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Statistical parameters of AM and PSD functions for the generation of site-specific strong ground motions

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ABSTRACT: This paper presents the statistical results from a characterization analysis of some 158 recorded strong-motion accelerograms. The nonstationarity in the time domain is described by a new energy-based amplitude-modulating function which is developed in a normalized form for both duration and intensity. The frequency content is modeled with a sectionally stationary process and is characterized through the use of normalized Kanai-Tajimi power spectral density functions. Correlations between the function parameters and the peak ground acceleration, epicentral distance, strong-motion duration, and local geologic condition are investigated; and parametric values are recommended for the generation of site-specific strong ground motions.

1 INTRODUCTION

In the random vibration analysis approach to structural safety assessment, the ground motion representation typically requires two functions — the amplitude-modulating (AM) and the power spectral density (PSD) functions. Despite the engineering importance of these functions, there is, however, a general lack of statistics on the parameters describing these functions. Even more scarce is the statistical information relating these function parameters to the various site or event specific parameters such as epicentral distance, local geologic condition, peak ground acceleration (PGA) and strong-motion duration. The aim of this paper is to address this lack of statistical information for the generation of site-specific strong ground motions. In order to facilitate the statistical characterization of earthquake ground motions in the time domain, a new amplitude-modulating function is proposed. In developing this function, the main shortcomings of the AM functions in current use are addressed. In particular, the dependency of the current AM functions on strong-motion duration which makes any statistical characterization based on an ensemble of different earthquakes infeasible. For the characterization of ground motion in the frequency domain, a sectionally stationary process is assumed for the strong-motion. The frequency content is then characterized through normalized time-dependent Kanai-Tajimi (K-T) PSD functions.

2 A NEW AM FUNCTION

The nonstationarity of earthquake acclerations has been incorporated by many (e.g., Saragoni and Hart, 1972) using a deterministic amplitude-modulating function superimposed on a random process, i.e.,

$$a(t) = \Psi(t)\, x(t) \tag{1}$$

where $a(t)$ is the simulated ground acceleration, $\Psi(t)$ is a slowly varying deterministic function, and $x(t)$ is a generated random process. A number of different amplitude-modulating functions (e.g., Amin and Ang, 1968) have been suggested in the past for use in Equation 1. However, they are all duration dependent. The proposed new amplitude-modulating function is duration-normalized and has a trigonometric form with just two parameters. The function is

$$\Psi(t) = Z\, \sin^{\alpha}[\pi(\frac{t}{t_d})^{\beta}] \tag{2}$$

where α and β are the shpae parameters and t_d is the strong motion duration. This study uses the definition of strong-motion duration by Trifunac and Brady (1975) which is the time required for the energy to accumulate from 5% to 95% of the total Arias intensity. The term Z represents a normalizing factor which is defined later. The basic shape of the function is given by $\sin^{\alpha}[\pi(t/t_d)]$ and Figure 1 shows two sets of plots for different values of

α and β. It clearly demonstrates the versatility of the modulating function in characterizing a variety of nonstationary intensity shapes.

To determine the values of parameters α and β from a given actual earthquake accelerogram, the concept of the expected cumulative energy function of acceleration is used. The expected cumulative energy function for a strong motion time history is defined as

$$\mathrm{E}[W(t)] = \int_0^t \mathrm{E}[a^2(\tau)]\, d\tau \approx \int_0^t a^2(\tau)\, d\tau \quad (3)$$

in which $\mathrm{E}[a^2(\tau)]$ is the expected mean square acceleration at time τ. The approximation on the right side in Equation 4 indicates the fact that the expected mean square acceleration is estimated from one sample record instead of from an ensemble of records. Since the relative magnitudes of $\Psi(t)$ and $x(t)$ in Equation 2 are arbitrary, it is always possible to let $\mathrm{E}[x^2(t)] = 1$ such that the time dependent intensity variance is only represented by the square of the modulating function as follows:

$$\sigma_a^2(t) = \Psi^2(t)\, \mathrm{E}[x^2(t)] = \Psi^2(t) = RMS^2(t) \quad (4)$$

where $RMS(t)$ is the root mean square of the generated acceleration. The simulated ground acceleration $a(t)$ is then defined in the frequency domain by a unit area PSD function $S_x'(\omega)$, where the superscript $'$ indicates that the function is normalized. Then, it follows that

$$\mathrm{E}[W_N(t_n)] = \frac{\int_0^{t_n} \mathrm{E}[a^2(\tau)]\, d\tau}{\int_{t_d} \mathrm{E}[a^2(\tau)]\, d\tau} \approx \frac{\int_0^{t_n} \sin^{2\alpha}(\pi\tau^\beta)\, d\tau}{\int_0^1 \sin^{2\alpha}(\pi\tau^\beta)\, d\tau} \quad (5)$$

where t_n is the normalized strong-motion duration ($t_n = t/t_d$) and $0 \leq t_n \leq 1$. This equation represents the normalized expected cumulative energy function, $\mathrm{E}[W_N(t_n)]$, of an acceleration time history between the 5% and 95% Arias intensity. The α and β parameter values are obtained by fitting the envelope function (right hand side of Equation 6) to the normalized expected cumulative energy of the actual record. The least square error method is used for this purpose.

