Random field models of spatially varying ground motions and the estimation of differential ground motions

Keichi Tamura  
Public Works Research Institute, Ministry of Construction, Tsukuba, Japan

Steven R. Winterstein & Harsh C. Shah  
Stanford University, Calif., USA

ABSTRACT: Two simple stochastic models of spatially varying ground motions are established and calibrated from strong motion array data at four sites. The first model chooses a coherence function that varies with separation distance but is independent of frequency, employing a coherence value estimated at the ground motion predominant frequency as a representative. The second model assumes the infinite propagation velocity of seismic waves, so that the time-space covariance function of ground displacement is a separable function of time and space. These models are applied to estimate the maximum relative displacement between two points on the ground surface, using conventional Poisson models of temporal and spatial extremes. Good agreement is found between the estimated and observed maximum relative displacement.

1. INTRODUCTION

Various studies on the spatial variation of ground motions have been conducted with the recent development of closely spaced arrays of accelerographs. Particularly, a number of studies (e.g., Harada, 1984; Loh and Yeh, 1988) discuss the differential ground motion characteristics, which may have significant influence on the dynamic behavior of buried lifeline facilities, multiple-span bridges, widely spread tanks, and so on.

Two simple variants of standard ground motion spectral models are here established (Tamura et al., 1990, 1991). They are calibrated from strong motion array data at four sites in Shizuoka Prefecture, Japan. Since an attention is focused on the strong motion duration on the nearly homogeneous ground, the proposed models adopt a stationary Gaussian random field model of each event's ground displacement in time and space.

The first model employs a coherence function that varies with separation length but is independent of frequency. A coherence value is estimated at the ground motion predominant frequency in this model. The second model assumes the apparent horizontal propagation velocity of seismic waves to be infinite, so that the time-space covariance function of ground motion is a separable function of time and space.

The proposed two models are applied to estimate the maximum relative displacement between two points on the ground surface, using a Poisson arrival process model of extremes. Both the temporal maximum over any fixed distance and the spatial maximum at any fixed time can be estimated by the present procedure.

2. PROPOSED GROUND MOTION SPECTRAL MODELS

The time-space cross spectral density function of ground motions at two points separated by a distance $\eta$ may be expressed as (Harichandran, 1988)

$$S(f, \eta) = S_0(f) \eta f \exp(-i 2 \pi f \eta / c)$$

(1)

where $S_0(f)$ is the power spectral density function of displacement at any fixed location, $\eta f$ is the coherence function, and $c$ is the apparent propagation velocity of seismic waves. In the first model, a coherence value is estimated at the predominant frequency of ground motion, and it is assumed that the coherence function $\eta f$ is independent of frequency. Then, Eq.(1) can be reduced to

$$S(f, \eta) = S_0(f) \eta f \exp(-i 2 \pi f \eta / c)$$

(2)

Let us call this model as the "Frequency Independent Coherence (FIC) model". The time-space covariance function for this model can be easily obtained by taking a Fourier transform of Eq.(2).

The time-space covariance function of ground displacement, which is a counterpart of Eq.(1) in the time lag domain, may be described as

$$C(t, \eta) = C_0(t) \eta \exp(-2 \pi f_2 \eta)$$

(3)
where $C_0(t)$ is the auto-covariance function of
displacement at any location, and $\rho(t)$ is the
correlation function in space at any fixed time. The
second model assumes $c$ in Eq.(3) to be infinite, then
Eq.(3) can be reduced to a separable function of
time and space as

$$C(t,\eta)=C_0(t)\rho_0(\eta)=\sigma_0^2 \rho(t)\rho(\eta)$$

(4)

where $\sigma_0$ is the root mean square value of
displacement, and $\rho(t)$ is the correlation function in
time at any fixed spatial location. Let us call this
model as the "Time-Space Separable Correlation
(TSSC) model". While FIC model required $S(\eta)$,
$\rho(t)$ and $c$ to be determined, this model requires
only either $C_0(t)$ or $S(\eta)$, and $\rho(t)$.  

