

Spectral analysis of earthquake accelerations as realizations of a non-stationary stochastic process

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ABSTRACT: The aim of this work is to obtain the evolutionary spectrum of an accelerogram by applying Priestley's theory. This procedure serves to analyze non-stationary random processes whose frequency content varies "slowly" over time. In spite of the fact that this theory is well-established, its practical application involves difficulties concerning the stationary case, in addition to those problems which are inherent in its time dependency. However, the theory is useful in giving an "interpretation" and "assessment" of the precision of the quantities that are actually measured. This work analyzes all these questions, and the multifilter technique is applied in order to estimate a "smoothed" version of the evolutionary spectral density function of an accelerogram.

1 INTRODUCTION

Since 1947, when Housner first suggested modelling earthquakes as realizations of a random process, the probabilistic models have been used as being the most suitable and rational way of analyzing and characterizing seismic motion from an engineering perspective.

The first models were stationary: series of pulses distributed randomly in time, white noise and filtered white noise. The use of stationary models has been justified on the basis of the fact that accelerograms simulated in this way, give rise to response spectra with damping that is similar to that obtained from the real earthquakes of great magnitude in a limited frequency range.

The need to modelize seismic motion by means of non-stationary stochastic processes arises when the ground behaviour and/or the structure is clearly non-linear because, in this case, the system might undergo deterioration with a strength loss during the major motion phase (intermediate phase) of the earthquake, and this would result in a reduction in its natural frequencies. In such a situation, the arrival of low frequency wave motion at the end of the earthquakes, can induce fatigue and/or structural resonance in the materials (Kameda (1975), Tiliouine (1982) and Shinozuka (1988)).

As the non-stationary state can be caused in the amplitude domain (time varying mean square value) and in the frequency domain (time varying power spectral density function), it is not surprising that the locally stationary, also called uniformly modulated or separable models, -amplitude domain- are the ones which have most often been used in practice, by the majority of authors.

Two lines of research can be distinguished from among the models that consider the variation of the frequencies in time: those which use the "physical spectrum" (Hoshiya (1980), Tiliouine (1982) and Bendimerad and Gere (1984)) and those which use the

"evolutionary spectrum" (Brant and Shinozuka (1969), Kameda (1975), Scherer, Riera and Schuëller (1982), Sugito and Kameda (1984), Spanos, Roesset and Donley (1987) and Sawada and Kameda (1988)). Since the second approach may be considered superior, at least from a theoretical point of view, it has been chosen for this work.

2 PRIESTLEY'S EVOLUTIONARY SPECTRAL DENSITY FUNCTION

The concept of "evolutionary spectra" was introduced by Priestley (1965) and is applied to a class of non-stationary processes known as oscillatory processes.

It is said that the random process $\{X(t)\}$ is oscillatory if there is a family of determinist "oscillatory" functions, $\phi(t, f)$:

$$\phi(t, f) = A(t, f) e^{i2\pi ft} \quad (1)$$

where $A(t, f)$ is a modulating function that varies slowly over time, so that $\{X(t)\}$ can be expressed as:

$$X(t) = \int_{-\infty}^{\infty} A(t, f) e^{i2\pi ft} dZ(f) \quad (2)$$

where $Z(f)$ is a random orthogonal process, in the sense that the increments $dZ(f)$, $dZ(f')$ are uncorrelated random variables. The most convenient way to define the slow variation of $A(t, f)$, is the one which assumes that the Fourier transform of $A(t, f)$ is highly concentrated in the zero frequency region, i.e.:

$$A(t, f) = \int_{-\infty}^{\infty} e^{i2\pi f\theta} dH(f, \theta) \quad (3)$$

with $|dH(f, \theta)|$ having an absolute maximum at $\theta = 0$. This restriction in the selection of

