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Simulation of seismic ground motion in the New Madrid area using analytically derived frequency-wave number Fourier amplitudes

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ABSTRACT: In this paper, analytic expressions of the frequency-wave number Fourier amplitudes of the displacement field at the free surface of a layered half-space are used to synthesize seismic ground motion. The generation of the displacement field is carried out numerically using the Fast Fourier Transform technique to perform the inversion from the frequency-wave number domain to the time-space domain. The earthquake that is generated in this study is believed to be representative of the earthquake sequence of 1811-1812 that struck the New Madrid area in the central Mississippi valley in USA. Using the numerically computed displacement field, first the frequency-wave number spectra and then the frequency-dependent autocorrelation function are estimated over an area and a time window where the seismic wave is approximately stationary and homogeneous.

1. DISPLACEMENT FIELD

Analytic expressions for the Fourier amplitudes of the displacement field at the free surface of a layered half-space were established in Theoharis, Deodatis and Shinozuka (1992) and in Theoharis, Deodatis and Papageorgiou (1992). The displacement field was produced by either a pure dip slip or a pure strike slip Haskell-type source having arbitrary orientation. When the slip had other orientation than parallel or perpendicular to the fault, the displacement field was obtained by summing up its pure strike slip and pure dip slip components (Theoharis, Deodatis and Papageorgiou, 1992):

$$U_n^T(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{U}_n^T(\kappa_x,\kappa_y,\omega)\cdot \cdot \mathrm{I}\cdot d\kappa_x d\kappa_y d\omega \quad ; \quad n = x,y,z \qquad (1)$$

where:

$$\mathrm{I} = \exp[-i\kappa_x x - i\kappa_y y + i\omega t] \qquad (2)$$

and:

$$\tilde{U}_n^T(\kappa_x,\kappa_y,\omega) = \tilde{U}_n^s(\kappa_x,\kappa_y,\omega) + \tilde{U}_n^d(\kappa_x,\kappa_y,\omega) \qquad (3)$$

In the above equations, subscript $n = x, y, z$ denotes the component of the displacement field, superscript s denotes a pure strike slip, superscript d a pure dip slip and superscript T the total displacement. A cartesian coordinate system is considered with z indicating the vertical direction (z=0 is the ground surface – positive z is facing downwards). κ_x and κ_y are the wave numbers and ω is the frequency. Closed-form analytic expressions for $\tilde{U}_n^s(\kappa_x,\kappa_y,\omega)$ and $\tilde{U}_n^d(\kappa_x,\kappa_y,\omega)$ can be found in Theoharis, Deodatis and Shinozuka (1992).

At this juncture it is mentioned that the computational method used to establish the expressions for $\tilde{U}_n^s(\kappa_x,\kappa_y,\omega)$ and $\tilde{U}_n^d(\kappa_x,\kappa_y,\omega)$ provides a complete description of ground motion (i.e. it accounts for all types of waves and there are no approximations involved) at any point of the ground surface (i.e. ground motion is computed both in the near-field as well as in the far-field).

Finally, it can be seen from Eq. 1 that the displacements in the space-time domain can be obtained by performing a triple Fourier transform of their Fourier amplitudes in the frequency-wave number domain. This is done with great computational efficiency using the Fast Fourier Transform (FFT) technique.

2. THE NEW MADRID FAULT ZONE

The New Madrid region is located in the southern part of the Mississippi embayment. During the winter of 1811-1812, three great earthquakes struck the central Mississippi River valley and virtually destroyed the town of New Madrid, Missouri. These three major events together with some large aftershocks represent the greatest historical seismic activity in the portion of the United States east of the Rocky mountains.

Figure 1 (taken from Herrman et al., 1978) shows the earthquake activity in the New Madrid fault zone for the period July 1974 through December 1977, indicating that the fault continues to be active.

