The extended quasi-three-dimensional ground model

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ABSTRACT: The extended quasi-three-dimensional ground model (EXQ3D model) is presented in this paper, for the analysis of seismic responses of irregularly bounded 3-D surface ground. The quasi-three-dimensional ground model (Q3D model) proposed by Tamura and Suzuki is extended to EXQ3D model so that the new model can take fundamental through arbitrary n-th mode of shear vibrations into consideration. The formulation of the model is described first. Then, earthquake response analyses of a 3-D model ground are carried out using both 3-D finite element method and the proposed model. The validity of the model is verified through the comparison of the results of analyses obtained using both models. It is pointed out, finally, that EXQ3D model is effective in evaluating the effect of surface geology on seismic motions and seismic zonation.

1 INTRODUCTION

Recent large earthquakes, for example, the Mexico Earthquake of 1985 (Suzuki 1986), the Loma Prieta earthquake of 1989 (Seed et al. 1990) and the Luzon earthquake of 1990 (Tamura et al. 1990), posed us important lessons suggesting that there exists a close relationship between severe damages and surface geology. There are a lot of attenuation equations and methods to estimate response spectra to evaluate the effects of local soil and geological condition (Trifunac 1989). Based on these measures, much efforts have been taken for seismic zonation. In case that a specific superstructure or an underground tunnel is planned, the prediction of the seismic ground motions due to large earthquakes is necessary to evaluate the earthquake resistance of the structure. In this aspect, much attention has been concentrat-ed on how to evaluate the effect of surface geology on seismic motions and various approaches have been proposed to model surface ground.

The quasi-three-dimensional ground model, Q3D model, was proposed (Tamura & Suzuki 1987) to analyze horizontal ground displacement responses of 3-D surface ground as the input data for the earthquake response analysis of an underground tunnel. Q3D model only takes fundamental shear vibration of surface ground into consideration, which is considered to be valid on the ground where the fundamental shear vibration mode of the surface layer is predominant. Because the vibration higher than fundamental one does not affect ground displacement response so much especially at the shallow part of the layer. This model was applied to simulate the earthquake response of a shield-driven tunnel in soft ground at the site where the earthquake observation was conducted (Suzuki 1988).

Since Q3D model treats the effect of irregular boundaries of 3-D surface ground, its extension to the model, which can deal with fundamental through arbitrary order of shear vibration modes which are predominant at the site, enables us to simulate not only ground displacement response but also ground acceleration and velocity response of 3-D ground during earthquakes. Thus, the extended model, EXQ3D model, will play an important role in seismic zonation or microzonation. This paper describes the formulation of EXQ3D model first. Then, the validity of the proposed model is shown based on the comparison of the results of earthquake response analyses using this model and 3-D finite element approach.

2 FORMULATION OF EXQ3D MODEL

The formulation for a linear analysis using EXQ3D model is described in this chapter. The surface ground is divided into a number of soil columns in the first step. Each column is replaced by a set of N coupled spring-mass systems or two degree of freedom systems, each of which corresponds with each of the shear vibration mode of the column. Fig.1 illustrates concretely how to model the soil column, replacing it by...
spring-mass-damper system. The mass of the n-th mode of vibration system \( \mathbf{M}^{(n)} \) is given by the following equation:

\[
\mathbf{m}^{(n)} = \frac{\int_{x_1}^{x_2} \mathbf{a}^{(n)}(z) \mathbf{a}^{(n)}(z)^T dz}{\int_{x_1}^{x_2} \mathbf{a}^{(n)}(z) \mathbf{a}^{(n)}(z) dz}
\]

(1)

in which: \( \mathbf{m}^{(n)} \) denotes the effective mass of soil column \( i \) on n-th shear vibration mode; \( \mathbf{a}^{(n)}(z) \) mass of soil column \( i \) at depth \( z \); \( f_{ij}^{(n)}(z) \) normalized modal vector on n-th shear vibration mode; \( H_i \) thickness of soil column \( i \); and a depth below ground surface.

