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Random field models of spatially varying ground motions and the estimation of differential ground motions

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ABSTRACT: Two simple stochastic models of spatially varying ground motions are established and calibrated from strong motion array data at four sites. The first model chooses a coherence function that varies with separation distance but is independent of frequency, employing a coherence value estimated at the ground motion predominant frequency as a representative. The second model assumes the infinite propagation velocity of seismic waves, so that the time-space covariance function of ground displacement is a separable function of time and space. These models are applied to estimate the maximum relative displacement between two points on the ground surface, using conventional Poisson models of temporal and spatial extremes. Good agreement is found between the estimated and observed maximum relative displacement.

1. INTRODUCTION

Various studies on the spatial variation of ground motions have been conducted with the recent development of closely spaced arrays of accelerographs. Particularly, a number of studies (e.g., Harada, 1984; Loh and Yeh, 1988) discuss the differential ground motion characteristics, which may have significant influence on the dynamic behavior of buried lifeline facilities, multiple-span bridges, widely spread tanks, and so on.

Two simple variants of standard ground motion spectral models are here established (Tamura et al., 1990, 1991). They are calibrated from strong motion array data at four sites in Shizuoka Prefecture, Japan. Since an attention is focused on the strong motion duration on the nearly homogeneous ground, the proposed models adopt a stationary Gaussian random field model of each event's ground displacement in time and space.

The first model employs a coherence function that varies with separation length but is independent of frequency. A coherence value is estimated at the ground motion predominant frequency in this model. The second model assumes the apparent horizontal propagation velocity of seismic waves to be infinite, so that the time-space covariance function of ground motion is a separable function of time and space.

The proposed two models are applied to estimate the maximum relative displacement between two points on the ground surface, using a Poisson arrival process model of extremes. Both the temporal maximum over an any fixed distance and the spatial maximum at any fixed time can be estimated by the presenting procedure.

2. PROPOSED GROUND MOTION SPECTRAL MODELS

The time-space cross spectral density function of ground motions at two points separated by a distance η may be expressed as (Harichandran, 1988)

$$S(f,\eta)=S_0(f)\gamma(f,\eta)\exp(-i2\pi f\eta/c) \qquad (1)$$

where $S_0(f)$ is the power spectral density function of displacement at any fixed location, $\gamma(f,\eta)$ is the coherence function, and c is the apparent propagation velocity of seismic waves. In the first model, a coherence value is estimated at the predominant frequency of ground motion, and it is assumed that the coherence function $\gamma(f,\eta)$ is independent of frequency. Then, Eq.(1) can be reduced to

$$S(f,\eta)=S_0(f)\gamma(\eta)\exp(-i2\pi f\eta/c) \qquad (2)$$

Let us call this model as the "Frequency Independent Coherence (FIC) model". The time-space covariance function for this model can be easily obtained by taking a Fourier transform of Eq.(2).

The time-space covariance function of ground displacement, which is a counterpart of Eq.(1) in the time lag domain, may be described as

$$C(\tau,\eta)=C_0(\tau-\eta/c)\rho_S(\eta) \qquad (3)$$

where $C_0(\tau)$ is the auto-covariance function of displacement at any location, and $\rho_S(\eta)$ is the correlation function in space at any fixed time. The second model assumes c in Eq.(3) to be infinite, then Eq.(3) can be reduced to a separable function of time and space as

$$C(\tau,\eta)=C_0(\tau)\rho_S(\eta)=\sigma_u^2\rho_T(\tau)\rho_S(\eta) \quad (4)$$

where σ_u is the root mean square value of displacement, and $\rho_T(\tau)$ is the correlation function in time at any fixed spatial location. Let us call this model as the "Time-Space Separable Correlation (TSSC) model". While FIC model required $S_0(f)$, $\gamma(\eta)$ and c to be determined, this model requires only either $C_0(\tau)$ or $S_0(f)$, and $\rho_S(\eta)$.

3. MAXIMUM RELATIVE DISPLACEMENT

The time-space covariance function of relative displacement between two points with a separation distance ξ is expressed in terms of corresponding displacement covariance function as

$$C_d(\tau,\eta;\xi)=2C(\tau,\eta)-C(\tau,\eta+\xi)-C(\tau,\eta-\xi) \quad (5)$$

The mean square value of relative displacement can be obtained by setting $\tau=\eta=0$ in Eq.(5):

$$\sigma_d^2=C_d(0,0;\xi) \quad (6)$$

Assuming that the crossings of a specified threshold occur as a Poisson arrival process, the p-fractile of the maximum relative displacement over a temporal or spatial window length B, may be estimated as (Vanmarcke and Lai, 1980)

$$\frac{d_{max}}{\sigma_d}=\begin{cases}\sqrt{2\ln(-2B/L_D\ln p)} \cdots -2B/L_D\ln p\geq e\\ \sqrt{2} \cdots\cdots\cdots\cdots \text{otherwise}\end{cases} \quad (7)$$

in which σ_d is given by Eq.(6), and L_D is estimated as

$$L_{DT}=2\pi\sqrt{\frac{C_d(\tau,\eta;\xi)}{-\partial^2 C_d(\tau,\eta;\xi)/\partial\tau^2}}\Bigg|_{\tau=\eta=0} \quad \text{for temporal maximum} \quad (8)$$

L_D for spatial maximum is derived by taking a differential with respect to η, instead of τ, in Eq.(8).

4. ANALYSIS OF ARRAY DATA

The array data obtained at four sites in Shizuoka Prefecture, Japan are used in this analysis. Fourteen to eighteen tri-axial 16 bits digital strong motion accelerographs are installed at each site. Figure 1 shows an example of array deployments. The array records here analyzed were obtained from four earthquakes shown in Table 1. Both radial and transverse components have been analyzed, however the results for radial components are here presented due to limitations of space.

