Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

The local effects of curved and parallel soil medium to seismic waves

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ABSTRACT : To obtain the Fourier spectra on free surface for parallel and curved layered media, we employ the generalized ray theory to calculate the responses for a seismic source of equivalent forces of double couple without moment. From the results obtained, we find that the Fourier amplitudes for wave propagating in circular media are much greater than those in parallel media, especially, there are high peaks in low frequency for the responses generated in circular medium.

1 INTRODUCTION

The geometry of surface layers at some site location significantly affect the ground motions during the passage of seismic waves, and strongly influence the dynamics of foundation for the structure [Porceski, 1969]. In this study, the seismic source was presumably located in the half space overlaid with a parallel medium or a circular basin-type medium, then, based on the given conditions for concentrated forces, we derived the source functions in infinitive space for equivalent body force of double couple without moment, and to interpret the influence of the existing parallel or circular interface to the ground motions, we incorporate the transmission and reflection coefficients for this interface into the ray integrals, of which the final mathematical form are expressed in Laplace transform, and we use the well-known Cagniard method to simplify the procedures to inverse Laplace transform to obtain the responses in time domain.

2 THE GENERALIZED RAY INTEGRTAL

The emitted wave without impinging upon the boundary is referred to as the source ray, of which the motion of a particle in an elastic space could be governed by the following equation

$$(\lambda_2 + \mu_2)\nabla\nabla\cdot\vec{u}_2 + \mu_2\nabla^2\vec{u}_2 + \rho_2\vec{B} = \rho_2\frac{\partial^2\vec{u}_2}{\partial t^2}, \quad (1)$$

where ∇^2 is the Laplace operator, λ_2, μ_2 are Lame contants for half space, $\vec{u}_2$ and $\vec{B}$ denote the displacement and the body force vector respectively, and ρ_2 is the mass density.

If the Helmhotz decompositions are introduced as follows,

$$\vec{u}_2 = \nabla\phi_2 + \nabla\times\vec{\psi}_2, \qquad \nabla\cdot\vec{\psi}_2 = 0, \quad (2)$$

$$\vec{B} = \nabla r + \nabla\times\vec{G}, \qquad \nabla\cdot\vec{G} = 0, \quad (3)$$

Then substituting Eqs. (2) and (3) into Eq. (1), yield

$$C_{p2}^2\nabla^2\phi_2 + p = \frac{\partial^2\phi_2}{\partial t^2}, \quad (4)$$

and

$$C_{v2}^2\nabla^2\vec{\psi}_2 + \vec{G} = \frac{\partial^2\vec{\psi}_2}{\partial t^2}, \quad (5)$$

where

$$C_{p2} = \left(\frac{\lambda_2 + 2\mu_2}{\rho_2}\right)^{1/2}$$

is the compression wave speed, and

$$C_{v2} = \left(\frac{\mu_2}{\rho_2}\right)^{1/2}$$

is the shear wave speed.

Since the propagation of a plane strain wave is assumed, hence we let

$$\vec{\psi}_2 = [0, \psi_2, 0], \quad (6)$$

and

$$\vec{G} = [0, q, 0], \quad (7)$$

where ψ_2 and q denote the y-component of vectors $\vec{\psi}_2$ and $\vec{G}$ respectively. Then substituting Eqs. (6) and (7) into (5), yields

$$C_{v2}^2\nabla^2\psi_2 + q = \frac{\partial^2\psi_2}{\partial t^2}, \quad (8)$$

Furthermore, by the substitution of Eq. (6) into (2), the displacement can be expressed in scalar form as

$$u_{x2} = \frac{\partial\phi_2}{\partial x} - \frac{\partial\psi_2}{\partial z}, \quad (9)$$

$$u_{z2} = \frac{\partial\phi_2}{\partial z} + \frac{\partial\psi_2}{\partial x}, \quad (10)$$

and stresses as

$$\tau_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}), \qquad i,j = x \text{ or } z, \quad (11)$$

where τ_{ij} is the stress tensor, δ_{ij} is the Kronecker delta. To solve the wave equations, we define the Laplace transform as

$$\overline{\phi}(x,z,s) = \int_0^\infty \phi(x,z,t)e^{-st}\,dt, \qquad (12)$$

and Fourier transform as

$$\tilde{\phi}(\xi,z,s) = \int_{-\infty}^{\infty} \overline{\phi}(x,z,s)e^{-is\xi x}\,dx, \qquad (13)$$