The spring constants and the constants of viscous dampers of n-th shear vibration system are, then, determined using the circular frequency of n-th mode of the soil column as:

\[
\mathbf{k}_{ix}^{(n)} = \omega_{ix}^{2} \mathbf{m}_{ix}^{(n)}
\]

(2)

\[
\mathbf{c}_{ix}^{(n)} = \frac{20 \pi}{\omega_{ix}^{2} \mathbf{m}_{ix}^{(n)}}
\]

(3)

in which: \( \mathbf{k}^{(n)} \) and \( \mathbf{c}^{(n)} \) denote shear spring constant and connecting mass \( M_i^{(n)} \) with the base ground in the horizontal \( x \) and \( y \) direction, respectively; \( \mathbf{C}^{(n)} \) and \( \mathbf{C}^{(n)} \) constants of viscous dampers connecting mass \( M_i^{(n)} \) with the base ground in the horizontal \( x \) and \( y \) direction, respectively; \( \omega_{ix} \) circular frequency of n-th mode; and \( \mathbf{m}_{ix}^{(n)} \) damping factor of soil column \( i \) on n-th mode of shear vibration.

\( D_0 \) is the weighted mean calculated by the damping factor of soil column \( i \), \( h_{ix}(z) \), and the displacement function \( \mathbf{f}_{ix}(z) \), given as follows:

\[
h_{ix}(z) = \frac{\int_{x_1}^{x_2} h_{ix}(z) f_{ix}(z) dz}{\int_{x_1}^{x_2} f_{ix}(z) dz}
\]

(4)

\[
\mathbf{f}_{ix}(z) = \frac{1}{\mathbf{m}_{ix}^{(n)}} \mathbf{a}^{(n)}(z)
\]

(5)

in which, \( \beta_i \) denotes the participation factor of soil column \( i \) on n-th mode of shear vibration.

Each two degree of freedom system composed of a mass, 2 springs and 2 dampers is connected with each other by finite plate elements with unit thickness as illustrated in Fig. 2. Fig. 2(b) is the schematic representation of the n-th mode of shear vibration system when the 3-D surface ground illustrated as Fig. 2(a) is replaced by EXQ3D model. The elastic modulus of plate element J, \( E_j \), is given by the following equation. which the element connects 4 masses with a rectangular shape.

\[
E_{i,j} = \frac{1}{4} \mathbf{e}_{j,n} \cdot \frac{x_2}{x_1}
\]

(6)

\[
E_{i,j} = \sum E_j(z) f_{ij}^{(n)}(z) dz
\]

(7)

in which: \( E_j(z) \) denotes the elastic modulus of soil column \( i \) at depth \( z \) below surface; and \( h_{1,0} \) denotes the equivalent rigidity of

the nodal point \( i \) in consideration of n-th shear vibration mode, and can be defined by integrating the product of \( E_j(z) \) and the absolute form of \( f_{ij}^{(n)}(z) \) over the thickness of the layer at nodal point \( i \). Another parameter, which is necessary to form two-dimensional elasticity matrix represented as \( D \) is Poisson’s ratio of plate element \( J \). \( c_{ij,n} \) defined as follows:

\[
\mathbf{c}_{ij,n} = \frac{1}{4} \mathbf{e}_{j,n} \cdot \mathbf{f}_{ij}(z)
\]

(8)

\[
\mathbf{f}_{ij}(z) = \frac{\int_{x_1}^{x_2} f_{ij}(z) f_{ij}(z) dz}{\int_{x_1}^{x_2} f_{ij}(z) dz}
\]

(9)

in which: \( f_{ij}(z) \) represents the Poisson’s ratio of the layer at nodal point \( i \); and \( D_{ij,n} \) is Poisson’s ratio at nodal point \( i \) in n-th mode of shear vibration system, and is given as the weighted mean obtained using the same manner as that used in equation (4). Thus, the element-stiffness matrix of plate element \( J \), \( \mathbf{K}_{ij}^{(n)} \), can be given, using finite element procedure for a plate element with unit thickness under plane-stress condition:

\[
\mathbf{K}_{ij}^{(n)} = \int_0^1 \mathbf{b}_{ij} \, d\mathbf{b}
\]

(10)

in which, \( \mathbf{b} \) and \( D \) denote strain matrix and two-dimensional elasticity matrix respectively under plane-stress condition. The damping matrix of plate element \( J \) in n-th mode of shear vibration system is, therefore, given by the following equation:

\[
\mathbf{C}_{ij}^{(n)} = \int_0^1 \mathbf{c}_{ij,n} \, d\mathbf{c}
\]

(11)

\[
\mathbf{c}_{ij,n} = \frac{1}{4} \mathbf{e}_{j,n} \cdot \frac{x_2}{x_1}
\]

(12)

\[
\mathbf{e}_{j,n} = \frac{1}{4} \mathbf{e}_{j,n} \cdot \frac{x_2}{x_1}
\]

(13)