In this paper, an event that is considered to be representative of the large earthquakes of 1811-1812 in the New Madrid area will be examined. Following the notation in Theoharis, Deodatis and Papageorgiou (1992), the source parameters for such an event are chosen as follows (Nuttli-Herrmann, 1984; Nuttli, 1973; Herrman and Jost, 1988):

Seismic Moment : $M_0 = 2 \cdot 10^{20}$ N $\cdot$ m
Rise Time of Ramp Function : $t_r = 3$ sec
Length Of Fault : $L = 52$ km
Width Of Fault : $W = 14$ km
Velocity Of Rupture : $v = 2.84$ km/sec
Depth Of Upper Edge Of Fault : $z = 6$ km
Strike Angle : $\phi = 220^o$
Dip Angle : $\delta = 70^o$
Slip Angle : $\lambda = 150^o$

From the above values, it is obvious that this event has both a strike slip and a dip slip component. The earth model to be used is that of a soft layer of 1 km depth overlaying the half-space. The properties of the soft layer are chosen as: elastic P-wave velocity $c_p = 2.0$ km/sec, elastic S-wave velocity $c_s = 1.3$ km/sec, mass density $\rho = 2.0$ gr/cm^3, attenuation factor $Q = 100$. The corresponding properties of the half-space are: $c_p = 6.15$ km/sec, $c_s = 3.55$ km/sec, $\rho = 2.8$ gr/cm^3 and $Q = 400$.

3. SIMULATION OF A NEW MADRID EARTHQUAKE

The displacement field at the free surface is retrieved using Eq. 1. The triple Fourier inversion is performed very efficiently using the FFT technique. Figure 2 shows the horizontal x and y axes on the ground surface and the 128×128 point grid that was used, covering an area of 447 km × 447 km.

Figures 3-5 display the computed x, y and z components of the displacement field at four time instants. As can be easily observed in these figures, the displacement field includes both propagating waves and permanent ground displacements. It is interesting to note that these permanent displacements (static part of the solution) change sign at specific subsequent points. For the z-component, for example, a significant uplift is immediately followed by a significant subsidence. This means that surface ground displacements of considerable magnitude will be visible as a gap after the earthquake ends. According to contemporary reports of the New Madrid 1811-1812 earthquake sequence (Herrmann et al., 1978), this was indeed the case. Note also that the static part of the solution is concentrated in the area above and around the fault and its maximum is located in the epicentral area. As far as the dynamic part of the solution is concerned, the generated waves propagate mainly in the same direction as the rupture.

4. ESTIMATION OF SPECTRA AND CORRELATION

The estimation of the frequency-wave number (F-K) spectra $S_{UU}(\kappa_x, \kappa_y, \omega)$ over an area $L_x \times L_y$ and a time window T where the seismic wave can be considered to be stationary and homogeneous is performed as follows:

$$S_{UU}(\kappa_x, \kappa_y, \omega) = \frac{1}{L_x L_y T} |D_{UU}(\kappa_x, \kappa_y, \omega)|^2 \tag{4}$$

where:

$$D_{UU}(\kappa_x, \kappa_y, \omega) = \frac{1}{(2\pi)^3} \int_0^{L_x} \int_0^{L_y} \int_0^T U(x, y, t) \cdot \mathrm{I} \cdot dx\, dy\, dt \tag{5}$$

where I was defined in Eq. 2. In Eq. 5, $U(x, y, t)$ is the computed displacement field. The FFT technique is used again to perform the triple Fourier transform shown in Eq. 5. The $L_x \times L_y$ area over which the F-K spectra are estimated is shown in Fig. 6 for the x-component of the displacement field. The time window T chosen to approximately comply with the stationarity assumption is 20 sec. Figure 7 displays the computed F-K spectra for the x-component at three values of the frequency. Studying Fig. 7, it is easy to identify most of the information concerning the direction of propagation, the dominant frequencies and the dominant wave numbers of the seismic waves.

Once the frequency-wave number spectra are available, the frequency-dependent autocorrelation function $C_{UU}(\xi_x, \xi_y, \omega)$ can be calculated numerically using again the FFT technique as:

$$C_{UU}(\xi_x, \xi_y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{UU}(\kappa_x, \kappa_y, \omega) \cdot \exp[i\kappa_x \xi_x + i\kappa_y \xi_y]\, d\kappa_x d\kappa_y \tag{6}$$

where ξ_x and ξ_y are the separation distances in the x and y directions, respectively.

5. CONCLUSIONS

In this paper, the displacement field at the free surface of a layered half-space has been synthesized for a representative earthquake in the New Madrid area according to the analytic expressions for the corresponding Fourier amplitudes in the frequency-wave number domain derived in Theoharis, Deodatis and Shinozuka (1992) and in Theoharis, Deodatis and Papageorgiou (1992).