$A(t, f)$ maintains the physical interpretation of f as a frequency, a fundamental concept in non-stationary processes, Priestley (1971, 1981). In fact, for greater clarification of the question, it is first assumed that the process is stationary. The reason why the power spectral density function $-S(f)-$ of a stationary process can be interpreted as a power-frequency distribution i.e.:

$$E\{X^2\} = \int_{-\infty}^{\infty} S(f) df \quad (4)$$

lies in the fact that if $\{X(t)\}$ is a stationary process, the process has a spectral representation as follows:

$$X(t) = \int_{-\infty}^{\infty} e^{i2\pi ft} dZ(f) \quad (5)$$

that is to say, a stationary process can be represented, taking into account the theory of generalized harmonic analysis, by the "sum" of sine and cosine waves of different frequencies ($e^{i2\pi ft}$) and random amplitudes and phases ($dZ(f)$). In this sense, the concept of frequency has the conventional meaning associated with the harmonic functions.

On the other hand, if the process $\{X(t)\}$ is non-stationary, the functions ($e^{i2\pi ft}$) cannot be used as "basic elements", because they are intrinsically stationary. So, if one wishes to introduce the frequency notion into the analysis of the non-stationary processes, other "basic elements" will have to be chosen which, although non-stationary, have an oscillatory form, and in which the notion of frequency is fundamental. These oscillatory functions are the ones defined in (1), with the restriction that appears in (3). The general concept of "frequency" upon which the theory of "evolutionary spectra" is based, and the meaning of (3), can be better understood with the following example (Priestley, 1981). Suppose $X(t)$ is a deterministic function which has the form of a damped sine wave, say:

$$X(t) = Ae^{-\alpha t} \cos(2\pi f_0 t + \varphi) \quad (6)$$

If a conventional Fourier analysis of $X(t)$ is made, it can be seen that $X(f)$ contains components of all the frequencies, because there are two Gaussian functions centred on $\pm f_0$. In other words, if one wishes to represent $X(t)$ as the sum of the sines and cosines with constant amplitudes, an infinite number of frequency components are needed. However, $X(t)$ can be described perfectly by saying that it consists of just two frequency components ($\pm f_0$), each component having a time varying amplitude ($Ae^{-\alpha t}$). In fact, if the "local" behaviour of $X(t)$ is examined in the neighbourhood of an instant t_0 , the latter is what is observed, given that if the interval of observation were short, compared to α , $X(t)$ would always appear as a cosine function with a frequency of f_0 and an amplitude ($Ae^{-\alpha t_0}$). Therefore, if there is a non-periodical function $X(t)$ whose Fourier transform has an absolute maximum at point f_0 (or at $\pm f_0$, which is the actual case), f_0 can be defined as "the frequency" of this function, because "locally", $X(t)$ behaves like a sine wave with "conventional"

frequency f_0 , modulated by a "smoothly" varying amplitude function.

This type of reasoning provides the interpretational basis of the "evolutionary spectra", the spectral representation (2) of a non-stationary process being virtually a direct generalization of (6). Expression (2), can be interpreted as the limit of a "sum" of sines and cosines with different frequencies and time-varying random amplitudes, $\{A(t, f) dZ(f)\}$, in such a way that the mean square obtained "across" all realizations is;

$$E\{X^2(t)\} = \int_{-\infty}^{\infty} G(f, t) df \quad (7)$$

where $G(f, t)$ is the evolutionary spectral density function. In this latest expression, $G(f, t)$ describes -for each $t-$ the distribution in frequencies of the mean square in the neighbourhood of the time instant t ("local power").

In spite of the fact that Priestley's theory is well-established, its practical application entails all the difficulties associated with the stationary case, plus those inherent in its dependence on time. $G(f, t)$ is determined below, and all these questions are analyzed.

