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Non-stationary models of ground motion

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ABSTRACT: A stochastic model of earthquake ground motion, with non-stationary amplitude and frequency content, is presented. This model is quantified on the basis of the information available from seismic hazard studies and strong ground motion seismology with the purpose of generating artificial accelerograms to be used in nonlinear analysis and design of structures. The stochastic model is defined by a 'reference' power spectrum and a frequency-dependent modulating function. The realizations of the stochastic model are obtained from the realizations of a stationary gaussian process, having the reference power spectrum, by modulating each sinusoidal component with the modulating function. Given a source-zones seismicity model the design eathquake is defined in terms of a return period and is characterized by its response spectrum, focal distance and magnitude. An application to the portuguese situation is presented.

1 INTRODUCTION

The usefulness of a stochastic model of strong earthquake vibration depends on two aspects: the easiness of its numerical implementation and the availability of seismological data for the quantification of its parameters. Some years ago a model should mainly be suitable to the programming skills and computer power available. With the development of computers this aspect is becoming less relevant and the possibility of quantifying a model through the optimal use of the existing data becomes the more important aspect. This means that, in principle and in regard to stochastic models, the randomness of the model should reflect only our uncertainty about the characteristics of the modeled ground motion, and the deterministic structure of the model should reflect our existing knowledge. To implement this orientation a detailed model of ground motion is needed; such a model is presented in this paper.

2 THE SPECTRUM-MODULATING FUNCTION MODEL

The spectrum-modulating function (SMF) model is a non-stationary stochastic model which is specified by a power spectrum $S(\omega)$ and a modulating function $m(\omega, t)$ and defined from the realizations (sample functions) of a gaussian stationary process $y_\epsilon(t)$ corresponding to the spectrum $S(\omega)$ and to the random variable ϵ. It is assumed that the non-stationary model is of finite duration i.e. $m(\omega, t) \equiv 0$ for $t \leq t_i$ and $t_f \leq t$, where t_i and t_f are the initial and final instants. Let $C_\epsilon(\omega)$ be the Fourier transform of $y_\epsilon(t)$ for the interval (t_i, t_f):

$$C_\epsilon(\omega) = \frac{1}{2\pi} \int_{t_i}^{t_f} y_\epsilon(t) e^{-i\omega t} dt \tag{1}$$

Then the corresponding realization of the SMF model $x_\epsilon(t)$ is defined as:

$$x_\epsilon(t) = \int_{-\infty}^{\infty} C_\epsilon(\omega) m(\omega, t) e^{i\omega t} d\omega \tag{2}$$

Those realizations correspond to an evolutionary power spectrum defined by:

$$S(\omega, t) = m^2(\omega, t) S(\omega) \tag{3}$$

The construction of a stochastic process directly from the σ - algebra of its realization is presented in Ibrahimov and Rozanov (1974).

The operationality of this model depends obviously on the type of specification of the modulating function. The following specification has given good results (Campos-Costa and Duarte, 1989):

(a) the modulating function is specified by a matrix m_{ij} defined for sequences of frequencies $(\omega_1, \omega_2, ...\omega_M)$ and instants $(t_1, t_2, ...t_N)$ with $\omega_{i+1} > \omega_i$ and $t_{i+1} > t_i$. Usually $\omega_1 = 0, t_1 = t_i$ and $t_N = t_f$.

(b) the values of the modulating function are given by

$$m(\omega, t) = \sqrt{m_1^2 + \frac{t-t_j}{t_{j+1}-t_j} \ (m_1^2)} \qquad (4)$$

where

$$m_1 = m_{ij} + \frac{\omega-\omega_i}{\omega_{i+1}-\omega_i} \ (m_{i+1j} - m_{ij}) \qquad (5)$$

$$m_2 = m_{ij+1} + \frac{\omega-\omega_i}{\omega_{i+1}-\omega_i} \ (m_{i+1j+1} - m_{ij+1}) \qquad (6)$$

and i and j are such that

$$\omega_i \leq \omega \leq \omega_{i+1} \qquad (7)$$

and

$$t_j \leq t \leq t_{j+1} \qquad (8)$$

This specification equates with a linear interpolation in frequency and a quadratic interpolation in time.