With reference to literature [Niazy, 1975], the horizontal and vertical forces F_x and F_z, when acting at $(0, z_0)$ can be mathematically described as

$$\left[\tau^+_{xz2} - \tau^-_{xz2}\right] = -F_x f(t)\delta(x), \qquad \text{at } z = z_0, \quad (15)$$

$$\left[\tau^+_{zz2} - \tau^-_{zz2}\right] = -F_z f(t)\delta(x), \qquad \text{at } z = z_0, \quad (16)$$

$$\left[u^+_{x2} - u^-_{x2}\right] = 0, \qquad \text{at } z = z_0, \quad (17)$$

$$\left[u^+_{z2} - u^-_{z2}\right] = 0, \qquad \text{at } z = z_0, \quad (18)$$

where $f(t)$ is the time function, and $\delta(x)$ is the Dirac delta function. With reference to literature [Chen, 1992] and boundary conditions (15) to (18), we apply Eqs. (12) and (13) to solve Eqs. (4) and (8) by ignoring the body forces, then, it yields

$$\overline{\phi}^-_2(x,z,s) = \overline{F}(s)\int_{-\infty}^{\infty} S_p(\xi)e^{s[i\xi x + \eta_{p2}(z-z_0)]}\,d\xi, \qquad (23)$$

and

$$\overline{\psi}^-_2(x,z,s) = \overline{F}(s)\int_{-\infty}^{\infty} S_v(\xi)e^{s[i\xi x + \eta_{v2}(z-z_0)]}\,d\xi, \qquad (24)$$

where

$$S_p(\xi) = F_z + \frac{i\xi}{\eta_{p2}}F_x, \qquad (25)$$

$$S_v(\xi) = \frac{i\xi}{\eta_{v2}}F_z - F_x, \qquad (26)$$

are referred to as the P and SV source functions respectively, $a_{p2} = 1/C_{p2}$, $\eta^2_{p2} = \xi^2 + a^2_{p2}$, $a_{v2} = 1/C_{v2}$, $\eta^2_{v2} = \xi^2 + a^2_{v2}$, $\overline{F}(s) = \overline{f}(s)/4\pi\mu a^2_{v2}s$, and $\overline{f}(s)$ is the Laplace transform of time function $f(t)$, whereas the superscript "-" denotes the upward source rays. If we denote $\overline{\phi}_x$, $\overline{\psi}_x$ as the potential functions obtained by $F_x \neq 0, F_z = 0$ and $\overline{\phi}_z$, $\overline{\psi}_z$ as those obtained by $F_x = 0, F_z \neq 0$, then with reference to literatures [Burridge 1964, Pao 1977], the displacement due to a double couple without moment $\vec{u}^d(\vec{x},t)$ can be deduced from the displacement due to a concentrated force $\vec{u}(\vec{x},t)$ as follows

$$\vec{u}^d(\vec{x},t) = M(\vec{b}\cdot\nabla)\vec{u}(\vec{x},t;\vec{a}) + M(\vec{a}\cdot\nabla)\vec{u}(\vec{x},t;\vec{b}), \quad (27)$$