3 ESTIMATED EVOLUTIONARY SPECTRAL DENSITY FOR EARTHQUAKE ACCELERATIONS

3.1 Application of the Multifilter Technique to Estimate the Evolutionary Spectra.

The multifilter technique has been applied in order to: find the power spectral density function of random stationary processes and of transient processes, analyze the dispersion of the surface waves, determine the "response envelope spectrum" of accelerograms and, finally, to calculate the evolutionary power spectral density function (Kameda, (1975) and Scherer, Riera and Schueller, (1982)).

The aim of this work is to estimate $G(f, t)$ by means of the technique used by the last-mentioned authors. To do this, consider the discrete system with a single degree of freedom (S.D.O.F.) as a non-recursive causal filter. The movement equation of a S.D.O.F. whose base is subjected to excitation is:

$$\ddot{z}(t) + 2\xi(2\pi f_n)\dot{z}(t) + (2\pi f_n)^2 z(t) = -x(t) \quad (8)$$

where:

$\{x(t)\}$ = non-stationary random base acceleration (input)
 $\{z(t)\}$ = non-stationary random relative displacement response (output).
 ξ = relative damping of the oscillator.
 f_n = natural frequency of the oscillator.

As $\xi \ll 1$, the S.D.O.F. acts as a narrow band-pass filter, $z(t)$ can therefore be approached through the following expression:

$$z(t, f_n) = R(t, f_n) \cos[2\pi f_n t + \varphi(t)] \quad (9)$$

with $R(t, f_n)$ equal to:

$$R^2(t, f_n) = z^2(t, f_n) + \frac{[z(t, f_n)]^2}{(2\pi f_n)^2} \quad (10)$$

Taking the average over all realizations of expression (10), the following is obtained:

$$E[R^2(t, f_n)] = E[z^2(t, f_n)] + \frac{E[z^2(t, f_n)]}{(2\pi f_n)^2} \quad (11)$$

However, Priestley (1965, 1966), shows that:

$$E[z^2(t, f_n)] = \int_{-\infty}^{\infty} |H(f)|^2 S_x(t, f + f_n) df + O\left(\frac{B_n}{B_x}\right) \quad (12)$$

where $H(f)$ is the transfer function of the filter, B_n is the "time domain bandwidth" of the S.D.O.F. and B_x is a "characteristic width" of the process $\{x(t)\}$ that can be interpreted as the maximum time interval over which functions $A(t, f)$ are approximately constant and hence may be treated as stationary. Likewise, if:

$$B_n \ll B_x \quad (13)$$

an approximation of relation (12) can be obtained through:

$$E[z^2(t, f_n)] \approx \int_{-\infty}^{\infty} |H(f)|^2 S_x(t, f + f_n) df \quad (14)$$

If it is assumed that, for each t , $S_x(f, t)$ is smooth compared with $|H(f)|^2$, i.e. that its bandwidth is substantially longer than the bandwidth of $|H(f)|^2$, (14) can be expressed as follows:

$$E[z^2(t, f_n)] \approx \frac{S_x(t, f_n)}{(2\pi f_n)^3} \quad (15)$$

This relation (15) is exact for the particular case in which the input is "white noise", is a good approximation if the input is a stationary process and constitutes only a gross approximation in cases where the input is a non-stationary process. Substituting (15) and its derivative in (11), and taking into account the relationship between $S_x(t, f_n)$ and $G_x(t, f_n)$, the following is obtained:

$$G_x(t, f_n) \approx 4 \int (2\pi f_n)^3 E[R^2(t, f_n)] \quad (16)$$

Finally, the envelope process $R(t, f_n)$ is carried out taking Fig. 1 and expression (11) into account, so that $R(t, f_n)$ is the module of the complex function $\hat{z}(t)$. It is easy to obtain a "raw estimate" of $G_x(t, f_n)$ of an accelerogram from (16), because:

$$G_x(t, f_n) \approx 4 \int (2\pi f_n)^3 R^2(t, f_n) \quad (17)$$

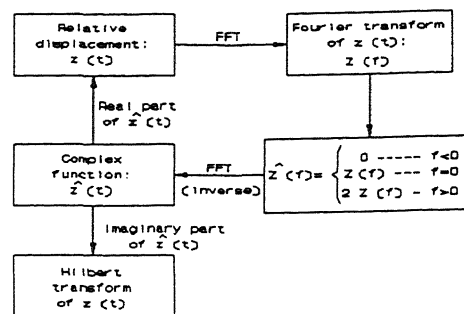


FIG. 1: Flow chart to calculate $\hat{z}(t)$

Therefore, the ensemble average of $R^2(t, f_n)$ has been eliminated in (17), because there was only one; so, to reduce sampling fluctuations, Priestley suggested "smoothing" the values of $G_x(f_n, t)$ for neighbouring values of t . In so doing, the precision of the estimate is increased, by sacrificing some degree of resolvability in the time domain. This smoothing is achieved with a weight function $W_T(t)$, in such a way that the smoothed spectrum is obtained through the new expression:

$$G_x(f_n, t) \approx \int_{-\infty}^{\infty} W_T(u) G_x(f_n, t-u) du \quad (18)$$

where the parameter T_p must satisfy the relation:

$$B_n \ll T_p \ll T \quad (19)$$

i.e., while $h(t)$ operates "locally" upon the process $\{x(t)\}$, $W_T(u)$ does so upon a considerably greater time interval. This work has taken as function $W_T(u)$ the rectangular one. Once $G_x(f_n, t)$ has been obtained through expression (18), it is important to find the precision of the spectral estimation. With this in mind, and for guidance purposes, the relative mean-square error $\eta^2(w)$ has been taken to measure the precision, and it is equal to:

$$\eta^2(w) = \epsilon^2(w) + \epsilon_b^2(w) \quad (20)$$

where, as is already known, ϵ , is the relative random error and ϵ_b is the relative bias error. The last-mentioned, can be calculated, Priestley (1966), by using the following approximate expressions:

$$\epsilon^2 \approx \frac{C}{T_p} \int_{-\infty}^{\infty} |H_n(w)|^4 dw \quad (21)$$

$$\epsilon_b^2 \approx \frac{1}{4} \left[\frac{B_w^2}{B_{ST}^2} + \frac{B_H^2}{B_{SF}^2} \right] \quad (22)$$

where: B_n = frequency domain bandwidth of the S.D.O.F., B_{ST} = time domain bandwidth of

$G_x(t, f_x)$, B_{SF} = frequency domain bandwidth of $G_x(t, f_x)$, B_w = time domain bandwidth of the weight function which is equal to $T_p/\sqrt{12}$ for the rectangular window and $C = 2\pi$ for the rectangular window.

An approximation of expression (21) can be obtained through:

$$\epsilon^2 \approx \frac{4\pi}{25T_p w_x \xi} \quad (23)$$

3.2 Analysis of the Results

The N-S components of the accelerogram from El Centro have been used as an example of application to determine the parameters necessary for the calculation of $G(f, t)$.

The parameters that would have to be quantified are as follows: filter (ξ) weight window (T_p) and non-stationary process (B_x , B_{ST} and B_{SF}).

Nevertheless, in practice, only ξ and T_p are determined, because $G(f, t)$ has to be known in order to be able to calculate B_{ST} and B_{SF} , and there is no method for finding B_x . Here lies the greatest source of difficulty in the rational application of the method: the estimation of parameters B_x , B_{ST} and B_{SF} . The first two are inherent to the non-stationary processes, whereas the third one (B_{SF}) is - in part - common to the stationary processes. In these last-mentioned processes, the frequency domain resolution (B_H/B_{SF}) is in no way restricted. However, if the process is non-stationary and B_H is reduced, B_x increases and this parameter is limited by restriction (13). In other words, in the non-stationary processes, as the filter operates only "locally" on $\{x(t)\}$, assuring a high degree of resolution in this domain, a certain degree of resolution has to be "sacrificed" in the frequency domain.

• Determination of Filter Damping.