The stationary root mean square RMS_d over the strong motion duration t_d is defined as

$$RMS_d^2 = \frac{\int_{t_d} a^2(\tau)\, d\tau}{t_d} \quad (6)$$

The expected total energy, or the product of RMS_d and t_d, of the excitation process, estimated from a single time history is $RMS_d^2\, t_d \approx \int_{t_d} \mathrm{E}[a^2(\tau)]\, d\tau$

Thus

$$\Psi^2(t) = RMS(t) \approx RMS_d^2 \frac{\sin^{2\alpha}[\pi(t/t_d)^\beta]}{\int_0^1 \sin^{2\alpha} \pi\tau^\beta\, d\tau} \quad (7)$$

In the above equation, the square of the modulating function is derived for the strong motion duration t_d (contains only 90% of the total energy). A scaling factor can be used to adjust the intensity of $\Psi(t)$ accounting for the 10% total energy discarded at the beginning and the end of the record. The final form of the modulating function after adjustment is

$$\Psi(t) = Z \sin^\alpha[\pi(t/t_d)^\beta] \quad (8)$$

where

$$Z = \sqrt{\frac{10/9}{\int_0^1 \sin^{2\alpha}(\pi\tau^\beta)\, d\tau}}\, RMS_d \quad (9)$$

3 SECTIONALLY STATIONARAY POWER SPECTRAL DENSITY FUNCTION

The frequency content of a random motion may be characterized through the Kanai-Tajimi PSD function.´ The K-T PSD describes how the power is distributed among the frequencies of vibration:

$$S_{K-T}(\omega) = \frac{[1 + 4\xi_g^2(\omega/\omega_g)^2]}{[1 - (\omega/\omega_g)^2]^2 + 4\xi_g(\omega/\omega_g)^2}\, S_o \quad (10)$$

where $S_{K-T}(\omega)$ is the energy content at frequency ω, S_o is a measure of shaking intensity, and ω_g and ξ_g are commonly interpretated as the predominant ground frequency and ground damping respectively. The K-T functional form has been widely used to develop PSD functions by previous investigators (e.g., Lai, 1982). In the study performed by Lai, 140 stong-motion records were analyzed to establish the statistics and dependencies of the K-T parameters. However, the results from those studies are based on the assumption of stationarity in frequency content.

To account for the nonstationarity in frequency content using the K-T PSD funciton, the frequency and damping parameters have to be time-dependent. For simplicity, this work describes the temporal variation of frequency content by dividing the strong-motion duration t_d into three sections of equal time period, t_1, t_2, and t_3. Accordingly, the approach presented herein assumes a sectionally stationary process (in the frequency domain). A normalized PSD function $S'(\omega, t_i)$ is obtained separately for each section of t_i. Since $S'(\omega, t_i)$ always has unit area, the characterization of the nonstationarity of intensity content and frequency content can be performed independently. Under the assumption of sectional stationarity, the temporal PSD function $S'(\omega, t_i)$ for the complete ground excitation is expressed by using

$$S(\omega, t_i) = \Psi^2(t)\, S'(\omega, t_i) \tag{11}$$

where t_i is referenced to section t_1, t_2, or t_3.

The Fast Fourier Transformation (FTT) method was used to obtain the PSD function from the strong-motion records. The PSD functions estimated on this basis were fitted by smooth K-T PSD functions using the "spectral moments" method. This method has been widely used (e.g., Lai, 1982). As shown by Equation 12, there are three parameters required to define the K-T PSD function: S_o, ω_g, and ξ_g. The method of spectral moments computes these parameters in such a way that the zero, first and second spectral moments of the PSD function estimated from the accelerograms are identical to those of the fitted K-T PSD function.