Where $\vec{x}$ is the position vector, t represent the time variable, M is the applied moment, and we let the two perpendicular unit vectors as $\vec{a} = \vec{e}_z$ and $\vec{b} = \vec{e}_x$. Hence, the potential functions for the equivalent seismic source can be represented by

$$\overline{\Phi}^{d-}_{jh} = \frac{1}{4\pi\mu a^2_v}\overline{f}(s)\int_{-\infty}^{\infty} S^{d-}_{jh}(\xi)e^{s[i\xi x + \eta_{j2}(z-z_0)]}\,d\xi, \qquad j = p \text{ or } v \qquad (28)$$

where the subscript h means the couple acting in ' horizontal ' direction, and we denote the point source by lower case letter, in which $S^{d-}_{ph} = i\xi(M_x + M_z)$, $S^{d-}_{vh} = -\eta_{v2}M_x - (\xi^2/\eta_{v2})M_z$. Furthermore, if a fault with length L and left end locate at (x_0, z_0), ruptures with velocity V, the potential function must modified as

$$\begin{aligned}\overline{\Phi}^{d-}_{jH} =& \frac{1}{4\pi\mu a^2_{v2}}\overline{f}(s)\int_{-\infty}^{\infty} S^{d-}_{jh}(\xi)\,d\xi \int_0^L e^{s[i\xi(x-x_0-l)+\eta_{j2}(z-z_0)-l/V]}\,dl,\\ =& \frac{1}{4\pi\mu a^2_{v2}}\frac{\overline{f}(s)}{s}\int_{-\infty}^{\infty}\frac{S^{d-}_{jh}(\xi)}{i\xi + 1/V}\Big\{e^{s[i\xi(x-x_0)+\eta_{j2}(z-z_0)}\\ &- e^{s[i\xi(x-x_0-L)+\eta_{j2}(z-z_0)-L/V]}\Big\}\,d\xi, \quad (29)\end{aligned}$$

Here, we use capital 'H' to indicate that the solution is for the source of finite fault. For convenience in the latter representations, we would define $\overline{\Phi}_1$ as the potential function for the ray segment of upward P-wave in surface medium, and $\overline{\Phi}_2$, $\overline{\Phi}_3$, $\overline{\Phi}_4$ are respectively defined as that of downward P-wave, upward S-wave and downward S-wave in surface medium. In the like manner, we define those in the half space as $\overline{\Phi}_5$, $\overline{\Phi}_6$, $\overline{\Phi}_7$ and $\overline{\Phi}_8$. Hence, if we let

$$\overline{\Phi}_j(x,z,s;L) = \overline{F}(s)\int_{-\infty}^{\infty} S_j(\xi)e^{sg_j(x,z;\xi,L)}\,d\xi, \qquad j = 5, 7$$

where $S_j(\xi) = S^{d-}_{jh}(\xi)/(i\xi + 1/V)$, $S_5(\xi) = S_p(\xi)$, $S_7(\xi) = S_v(\xi)$, $g_5(x,z;\xi) = i\xi(x - x_0 - L) + \eta_{p2}(z - z_0) - L/V$ and $g_7(x,z;\xi) = i\xi(x - x_0 - L) + \eta_{v2}(z - z_0) - L/V$, then, Eq. (29) can be rewritten as

$$\overline{\Phi}^{d-}_{jH} = \overline{\Phi}_j(x,z,s;0) - \overline{\Phi}_j(x,z,s;L),$$

for convenience, we consider the term $\overline{\Phi}_j(x,z,s;0)$ only in the latter discussion, and simplify it as $\overline{\Phi}_j(x,z,s)$.

The potential functions so obtained in the above equations can be used to derive the formula for displacement by employing Eqs. (9) and (10), hence, the general Laplace transform for displacement is written as

$$\overline{u}_i = \overline{F}_1(s)\Big[\int_{-\infty}^{\infty} S_p D_{ip} e^{s\eta_{p2}(z-z_0)}e^{is\xi x}\,d\xi + \int_{-\infty}^{\infty} S_v D_{iv} e^{s\eta_{v2}(z-z_0)}e^{is\xi x}\,d\xi\Big], \qquad (30)$$

where $\overline{F}_1(s) = s\overline{F}(s)$, i denotes x or z. D_{ip} and D_{iv} are referred to as the receiver functions of $P-$ and $S-$wave respectively, and $D_{xp} = i\xi$, $D_{xv} = \epsilon_s \eta_v$, $D_{zp} = -\epsilon_s \eta_p$ and $D_{zv} = i\xi$