When the multifilter technique was applied, it was assumed that a narrow band-pass filter was used and that its transitory phase was negligible. Both hypotheses are controlled by the damping value ξ allocated to the filter, in the following way:

- ξ has to be as small as possible, if the filter is to be narrow band-pass.
- On the other hand, so that the transitory phase effect disappears rapidly, ξ has to be as large as possible.

The conflict is solved by taking a compromise solution. Kameda (1975), recommends the use of values of ξ that range from 0.05 to 0.20. For the purpose of this work, the value of ξ has been chosen in such a way that the accelerogram simulated from $G(f, t)$ is as close as possible to the real one in three aspects: maximum acceleration (a_{max}), pseudo-relative velocity response spectra (S_p) and Fourier spectrum (S_f). A value of $\xi = 0.05$ has been chosen, taking into account this comparison (Uriel, Blázquez and Valerio, 1987).

• Determination of parameter T_p of the Weight Function

Parameter T_p influences in the following two ways:

- If the resolution in the time domain (B_w/B_x) is to be small, T_p must also be small, because as has already been stated:

$$B_w \approx \frac{T_p}{\sqrt{12}} \quad (24)$$

- On the other hand, if the relative random error is to be small, T_p must take a large value (23).

The conflict is solved in the same way as in the case of ξ , with a compromise solution. It has been observed, that for $\xi = 0.05$, it is better to take a value that is variable with the frequency, rather than a constant value, because the maximum acceleration of the simulated accelerogram is closer to the real value. This variation of T_p with the frequency can also be "justified" by merely optimizing $\eta^2(w)$ with respect to T_p . In fact, by replacing (22) and (23) with (20) and making zero the derivative of $\eta^2(w)$ with respect to T_p , we get the following expression:

$$T_p \approx 3,37 \sqrt[3]{T_x B_{ST}} \quad (25)$$

The value of T_p taken for this work, coincides with the value recommended by Kameda (1975), and has been adopted because it gives an acceptable solution in the time domain ($B_x \approx 1$ sec.) and a better peak value on the simulated accelerogram ($a_{max} = 328$ cm/sec²); nevertheless, it does contain a considerable relative random error in the frequency range analyzed ($\epsilon_r \approx 0.73 - 3.2$).

Using the values of the parameters taken for this work: $\xi = 0.05$ and $T_p = \min \{3T_x, 1.5\}$, the evolutionary spectra of the three components of the El Centro earthquake (1940) have been calculated (Uriel, Blázquez and Valerio, 1987). Figure 2, shows a three-dimensional representation of the normalized evolutionary spectrum of the NS component of that particular earthquake. Likewise, the accelerogram and the normalized evolutionary spectrum of some of the frequencies calculated in the 0.1 - 10Hz range, can be observed in Figure 3. The total number of frequencies analyzed was 47, and this amounts to taking a bandwidth (B_H) of the S.D.O.F. filter that is equal to $2\xi f_x$.

An interesting application of the evolutionary spectrum, which forms part of this work, consists of examining the energy concentrations that can be observed in the evolution in time, of both the spectrum and the accelerogram. In fact, if the three components are analyzed as a whole, it can be deduced that:

- There are 6 energy peaks in the first 30 seconds, at about 2, 5, 9, 12, 21 and 25 seconds (see Table 1).
- The most representative frequency for the evolution of the intensity of the horizontal components happens to be 2 Hz, because at this frequency (see Fig. 3), the above-mentioned 6 peaks can be clearly seen as having spectral levels which are very high, when compared to the maximums obtained (see Table 1).
- The maximum value of $G(f, t)$ obtained from the component NS is 5986 [cm/sec.²]²/Hz for $f = 1.822$ Hz and $t = 2.22$ seconds (Fig. 3).