Using the numerically computed displacement field, first the frequency-wave number spectra and then the frequency-dependent autocorrelation function were estimated over an area and a time window where the seismic wave is approximately stationary and homogeneous.

6. ACKNOWLEDGMENTS

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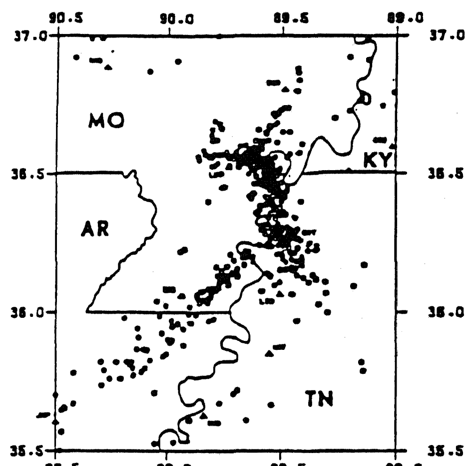


Fig. 1 Earthquake activity in the New Madrid area for the period July 1974 - December 1977.

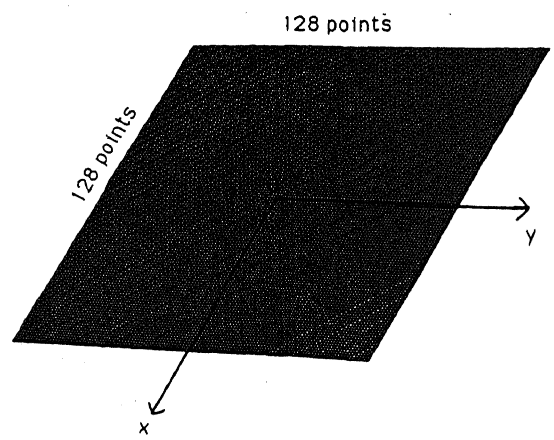


Fig. 2 Definition of axes for plots of displacement field.

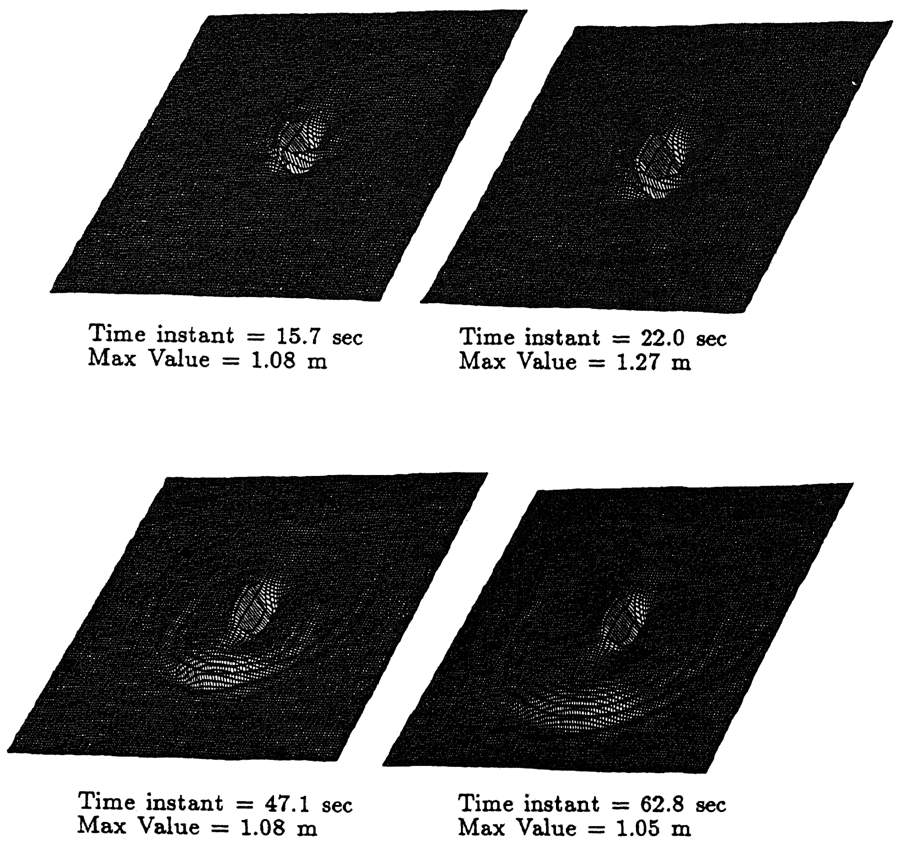


Fig. 3 The x-component of the displacement field at four time instants.