For free surface, the boundary conditions $\tau_{zz} = 0$ and $\tau_{xz} = 0$ should be satisfied, and the continuous conditions for interface $u_x = u_{x2}$, $u_z = u_{z2}$, $\tau_{zz} = \tau_{zz2}$ and $\tau_{xz} = \tau_{xz2}$ are also the same. With these conditions and reference to [Ewing 1957], we let $R_{pp}^{(0)}$ denotes the reflection coefficient for the incident $P-$wave reflected as $P-$wave on the free surface, also $R_{ps}^{(0)}$ as that for incident $P-$wave reflected as $S-$wave on the free surface, also, others are obtained in the same way. Hence, the receiver functions on a free surface can be modified as follows [Pao 1977],

$$D_{ip}^{(f)} = D_{ip} + R_{pp}^{(0)} D_{ip} + R_{ps}^{(0)} D_{iv},$$
$$D_{iv}^{(f)} = D_{iv} + R_{ss}^{(0)} D_{iv} + R_{sp}^{(0)} D_{ip}, \qquad (31)$$

where $D_{ip}^{(f)}$ and $D_{iv}^{(f)}$ denote the receiver functions on surface for $P-$ and $S-$waves respectively.

To illustrate the construction of a four segment ray integral of $P_2 P_1 s_1 S_1$ [Chen 1991], for which capital letters denote the upward wave and lower case letter means downward wave, then, the corresponding potential function is represented as $\overline{\Phi}_{5143}$ in the following form

$$\overline{\Phi}_{5143}(s) = \overline{F}(s) \int_{-\infty}^{\infty} S_5(\xi_5) T_{pp}^{(1)}(\xi'_{51}) R_{pp}^{(0)}(\xi_{514}) R_{(1)}^{ps}(\xi'_{5143}) e^{s g_{5143}(x,z;\xi_{5143})} d\xi_5, \qquad (32)$$

$$g_{5143}(x,z;\xi_{5143}) = -i\xi_5 x_0 - \eta_5 z_0 + (\eta'_5 - \eta'_{51} - \eta'_{514} - \eta'_{5143}) r + i\xi_{5143} x + \eta_{5143} z, \qquad (33)$$

Based on the above derived formulas for curved layered media, the relationship between horizontal slownesses for parallel layered media, as shown in Fig. 3, can be easily obtained from Snell's law by referring to literature [Pao 1977] and the phase function for the forgoing four segment ray is given as follows

$$g_{5143}(x,z;\xi_{5143}) = -i\xi_5 x_0 - \eta_5 z_0 + (\eta_5 - \eta_{51} - \eta_{514} - \eta_{5143}) h + i\xi_{5143} x + \eta_{5143} z. \qquad (34)$$

3 EVALUATION OF RAY INTEGRALS BY CAGNIARD METHOD

So far we will employ the well-known Cagniard method to inverse the ray integral in Eq. (30), which may be further simplified as

$$\overline{u}(s) = \overline{F}_1(s) \int_{-\infty}^{\infty} S(\xi) \Pi D e^{s g(\xi)} d\xi, \qquad (35)$$

where Π denotes the product of reflection and transmisstion coefficients and D is the receiver function on the free surface. Then Eq. (35) can be inverted by two steps. The first step is to invert

$$\overline{I}(s) = \int_{-\infty}^{\infty} S(\xi) \Pi D e^{s g(\xi)} d\xi, \qquad (36)$$

to obtain the response $I(t)$ in time domain, and the second step is the convolution of $I(t)$ with $F_1(t)$, the inverse Laplace transform of $\overline{F}_1(s)$.

By observing Eq. (36), if we let

$$-t = g(\xi), \qquad (37)$$

then the inverse transform can be obtained by inspection.

With reference to literature [Pao, 1977] and Fig. 4, we find the stationary point M can be obtained by

$$\frac{dt}{d\xi} = 0, \qquad (38)$$

the root of the above equation denoted as

$$\xi_M = i b_M. \qquad (39)$$

and $t_M = -g(\xi_M)$.