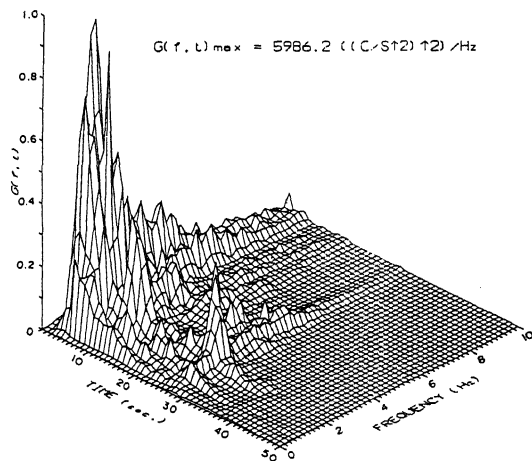


Fig. 2 Three-dimensional representation of normalized evolutionary spectrum of the earthquake accelerogram recorded at El Centro, NS-component, 1940.

- As for as the component EW is concerned, the maximum value of $G(f,t)$ is 3193 $[(\text{cm}/\text{sec}^2)^2/\text{Hz}]$ for $f = 0.818 \text{ Hz}$ and $t = 12.58 \text{ seconds}$.

- Finally, the maximum horizontal accelerations take place for $t = 2.12 \text{ seconds}$ (component NS) and $t = 11.44 \text{ seconds}$ (component EN), values which are roughly consistent with the position of the maximum absolutes of $G(f,t)$.

If each energy time concentration is associated with a packet of waves caused by an "event", the El Centro earthquake would have been caused by a sequence of 6 different events in the first 30 seconds (more seconds were not analyzed due to a lack of memory in the computer used). The result is partly consistent with the study carried out by Trifunac and Brune (1970). These authors analyzed the accelerograms for this earthquake and distinguished "at least 4 events", at 2, 8, 13 and 23.5 seconds, respectively. On the other hand, in a more recent survey carried out by Doser and Kanamori (1987), it has been calculated that this earthquake can be satisfactorily modeled through 4 to 6 point sources 17.5 Kms apart from each other, although the best

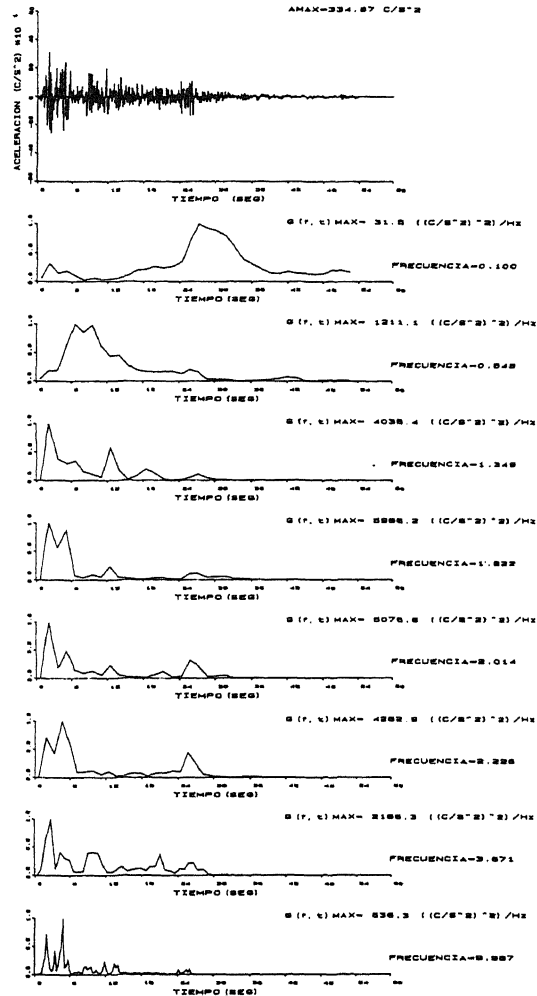


Fig. 3 The accelerogram and the normalized evolutionary spectrum of some of the frequencies calculated in the 0,1-10 Hz range (El Centro, NS-component, 1940)