The total element stiffness matrix, \( \mathbf{K}_{ij}^{(n)} \), the total damping matrix, \( \mathbf{C}_{ij}^{(n)} \), for plate elements in n-th mode of shear vibration system is given as follows:

\[
\mathbf{K}_{ij}^{(n)} = \sum \mathbf{K}_{ij}^{(n)}
\]

(14)

\[
\mathbf{C}_{ij}^{(n)} = \sum \mathbf{C}_{ij}^{(n)}
\]

(15)

in which, \( N \) denotes the total number of plate elements used in the model. Defining the stiffness matrix for the spring-mass system of the model constituted of \( \mathbf{M}^{(n)} \) and \( \mathbf{K}^{(n)} \) as \( \mathbf{K}^{(n)} \) and the damping matrix \( \mathbf{C}^{(n)} \) as \( \mathbf{C}^{(n)} \), the total stiffness matrix \( \mathbf{C}^{(n)} \) of n-th mode of shear vibration system of EXQ3D model can be written as:

\[
\mathbf{K}^{(n)} = \mathbf{K}_{ij}^{(n)} + \mathbf{K}_{ij}^{(n)}
\]

(16)

\[
\mathbf{C}^{(n)} = \mathbf{C}_{ij}^{(n)} + \mathbf{C}_{ij}^{(n)}
\]

(17)

Thus, the equation of motion for n-th mode of shear vibration system of EXQ3D model...
illustrated in Fig. 2(b) is written as:

\[
\begin{align*}
[U^\prime]^{(n)} + (C)^{(n)} [U]^{(n)} + (K)^{(n)} [U]^{(n)} &= -[K] [U]^{(n)} \\
\end{align*}
\]

in which: \([M]^{(n)}\) denotes the effective mass matrix of \(n\)-th mode of shear vibration system; \([C]^{(n)}\) damping matrix of \(n\)-th mode of shear vibration system; \([K]^{(n)}\) stiffness matrix of \(n\)-th mode of shear vibration system; \(X\) and \(Y\) horizontal displacement in \(x\) and \(y\) direction, respectively; and \(u, w\) input horizontal acceleration in \(x\) and \(y\) direction, respectively.

A set of \(N\) coupled above equations are induced when fundamental through \(N\)-th mode of shear vibration for FEMS includes 1-21 nodes, which is considerably rough compared with that for EQEQD. Surface layer is divided into 4 sublayers vertically corresponding to 4 soil types. Consequently, the number of nodes used for FEMS is 605, while that of freedom is 972, which is twice as much as that used for EQEQD.

Earthquake response analyses were conducted using NS and EW components of Hachinohe waves. Tokachi-oki Earthquake of 1968 as input ground motions in the horizontal \(y\) and \(x\) direction, respectively. The damping factor of surface layer was fixed to 0.1 in both models of analysis.

3.2 Comparison of results

The results of analyses obtained by EQEQD and FEMS are compared with respect to both acceleration and displacement responses. Table 1 summarizes the values of maximum acceleration and maximum displacement obtained by the two models. In terms of maximum responses, as shown in the table, results obtained by EQEQD give good approximations of finite element solutions except for cases of No. 137, GL-3.0 m. In consideration of the rough discretization in FEMS, it can be concluded that the results in Table 1 validate EQEQD sufficiently.

Table 1 Comparison of maximum responses

<table>
<thead>
<tr>
<th>Location</th>
<th>Max. Accel. (gal)</th>
<th>Max. Disp. (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQEQD</td>
<td>FEMS</td>
<td>EQEQD</td>
</tr>
<tr>
<td>137-X-0</td>
<td>306.6</td>
<td>306.6</td>
</tr>
<tr>
<td>137-Y-0</td>
<td>325.3</td>
<td>325.3</td>
</tr>
<tr>
<td>137-Y-5</td>
<td>245.4</td>
<td>245.4</td>
</tr>
<tr>
<td>137-Y-5</td>
<td>312.3</td>
<td>312.3</td>
</tr>
<tr>
<td>137-Y-10</td>
<td>197.0</td>
<td>197.0</td>
</tr>
<tr>
<td>137-X-10</td>
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<td>325.3</td>
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<td>137-X-15</td>
<td>165.3</td>
<td>165.3</td>
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<td>137-Y-15</td>
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<td>167.2</td>
</tr>
<tr>
<td>141-X-10</td>
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</tr>
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<td>200-Y-5</td>
<td>379.8</td>
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<td>204-X-0</td>
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<td>204-Y-0</td>
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<td>365.7</td>
</tr>
<tr>
<td>Location</td>
<td>Max. Accel. (gal)</td>
<td>Max. Disp. (cm)</td>
</tr>
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<td>306.6</td>
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<td>137-Y-0</td>
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<tr>
<td>204-Y-0</td>
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Fig. 1 A schematic representation of a set of N coupled spring-mass-damper systems