Then the solution can be obtained by inspection as follows

$$I(t) = 2H(t - t_M) \Re[S(\xi(t)) \Pi(\xi(t)) D(\xi(t)) \frac{d\xi(t)}{dt}]. \qquad (40)$$

thus, the argument t_M in Heaviside step function defined in Eq. (40) is referred to as the arrival time of the direct ray, and the final solution of displacement can be obtained by

$$u(t) = \int_0^t F_1(t - \tau) I(\tau) d\tau, \qquad (41)$$

Furthermore, if Eq. (41) is integratged by part and remapped onto the $\xi-$plane by virtue of Eq. (37), it yields

$$u(t) = H(t - t_M) \int_{t_M}^{t} F'_1(t - \tau) \Re[\int_{\xi_M}^{\xi_1(\tau)} S(\xi) \Pi(\xi) D(\xi) d\xi] d\tau, \qquad (42)$$

4 NUMERICAL RESULTS AND CONCLUSION

Eq. (42) denotes the displacement for each individual ray, hence, the total response is expressed as

$$u_T(t) = \sum_i^{t_{M_i} < t} u_i(t), \qquad (43)$$

where t_{M_i} and $u_i(t)$ are respectively the arrival time and response for the individual ray.

To calculate the accelerograms due to the slipping of fault, we let the focus distance $z_0 = 16km$ and layer radius $r = 10km$, or thickness $h = 10km$, as shown in Figs. 1 and 3, and select $C_p = 5.5km/sec$, $C_v = 3.3km/sec$ and $\rho = 1900kg/m^3$ as the compression and shear wave speed and mass density respectively for surface layer, and the corresponding value for the half space are respectively $C_{p2} = 13.75 km/sec$, $C_{v2} = 8.25 km/sec$ and $\rho_2 = 2850 kg/m^3$. As for the subshear rupture speed for the fault is $V = 2.5 km/sec$.

So far, we will take into consideration the time function in Eqs. (15) and (16), which is given in literature [Niazy, 1975] as follows

$$f(t) = \begin{cases} 0, & t < 0 \\ (t/t_0)^2, & 0 \leq t \leq t_0/2 \\ \sum_{j=0}^{3} a_j t^j, & t_0/2 \leq t \leq t_0, \\ & a_0 = 2,\ a_1 = -10/t_0, \\ & a_2 = 17/t_0^2,\ a_3 = -8/t_0^3 \\ 1 & t_0 \leq t \end{cases}, \quad (44)$$

and shown in Fig. 5.

The value '1' in Fig. 5 denotes the unit moment $M = 0.87 * 10^{27} dyne - cm$, thus, with a choice of fault area $S = 4000 km^2$ and crust rigidity $\nu = 3 * 10^{11} dyne/cm^2$, we get a fault slip $\Delta u = M/(\nu S) = 72.8 cm$.

In this study, we calculate the responses with four two-segment and sixteen four-segment rays, the more segment rays are not considered for great loss of energy via multiple reflections from the free surface and interface. The results shown in Figs. 6 and 7 are respectively the responses of $x-$ and $z-$component acceleration received on epicenter for parallel layered media with a horizontal fault of length $L = 2km$, and the corresponding results for circular layered media are shown in Figs. 8 and 9. Apparently, the accelerations obtained from the wave propagating in circular layered media are stronger than those for parallel layerd media, this may be attributed to the energy concentration for circular boundary. Furthermore, if we check the Fourier specra, as shown in Figs. 10 to 13, for the corresponding accelerograms in Figs. 6 to 9, we find that the Fourier amplitudes for wave propagating in curlar media are much greater than those in parallel media. Hence, we may conclude that those structures erected above curved media must take more precautions against energy concentration.