4 STATISTICAL ANALYSIS OF FUNCTION PARAMETERS

The statistical characterization of the earthquake ground motions using amplitude-modulating and PSD functions was performed using the horizontal components 158 strong motion accelerograms, 36 of them are from the 1989 Loma Prieta earthquake (Shakal et al., 1989) and the rest are taken from Volume II of the Caltech series of corrected data (Hudson, et al., 1971). For the characterization using amplitude-modulating function, all records were used and 96 were classified as soil site records and 62 were classified as rock site records. Due to the limitation of the numerical procedure (FFT) used, only 116 records were used to characterized the nonstationary nature of the strong-motion frequency content and 72 were classified as soil site records and 44 as rock sites records. A summary of the statistics on α, β, ω_g, and ξ_g is given in Table 1.

Histograms of the α and β parameters were constructed. Lognormal probability density functions (PDFs) were fitted to the histograms of α for soil and rock sites using the mean and standard deviation values in Table 1. For the parameter β, using the values in Table 1, exponential PDFs were fitted to the histograms. Figures 2 and 3 show the histograms and the fitted PDFs for α and β respectively.

The correlations in the time domain between α, β, and the other site and event specific parameters (PGA, epicentral distance, RMS, strong- motion duration) were also investigated. From the analysis results, it is observed that the modulating function parameter α has the strongest correlation with the strong-motion duration. This follows past observations that longer duration earthquakes tend to have a longer stationary portion of intense shaking. Therefore, the records appear to be more stationary overall and this is reflected by smaller α values as shown in Figure 1. It is also observed that the modulating function parameters α and β are correlated with a correlation coefficient of about 0.5 for both soil and rock site records.

Linear regression analyses were performed for the α versus strong-motion duration and β versus α relationships. The results are

$$\alpha(t_d) = \begin{cases} -0.020t_d + 1.04 & \text{soil sites} \\ 1.14 & \text{rock sites} \end{cases} \tag{12}$$

and

$$\beta(\alpha) = \begin{cases} 0.21\alpha + 0.05 & \text{soil sites} \\ 0.14\alpha + 0.15 & \text{rock sites} \end{cases} \tag{13}$$

where t_d is the strong-motion duration in seconds. The linear regression analyses results are shown in Figures 4 and 5 and the standard deviations of the linear regression estimates are also given in the plots.

The evolutionary characteristics in frequency content for both soil and rock site records are observed from the changing statistics of ω_g corresponding to time sections t_1, t_2, and t_3. Most of the K-T frequencies tend to decrease with time. This phenomenon is in agreement with the general belief that the longer the distance the seismic waves travel through the earth, the more the high frequency commponents in the motion will be filtered out. The K-T damping coefficients are also time varying quantities, however, they evolve with little observed regularity. The statistics obtained are different from those computed by Lai (1982). For example, ω_g (t_d) for soil and rock sites in this study were found as 13.8 and 21.9 rad/sec respectively but the corresponding quantities were 19.1 and 26.7 rad/sec by Lai.

The temporal variation of frequency content are better observed by inspecting the temporal variation of sample distributions of ω_g. The histograms and the fitted analytical PDFs of K-T frequencies (soil sites) are shown in Figure 6 for time sections t_1, t_2, and t_3. Histograms of ω_g for rock sites are similar. The Gamma function is adopted to fit the histograms and its parameters are derived by matching the mean and the variance of the analytical function to those of the histograms.

The correlations between the frequency domain parameters and the site or event specific parameters were investigated and they were found to be too weakly correlated to have any significant meaning.

5 CONCLUSIONS

In modeling earthquake ground motions, the amplitude-modulating and power spectral density functions are important parameters defining the characteristics of the strong-motions. The aim of this study is to analyze the statistics of the parameters of these functions used to characterize strong ground motions. In order to facilitate this statistical characterization analysis, in the time domain, a new amplitude-modulating funciton which is duration and intensity normalized is proposed. For the frequency domain analysis, the Kanai-Tajimi PSD and a pseudo-evolutionary or a sectionally stationary process are used to describe the temporal variation of frequency content. Useful statistics were obtained and recommended for the generation of site-specific strong ground motions.

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Table 1: Statistics on the parameters of the amplitude-modulating and the Kanai-Tajimi power spectral density functions (ω_g in rad/sec and all other parameters are unitless).