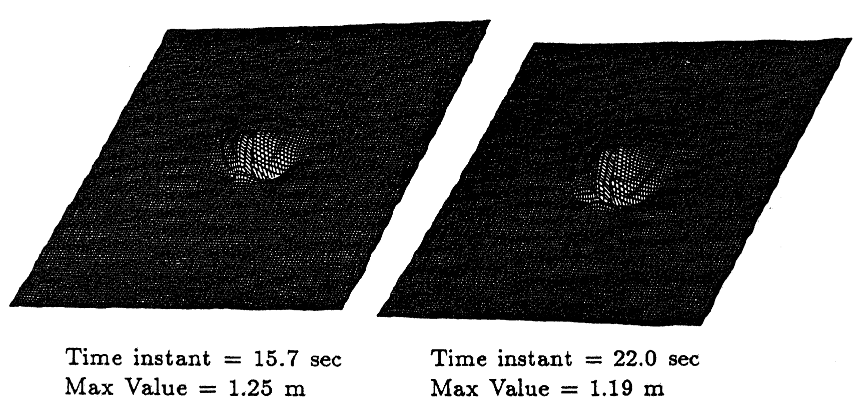
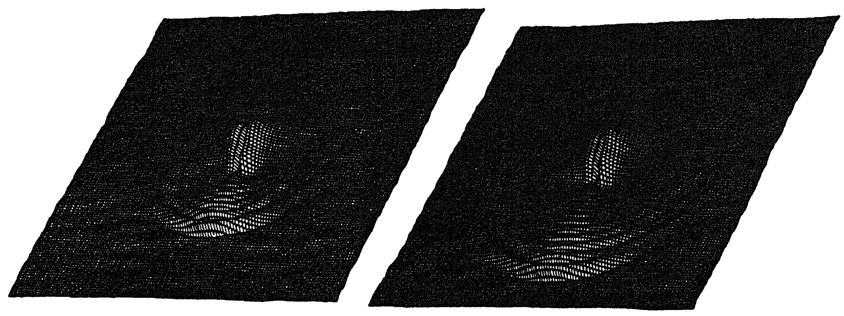


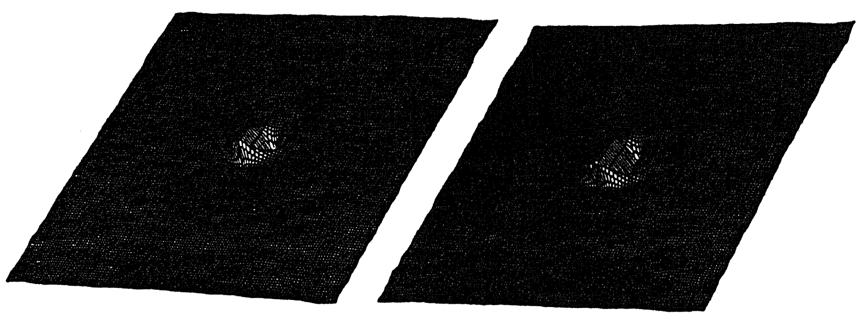
Fig. 4a The y-component of the displacement field at two time instants.



Time instant = 47.1 sec
Max Value = 0.80 m

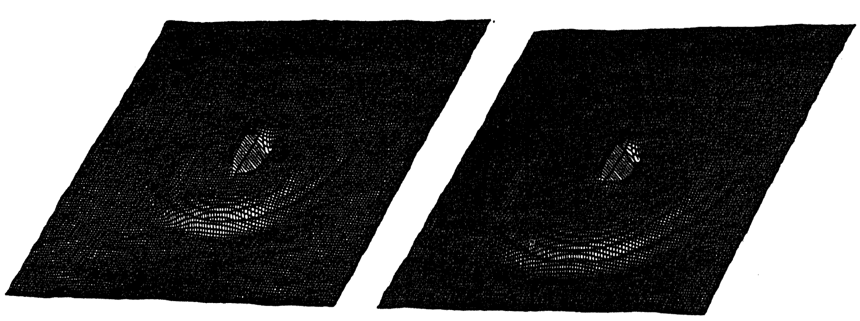
Time instant = 62.8 sec
Max Value = 0.77 m

Fig. 4b The y-component of the displacement field at two time instants.