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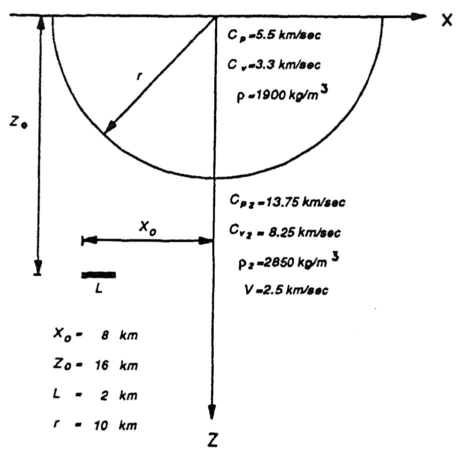

Fig. 1
Geometry of horizontal fault for circular media

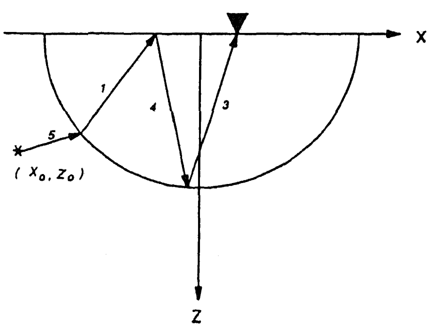

Fig. 2 Sketch of ray paths

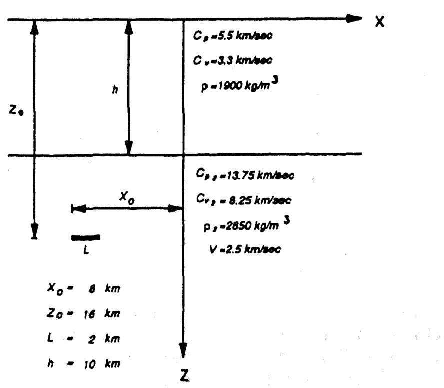

Fig. 3
Geometry of horizontal fault for parallel media

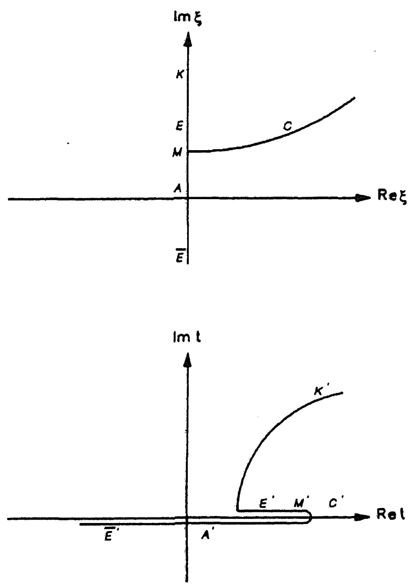

Fig. 4 The mapping of complex ξ onto real t

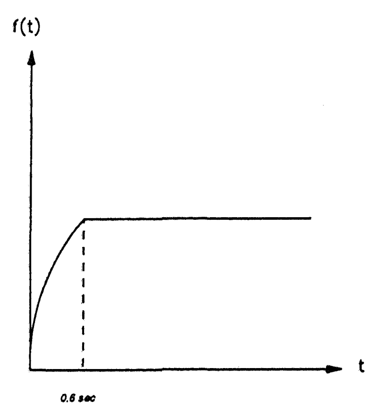

Fig. 5 The ramp time function for fault

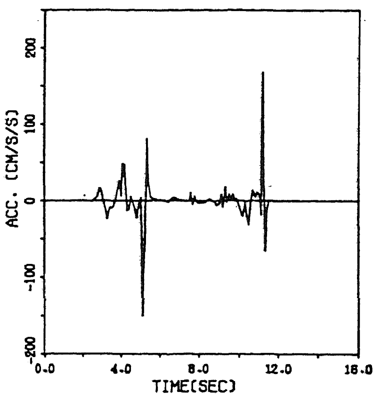

Fig. 6
x-acceleration at $x = 0km$ for parallel media

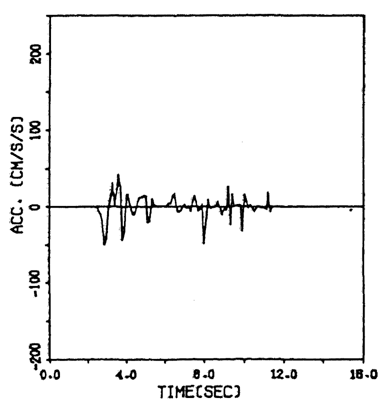

Fig. 7