(a) irregularly bounded 3-D surface ground  
(b) n-th mode of shear vibration system

Fig. 2 A schematic representation of EXQID model

Fig. 3 Three-dimensional surface ground used for earthquake response analyses

Fundamental Mode  
*Frequency = 1.74 Hz  
*Eff. Mass Rat. = 0.47

Second Mode  
*Frequency = 4.34 Hz  
*Eff. Mass Rat. = 0.18

Third Mode  
*Frequency = 5.55 Hz  
*Eff. Mass Rat. = 0.20

Soil Properties  
\( \frac{E}{G} (\text{kPa/m}) \)

<table>
<thead>
<tr>
<th>Layer</th>
<th>Young's Modulus</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>90</td>
<td>495</td>
</tr>
<tr>
<td>1.6</td>
<td>85</td>
<td>495</td>
</tr>
<tr>
<td>1.8</td>
<td>210</td>
<td>470</td>
</tr>
<tr>
<td>1.9</td>
<td>270</td>
<td>450</td>
</tr>
</tbody>
</table>

Fig. 4 Soil properties and shear vibration modes of surface layer used in analyses
Fig. 5: Discretization of plate element mesh used for the analysis by EXQ3D.

Fig. 6: Discretization of finite element mesh used for the analysis by FEM3.

Fig. 7: Comparison of time histories of acceleration between the analyses by EXQ3D and FEM3.

Fig. 8: Comparison of time histories of displacement between the analyses by EXQ3D and FEM3.
In order to compare time-dependent characteristics of responses obtained by the two models, time histories of acceleration and displacement were plotted at selected locations. Fig. 7 shows comparative plotting on time histories of acceleration. The upper two histories are acceleration responses at CL-10 m of No.131, where the effect of second and third modes of shear vibration is relatively large. The lower two time histories are, on the other hand, acceleration responses on the ground surface of No.141. As shown in the figure, EXQD gives good approximation of 3-D finite element solutions in terms of time-dependent characteristics of acceleration.

Fig. 8 shows comparative plotting on time histories of displacement. The selected locations for plotting are the same that are used in Fig. 7. Displacement responses near ground surface are mostly governed by fundamental mode of shear vibration. However, EXQD model also gives good simulation of ground displacement responses in the deep part, as shown in the figure. Therefore, EXQD model enables us easily to simulate earthquake response of underground structures deeply buried in surface ground.

4. LIMITATIONS OF EXQD MODEL

Since EXQD is the simplified model based on the incremental mode-superposition technique, there are some limitations in its application. The model is fully applicable to the surface ground consisted of a layer with uniform thickness and without any local irregularity. However, a large amount of error may generate, if the incremental mode-superposition technique is applied to the ground, where the frequency of fundamental shear vibration mode of a soil column is almost identical to that of the second or third mode of adjoining columns. Therefore, the following limitations should be kept in mind:

1. The discretization of a mesh should not be excessively rough. The difference in thickness of adjoining columns should be kept at the level less than 50 %.
2. The model can not be applied to the ground surface with excessively sharp slope of base rock. The slope angle should not exceed 45 degrees at maximum, and
3. The fifth mode will be the highest mode desirable to be incrementally superposed. Modes higher than the fifth one do not predominate in natural soil deposits in general. In such cases, the extent of abovementioned error becomes compatible to or larger than the increase in accuracy due to the incremental mode-superposition.

5. CONCLUSIONS

In this paper, the author proposed the extended quasi-three-dimensional ground model (EXQD model) for earthquake response simulation of surface ground under three-dimensional geological conditions. The validity of the model was verified, based on the comparison between the results of earthquake response analyses using both EXQD model and 3-D finite element model. Through the analyses, the followings are summarized as conclusions:

1. EXQD model enables us to simulate easily acceleration responses of 3-D surface ground, while QD model can not sufficiently simulate because it takes only fundamental mode of shear vibration into consideration.
2. Using EXQD model, the accuracy of earthquake response analyses of underground structures is improved, even if the structures are buried deeply in 3-D surface ground.
3. EXQD model not only eliminates laborious works for generating 3-D mesh in 3D-FEM, but also reduces computation time due to the reduction of number of freedoms needed.
4. In applying EXQD model, limitations of the model, based on the assumption that the incremental mode-superposition can be applied, should be kept in mind.

REFERENCES