3 QUANTIFICATION OF THE SMF MODEL

The SMF model may be quantified independently in terms of its time dependent characteristics and its power spectrum.

The time dependent characteristics are determined by the modulating function or, more specifically, by the matrix m_{ij}. The values of this matrix may be assembled from a general understanding of the time evolution of the frequency content of the earthquake vibration and eventually refined by trial and error (Campos-Costa and Duarte, 1989); they may also be quantified by data obtained from recorded ground motion, namely the evolutionary spectrum time varying functions presented by Kameda, Sugito and Asamura (1980).

The power spectrum of the SMF model may be quantified from a response spectrum. This quantification is performed through an iterative procedure which is based on the possibility of computing the response spectrum from a SMF model. This computation can be made by identifying the response spectrum values with the mean value of the maximum response of a linear one-degree-of-freedom oscilators. This mean value is obtained from the nonstationary spectral moments of the oscilator response (Corotis et al., 1972) considering that its distribution is a Gumbel type I distribution which is calibrated from the 5% and 95% fractiles of the corresponding Vanmarcke distribution (Duarte, 1987). The power spectrum is specified as a polygonal with vertices S_j at frequencies ω_j chosen in a geometric progression with a common ratio no greater than $1 + 2\varsigma$ where ς is the damping factor for the response spectrum considered. To begin the iterative procedure, values are chosen for the the S_j (e.g. $S_j = 1 \text{ cm}^2/\text{s}^3$). Then sucessive procedure cycles are carried out until a satisfactory approximation is obtained. Each cycle consists in multiplying the S_j values by a factor β_j given by

$$\beta_j = \frac{(R_j - R_j^s)}{(R_j^d - R_j^s)} \qquad (9)$$

where R_j is the desired response spectrum value for frequency ω_j, R_j^S is the "static" response due to the frequency content below $(1-\varsigma)\omega_j$ and R_j^d is the "dynamic" response due to the frequency content above $(1-\varsigma)\omega_j$. In general less than 10 iterations are necessary to obtain a response spectrum within $\pm$ 5% of the desired response spectrum.

4 GENERATION OF TIME SERIES

The engineering usefulness of a stochastic model of ground motion mainly lies in the possibility of generating artificial accelerograms to be used as the input in nonlinear dynamic analyses. In the present case accelerograms (realizations of the SMF model) are easily generated by modulating the "elementary" components of the underlying stationary process:

$$\alpha(t) = \sum_i \sqrt{2A_i} \ m(\omega_i, t)\cos(\omega_i t + \psi_i) \qquad (10)$$

where $S(\omega)$ is assumed to be divided into intervals $\Delta\omega_i$ with central frequency ω_i and the ψ_i are sample values of independent random variables uniformly distributed in [0, 2 π]. The amplitude A_i is given by

$$A_i = \int_{\Delta\omega_i} S(\omega)d\omega \qquad (11)$$

The corresponding realizations of velocity v(t) and displacement d(t) are easily calculated substituting in expression (10) the A_i by $V_i = A_i/\omega_i^2$ and $D_i = A_i/\omega_i^4$ respectively.

However when a(t) is discretized in a time series of acceleration, the single and double integrals of that time series do not match v(t) and d(t). In consequence it is necessary to apply a long period filtering so that the differences between realizations of the process and the corresponding time series of acceleration, velocity and displacement are minimized (Campos-Costa, 1992).

5 EXEMPLIFICATION

The concepts presented above are illustrated by an application to Lisbon (Duarte and Campos-Costa, 1991). Seismicity is idealized by a source-zones model with non-radial attenuation functions

(Oliveira and Campos-Costa, 1984) and results are obtained through a computer algorithm based on McGuire's.