	Section	Mean	Variance	Site Geology
α	t_d	0.706	0.433	Soil
α		1.050	0.691	Rock
β		0.250	0.193	Soil
β		0.199	0.194	Rock
ω_g	t_d	13.80	7.68	Soil
ω_g		21.90	12.33	Rock
ξ_g		0.346	0.175	Soil
ξ_g		0.353	0.133	Rock
ω_g	t_1	15.72	7.69	Soil
ω_g		23.57	11.99	Rock
ξ_g		0.343	0.152	Soil
ξ_g		0.352	0.130	Rock
ω_g	t_2	11.78	6.71	Soil
ω_g		21.12	12.98	Rock
ξ_g		0.333	0.140	Soil
ξ_g		0.394	0.145	Rock
ω_g	t_3	8.51	10.24	Soil
ω_g		18.38	12.28	Rock
ξ_g		0.327	0.144	Soil
ξ_g		0.417	0.162	Rock

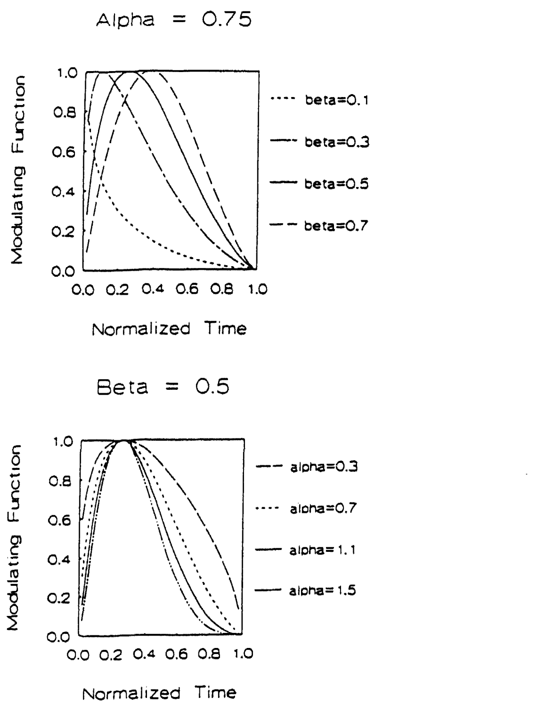

Figure 1: Two plots showing the effects of the proposed modulating function parameters α and β on the shape of the function.

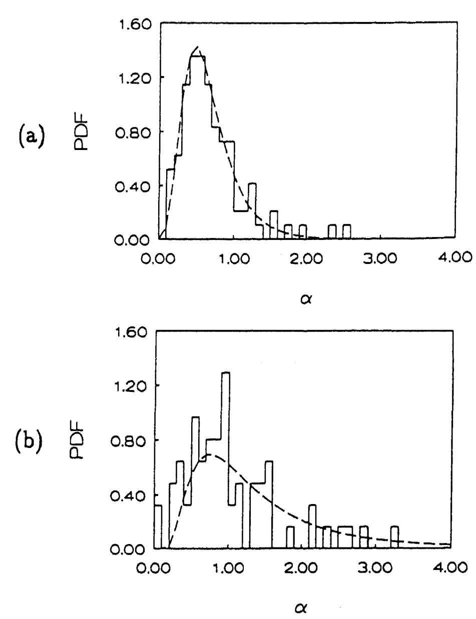

Figure 2: Histograms and fitted probability distribution functions for α: (a) soil and (b) rock.

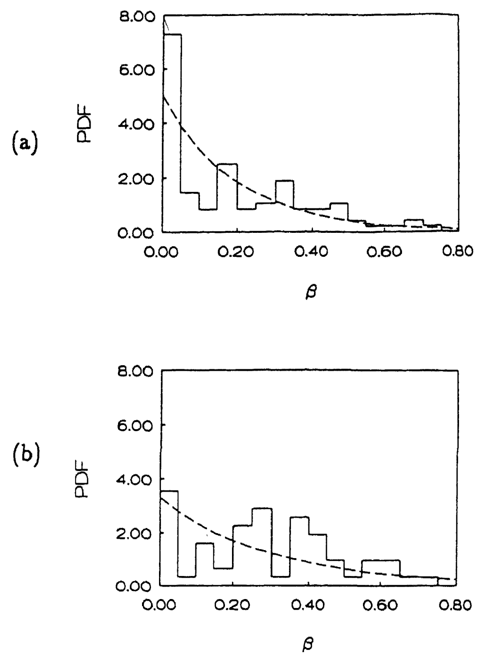

Figure 3: Histograms and fitted probability distribution functions for β: (a) soil and (b) rock.