Time instant = 15.7 sec
Max Value = 1.35 m

Time instant = 22.0 sec
Max Value = 1.33 m



Time instant = 47.1 sec
Max Value = 1.15 m

Time instant = 62.8 sec
Max Value = 1.18 m

Fig. 5 The z-component of the displacement field at four time instants.

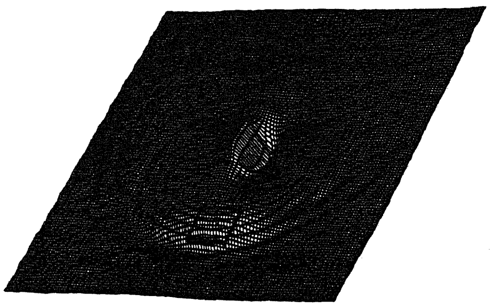


Fig. 6 Selected area over which frequency-wave number spectra are estimated.

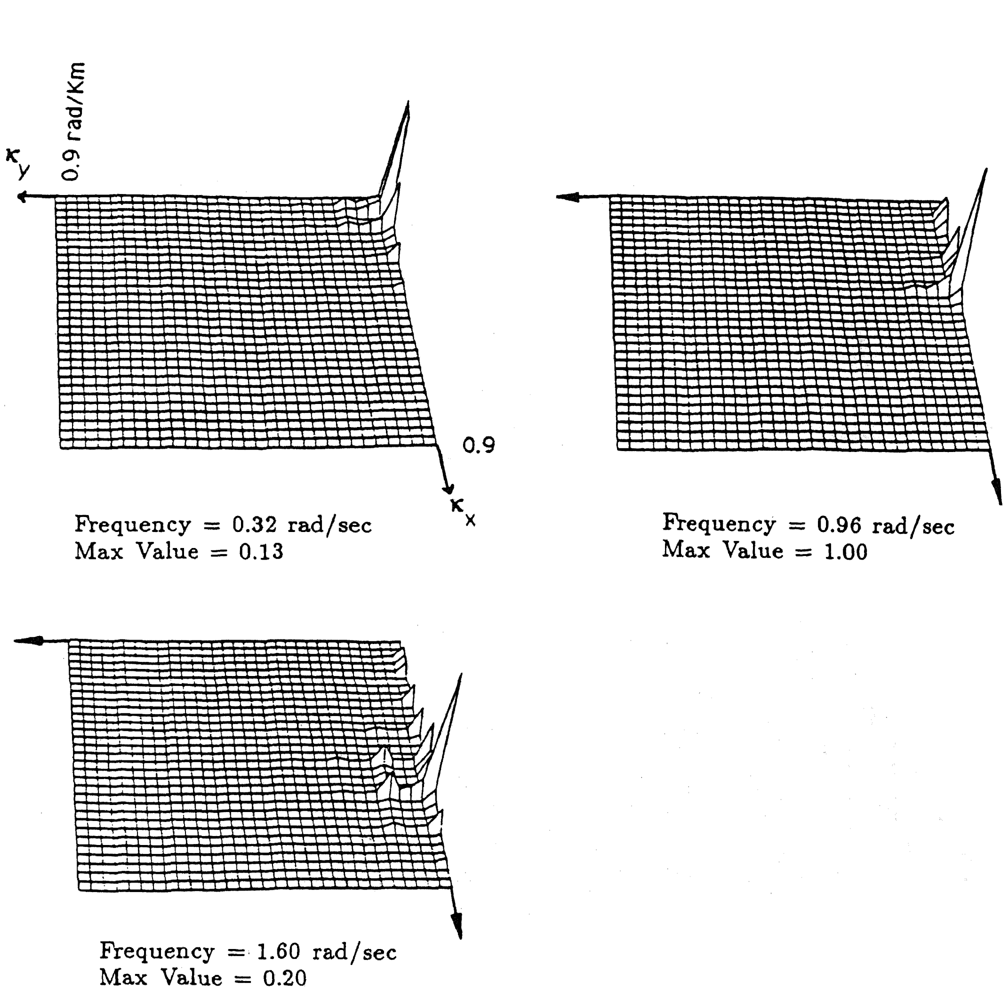


Fig. 7 Estimated frequency-wave number spectra over the area shown in Fig. 6, at three frequency values.