z-acceleration at $x = 0km$ for parallel media

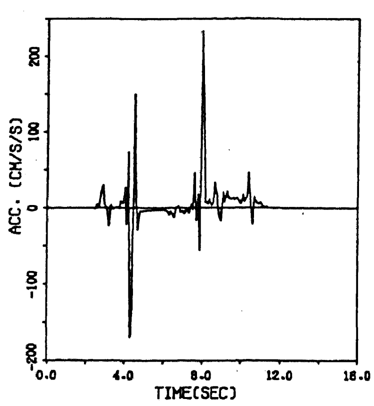

Fig. 8
x-acceleration at $x = 0km$ for circular media

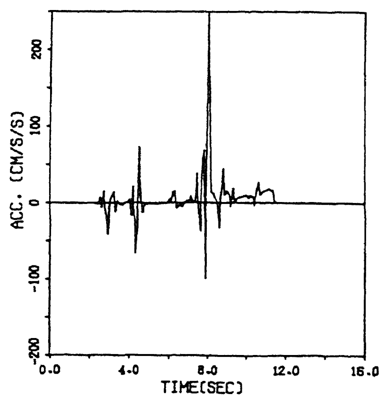

Fig. 9
z-acceleration at $x = 0km$ for circular media

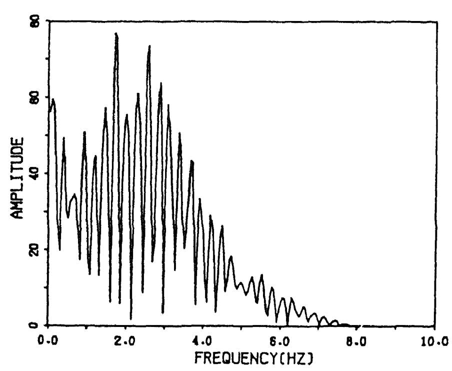

Fig. 12
Fourier spectrum of x-acceleration for circular media

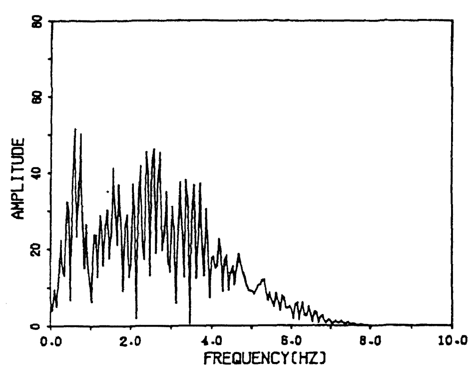

Fig. 10
Fourier spectrum of x-acceleration for parallel media

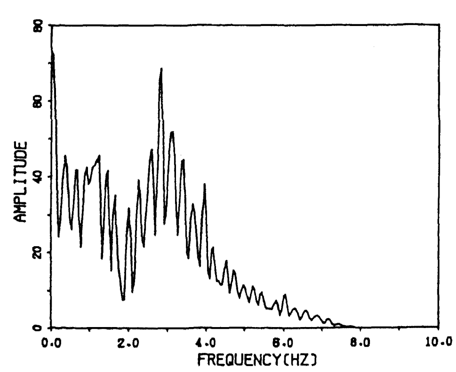

Fig. 13
Fourier spectrum of z-acceleration for circular media

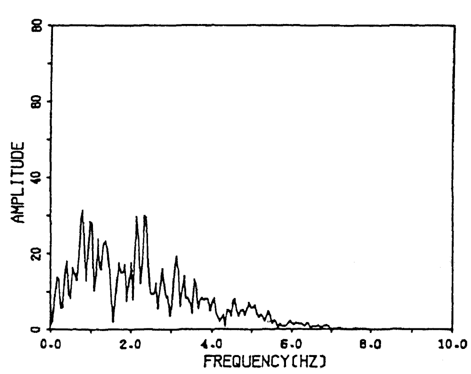

Fig. 11
Fourier spectrum of z-acceleration for parallel media