3. MAXIMUM RELATIVE DISPLACEMENT

The time-space covariance function of relative
displacement between two points with a separation
distance $\xi$ is expressed in terms of corresponding
displacement covariance function as

$$C_d(t,\eta,\xi)=2C(t,\eta)C(t,\eta+\xi)-C(t,\eta)-C(t,\eta+\xi)$$  

(5)

The mean square value of relative displacement can be
obtained by setting $t=\eta=0$ in Eq.(5):

$$\sigma_0^2=C_d(0,0,\xi)$$

(6)

Assuming that the crossings of a specified threshold
occurs as a Poisson arrival process, the $p$-fractile of
the maximum relative displacement over a temporal
or spatial window length $B$, may be estimated as
(Vanmaecke and Lai, 1980)

$$d_{p,B} = \frac{\sqrt{\ln(-2Bln p)} \cdots -2Bln p}{\sigma_0}$$

(7)

in which $\sigma_0$ is given by Eq.(6), and $L_D$ is estimated
as

$$L_D = 2\sigma_0 \sqrt{\frac{C_d(t,\eta,\xi)}{\partial^2 C_d(t,\eta,\eta)\partial^2 \tau^2}}$$

(8)

for temporal maximum

$L_D$ for spatial maximum is derived by taking a
differential with respect to $\eta$, instead of $t$, in Eq.(8).

4. ANALYSIS OF ARRAY DATA

The array data obtained at four sites in Shizuoka
Prefecture, Japan are used in this analysis. Fourteen
to eighteen tri-axial 16 bits digital strong motion
accelerographs are installed at each site. Figure 1
shows an example of array deployments. The array
records here analyzed were obtained from four
earthquakes shown in Table 1. Both radial and
transverse components have been analyzed, however
the results for radial components are here presented
due to limitations of space.

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Epicenter Region</th>
<th>Magnitude</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sagara</td>
<td>11/24/85</td>
<td>S Coast of Chubu</td>
<td>5.0</td>
<td>45</td>
</tr>
<tr>
<td>Yamanaka</td>
<td>9/14/84</td>
<td>Chubu</td>
<td>6.8</td>
<td>126</td>
</tr>
<tr>
<td>Numazu</td>
<td>9/5/88</td>
<td>E Yamanashi Pref</td>
<td>5.6</td>
<td>45</td>
</tr>
<tr>
<td>Matsuzaki</td>
<td>11/22/86</td>
<td>Near Izu-Oshima Is.</td>
<td>6.0</td>
<td>72</td>
</tr>
</tbody>
</table>

Figure 1. Array at Numazu site

4.1 Temporal and spatial correlation functions

The following function is assumed to represent the
temporal correlation function (auto-correlation function)
of ground displacement (Tamura et al., 1990, 1991):

$$\rho(t)=\cos(2\pi\tau T_0)/[(2\pi\tau T_0)^2+1]$$

(9)

where $T_0$ and $\alpha$ are the parameters determined by
least squares fit to the temporal correlation function
calculated from displacement data. An example of
least squares fit is shown in Figure 2. The solid and
dashed lines correspond to temporal correlation
functions calculated from the observed data and
Eq.(9), respectively. The parameters of ground
motion models are listed in Table 2. \( B_T \) and \( B_1 \) in Table 2 represent the temporal and spatial intervals in which the stationarity of ground motion is assumed, respectively.

<table>
<thead>
<tr>
<th>Size</th>
<th>( B_T ) (sec)</th>
<th>( B_1 ) (m)</th>
<th>( C ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saga-</td>
<td>8.5</td>
<td>0.70</td>
<td>1000</td>
</tr>
<tr>
<td>ra</td>
<td></td>
<td>0.25</td>
<td>740</td>
</tr>
<tr>
<td>Yai-</td>
<td>0.95</td>
<td>0.15</td>
<td>530</td>
</tr>
<tr>
<td>nu</td>
<td>0.80</td>
<td>0.30</td>
<td>590</td>
</tr>
<tr>
<td>Nmu-</td>
<td>8.5</td>
<td>0.70</td>
<td>380</td>
</tr>
<tr>
<td>Musi</td>
<td>13.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Parameters of ground motion models (radial component)

![Image: Temporal correlation function (Numazu, No.9, radial component)]

Figure 2. Temporal correlation function (Numazu, No.9, radial component)

For the spatial correlation function in TSSC model, the following function is adopted (Tamura et al., 1990, 1991):

\[
\rho_s(\eta) = [1-(\eta \sigma_s^2)] \exp\left[-(\eta \sigma_s^2)\right]
\]  

(10)

where \( \sigma_s \) is determined by least squares fit to the spatial correlation coefficients calculated from displacement records. Figure 3 compares the least squares fit spatial correlation function indicated by solid line and correlation coefficients calculated from the observed data. The dashed line represents the spatial correlation function inferred from FIC model. The separation distance in Figure 3 is the distance between two points projected to the radial direction. Both TSSC and FIC models give good approximation of ground motion spatial correlation, though the observed data show certain scatter.