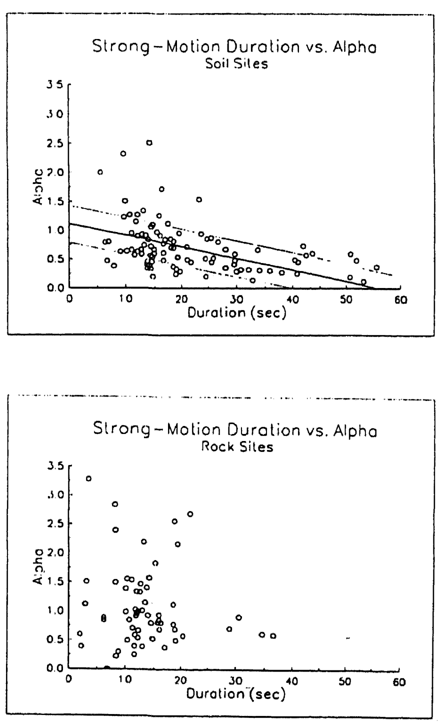

Figure 4: Scattergram of strong-motion duration versus α.

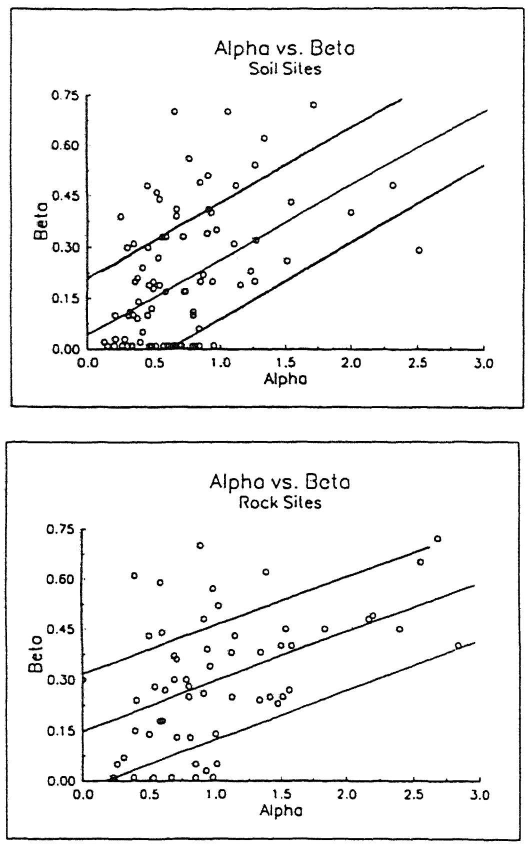

Figure 5: Scattergram of α versus β.

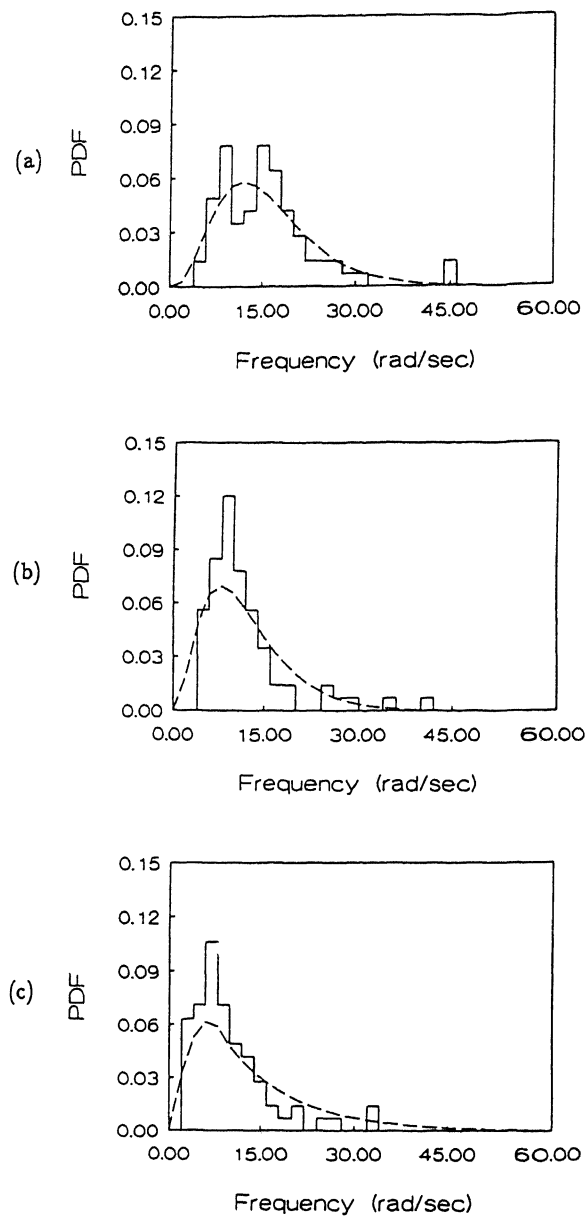

Figure 6: Histograms and fitted probability distribution functions for ω_g (soil sites): (a) t_1, (b) t_2, and (c) t_3.