. Eng. Struc. Dyn. 20: 471-481.

Suzuki, T. 1986. A summary report on earthquake damage in Mexico. Tech. Report of Kumagai Gumi 39: 99-109.

Suzuki, T. et al. 1988. Earthquake observation and response analysis of a shield tunnel. Proc. 9WCEE 6: 563-568.

Seed, R.B. et al. 1990. Preliminary report on the principal geotechnical aspects of the October 17, 1989 Loma Prieta Earthquake. Report No.UCB/EERC-90/05.

Tamura et al. 1990. Preliminary report of JSCE reconnaissance team on July 16, 1990 Luzon Earthquake: Earthq. Eng. Committ. JSCE.

Tamura, C. & Suzuki, T. 1987. A quasi-three-dimensional ground model for earthquake response analysis of underground structures. Monthly Jour. Inst. Indust. Sci., University of Tokyo 39-1: 37-40.

Trifunac, M.D. 1989. Dependence of Fourier spectrum amplitudes of recorded earthquake accelerations on magnitude, local soil conditions and on depth of sediments, Earthq. Eng. Struc. Dyn. 18: 999-1016.