Table 1. Energy peaks in the NS component of the Centro earthquake (1940)

ORDER NUMBER		1	2	3	4	5	6
ACELEROGRAM	t (sec)	2.12	4.82	8.94	11.84	20.74	25.74
	a (cm/sec ²)	334	240	166	199	-91	-113
MOST REPRESENTATIVE EVOLUTIONARY SPECTRUM	t (sec)	2.22	5.18	9.62	12.58	21.46	25.89
	f (Hz)	2	2	2	2	2	2
	G(f,t)	5076	2519	626	1165	601	1605
MAXIMUM VALUES ON EVOLUTIONARY SPECTRUM	t (sec)	2.22	5.18	9.62	12.58	21.20	25.74
	f (Hz)	1.822	1.822	0.548	1.349	3.671	2.222
	G(f,t)	5986	5274	1192	2364	801	1907

Expressed in $(\text{cm}/\text{sec}^2)^2/\text{Hz}$

adjustment was found to be with 5 events. In the light of these results, it is reasonable to point out that the technique used in this work is no less objective than the one used by Doser and Kanamori (1987) and Trifunac and Brune, (1970), and therefore it is possible that the El Centro earthquake of 1940, was a result of 5 or 6 events. In conclusion, Table 2 has been prepared with a view to partly endorsing this hypothesis. This table contains the number of absolute maximums found on the evolutionary spectrum and classified in time: peaks 1, 3, 4 and 6 are roughly consistent with those of Trifunac and Brune (1970), and 2 appears on many more occasions than 5, so it might correspond to the fifth event.

Table 2. Number of absolute maximums on the evolutionary spectrum corresponding to each one of the energy peaks (El Centro earthquake, 1940)

ORDER NUMBER	1	2	3	4	5	6
NS COMPONENT	15	24	-	-	1	3
EW COMPONENT	7	13	9	13	1	4
VERTICAL COMPONENT	15	9	7	15	1	-

N.B. The number of calculation frequencies is 47

4 CONCLUSIONS

In the light of the contents of this work, the following conclusions can be drawn:

- Priestley's theory is the most correct and powerful one that can be used, at present, for analyzing the non-stationary stochastic processes in the frequency domain, when their non-stationary characteristics are changing slowly over time.
- The practical application involves all the difficulties associated with the stationary case, plus the problems inherent in time dependency. However, the theory developed, serves to provide an "interpretation" and "to assess the precision" of the quantities that are actually measured.
- The evolutionary spectral density function, $G(f,t)$, was calculated using the multifilter technique and a "smoothed" version of $G(f,t)$ is the most that can be estimated, this being smoothed both in the time and frequency domains.
- The five parameters would have to be determined in order to estimate $G(f,t)$: damping of the filter (ξ), the averaging time of the weight function (T_p), characteristic width of the non-stationary process (B_t), analyzed and the bandwidths of the evolutionary spectrum in the time domain (B_{ST}) and frequency domain (B_{SF}). The greatest problems for a rational application of the theory, lie in the estimation of these problems, although from a practical perspective, these are reduced to two: ξ and T_p .
- With the values of ξ and T_p chosen, the estimation of $G(f,t)$ in the range of frequencies 0.1-10 Hz presents the following characteristics: $B_{ST} = 1$ sec., $B_{SF} = 2$ Hz, $\epsilon_t = 0.2$ and $\epsilon_f = 0.73-3.2$. This serves to show that the determination of $G(f,t)$ is fairly imprecise, although this should be used for guidance and in a qualitative way rather than quantitatively.
- Finally, given that $G(f,t)$ is capable of showing the evolution in time of each frequency, it is highly advisable to determine it in order to find out whether or not an earthquake is a sequence of "events" and, furthermore, so that the possible damage to Public Works can be estimated: either due to a degradation in the construction materials and/or as a result of the accumulative effects of successive events.

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