Due to the particularities of the portuguese seismicity, where nearby moderate magnitude and long distance large magnitude earthquakes may be expected, these two types of earthquakes were considered separately (Campos-Costa, 1992); if the focal distance is smaller or larger than 40 km earthquakes are considered to be near distance (type 1) or long distance (type 2). The hazard model adopted is constituted by the probability distribution of the maximum peak ground acceleration in a period of 50 years. This probability distribution is idealized as a Gumbel type I distribution:

$$f(\bar{a}) = \alpha(\exp - \alpha(\bar{a} - u) - \exp(-\alpha(\bar{a} - u))) \qquad (12)$$

The values of α and u were calibrated from numerical results obtained from the source-zones model and take the values $\alpha = 0.00225$ and $u = 87.36$ for the type 1 earthquakes and $\alpha = 0.0031$ and $u = 60.5$ for the type 2 earthquakes (with $\bar{a}$ expressed in cm/s^2).

In order to generate earthquake motions representative of those for which failures are likely to happen, the hazard study was carried out for a return period of 10 000 years. The corresponding peak ground accelerations are $317 cm/s^2$ for type 1 earthquakes and $233\ cm/s^2$ for type 2 earthquakes. The most likely earthquakes have a 7.1 magnitude and a 18 km focal distance and 8.3 magnitude and a 226 km focal distance; reference response spectra for those earthquakes were obtained through empirical models (Trifunac & Lee, 1989), and the time varying characteristics were defined from the spectral functions of Kameda, Sugito and Asamura (1980).

For illustration purposes, the time variation of the expected signal intensities of acceleration velocity and displacement are presented in figure 1; in figure 2 and 3 a realization of the type 1 and type 2 earthquakes are presented. The expected signal intensities and other time dependent statistics of the SMF model (Duarte and Campos-Costa, 1991) are particularly useful when the time variability of the model must be calibrated not from well defined data but from general trends of the ground motion parameters.

A more developed presentation of the identification of hazard-consistent ground motions may be found in Campos-Costa, Oliveira and Sousa (1992).

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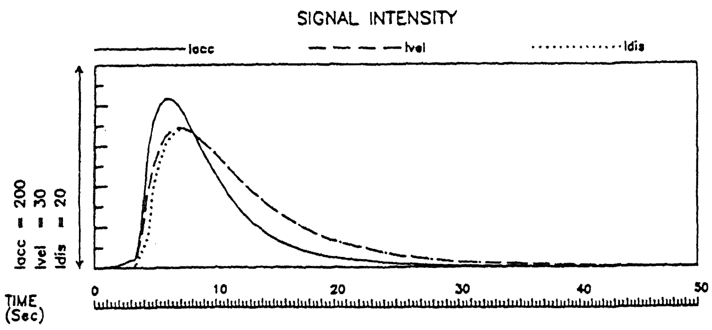

Figure 1: Time evolution of the expected signal intensity of acceleration (full lines), velocity (dashed lines) and displacement (dotted lines) of the type 1 earthquake model.

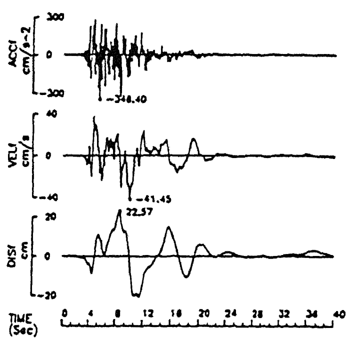

Figure 2: A realization of the type 1 non-stationary model.

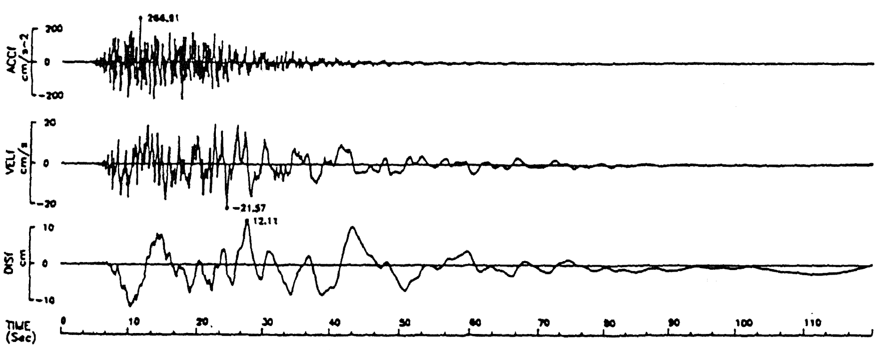

Figure 3: A realization of the type 2 non-stationary model.