![Image: Spatial correlation function (Numazu, radial component)]

Figure 3. Spatial correlation function (Numazu, radial component)

4.2 Spatial coherence and propagation velocity of seismic wave

The following frequency independent function is employed to represent coherence decay with separation (Tamura et al., 1990, 1991):

\[
\chi(\eta) = \exp\left[-(\eta \omega_s^2)\right]
\]

(11)

where \( \omega_s \) is determined by least squares fit. Figure 4 shows the comparison of least squares fit function and coherence calculated from the observed data, where coherence is calculated at the predominant frequency of ground motions. Propagation velocity of seismic waves is calculated by cross-correlation analysis (Sasaki et al., 1988). The calculated velocity is shown in Table 2.

![Image: Coherence decay (Numazu, radial component)]

Figure 4. Coherence decay (Numazu, radial component)

4.3 Maximum relative displacement

Figure 5 compares the temporal maximum relative displacement between any fixed locations estimated from TSSC model and FIC model, where only the median level (50%) is shown. The solid and dashed lines correspond to the former and latter models, respectively. The calculated values from the observed data are also shown in this figure. As seen from this figure, results from the two models agree fairly well.

![Image: Temporal maximum relative displacement (Numazu, radial component, p=0.5)]

Figure 5. Temporal maximum relative displacement (Numazu, radial component, p=0.5)

Figure 6 shows the temporal maximum relative displacement estimated from TSSC model. The solid line and dashed lines correspond to \( p=0.5 \) and \( p=0.84, 0.16 \), respectively. Though certain scatter can be seen, the theoretical formula of maximum relative displacement agrees well with the trend of observed data. Note that the range between 0.16 and 0.84 fractiles appears too narrow. This is because they include only random process uncertainty in the
Figure 6. Temporal maximum relative displacement estimated from TSSC model (radial component, $p=0.84, 0.5, 0.16$).

Gaussian model, not gross uncertainty in the model and its parameters.

The maximum ground strain between two points can be described as

$$\varepsilon_{max} = \frac{d_{max}}{d}$$

(12)

where $d$ is a separation distance between two points.

From the definition by Eq.(12), the maximum ground strain can be read from axes which go up from left to right with 45 degrees in Figure 6. The maximum temporal ground strains over an any fixed distance are $(25-35) \times 10^{-4}$ for Yaizu site and $(6-10) \times 10^{-4}$ for other sites, when $p$ is taken as 0.5.

Figure 7. Spatial maximum relative displacement (radial component, $p=0.5$).

Figure 7 shows the spatial maximum relative displacement estimated from TSSC model and FIC model, where the solid lines correspond to the former and latter model, and $p$ is taken as 0.5. Results are shown for Yaizu and Numazu sites.

5. CONCLUSIONS

Two ground motion spectral models are proposed and applied to estimate the maximum relative displacement over a temporal or spatial interval between two points. Numerical results are calibrated with array observation data. The following conclusions may be deduced from the present study:

1. Both TSSC and FIC models have been found to yield fairly accurate relative displacement statistics over separation distance up to 1-2 km.
2. Little difference exists between TSSC and FIC models, once the spatial correlation function in the former model and the spatial coherence in the latter model are suitably estimated.

An advantage of TSSC model is that conventional independent strong motion records can be incorporated into estimating the root mean square displacement and the temporal correlation function, because these quantities may be obtained by pooling ground motion records temporally. Taking account of this advantage, TSSC model has been further applied to estimate the attenuation characteristics of the maximum relative displacement, partly using the SMAC accelerograph records obtained in Japan (Tamura and Aizawa, 1992).

REFERENCES


Harichandran, R. S. 1988. Local Spatial Variation of Earthquake Ground Motion, Earthquake Engineering and Soil Dynamics 2, ASCE.