Table 1. Earthquakes analyzed

Site	Date	Epicentral Region	Magnitude	Distance [km]
Sagara	11/24/83	S Coast of Chubu	5.0	45
Yaizu	9/14/84	Central Chubu	6.8	126
Numazu	9/5/88	E Yamanashi Pref	5.6	45
Matsuzaki	11/22/86	Near Izu-Oshima Is.	6.0	72

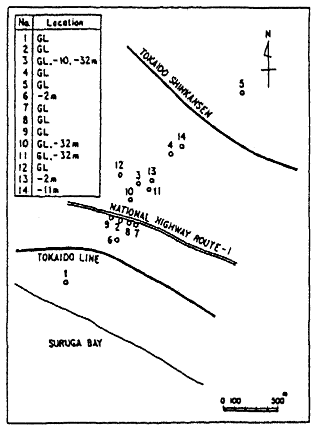


Figure 1. Array at Numazu site

4.1 Temporal and spatial correlation functions

The following function is assumed to represent the temporal correlation function (auto-correlation function) of ground displacement (Tamura et al., 1990, 1991):

$$\rho_T(\tau)=\cos(2\pi\tau/T_0)/\{(2\pi\alpha\tau/T_0)^2+1\} \quad (9)$$

where T_0 and α are the parameters determined by least squares fit to the temporal correlation function calculated from displacement data. An example of least squares fits is shown in Figure 2. The solid and dashed lines correspond to temporal correlation functions calculated from the observed data and Eq.(9), respectively. The parameters of ground motion models are listed in Table 2. B_T and B_S in Table 2 represent the temporal and spatial intervals in which the stationarity of ground motion is assumed, respectively.

Table 2. Parameters of ground motion models (radial component)

Site	B_T [sec]	T_0 [sec]	α	ξ_0 [m]	B_S [m]	a_0 [m]	c [m/s]
Saga-ra	3.0-8.5	0.70	0.25	470	1000	760	2635
Yai-zu	1.0-9.0	1.65	0.15	530	2000	960	1276
Nu-mazu	2.5-8.5	0.80	0.30	550	1500	650	2526
Matsu-zaki	7.0-13.0	1.00	0.15	380	1000	370	1573

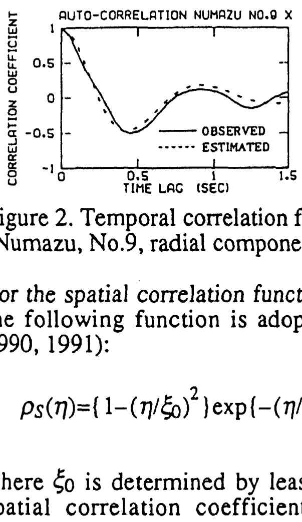


Figure 2. Temporal correlation function (Numazu, No.9, radial component)

For the spatial correlation function in TSSC model, the following function is adopted (Tamura et al., 1990, 1991):

$$\rho_S(\eta)=\{1-(\eta/\xi_0)^2\}\exp\{-(\eta/\xi_0)^2\} \tag{10}$$

where ξ_0 is determined by least squares fit to the spatial correlation coefficients calculated from displacement records. Figure 3 compares the least squares fit spatial correlation function indicated by solid line and correlation coefficients calculated from the observed data. The dashed line represents the spatial correlation function inferred from FIC model. The separation distance in Figure 3 is the distance between two points projected to the radial direction. Both TSSC and FIC models give good approximation of ground motion spatial correlation, though the observed data show certain scatter.

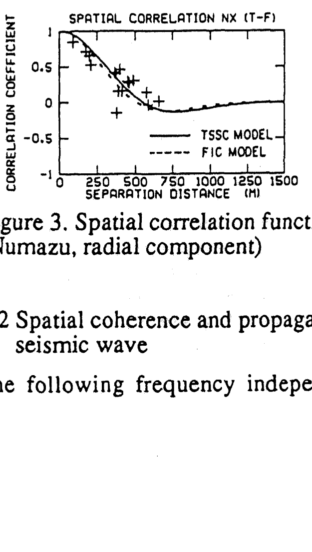


Figure 3. Spatial correlation function (Numazu, radial component)

4.2 Spatial coherence and propagation velocity of seismic wave

The following frequency independent function is employed to represent coherence decay with separation (Tamura et al., 1990, 1991):

$$\gamma(\eta)=\exp\{-(\eta/a_0)^2\} \tag{11}$$

where a_0 is determined by least squares fit. Figure 4 shows the comparison of least squares fit function and coherence calculated from the observed data, where coherence is calculated at the predominant frequency of ground motions.

Propagation velocity of seismic waves is calculated by cross-correlation analysis (Sasaki et al., 1988). The calculated velocity is shown in Table 2.

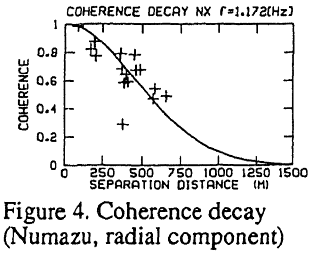


Figure 4. Coherence decay (Numazu, radial component)

4.3 Maximum relative displacement

Figure 5 compares the temporal maximum relative displacement between any fixed locations estimated from TSSC model and FIC model, where only the median level (p=0.5) is shown. The solid and dashed lines correspond to the former and latter models, respectively. The calculated values from the observed data are also shown in this figure. As seen from this figure, results from the two models agree fairly well.

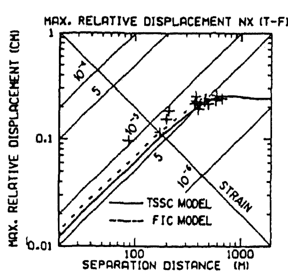
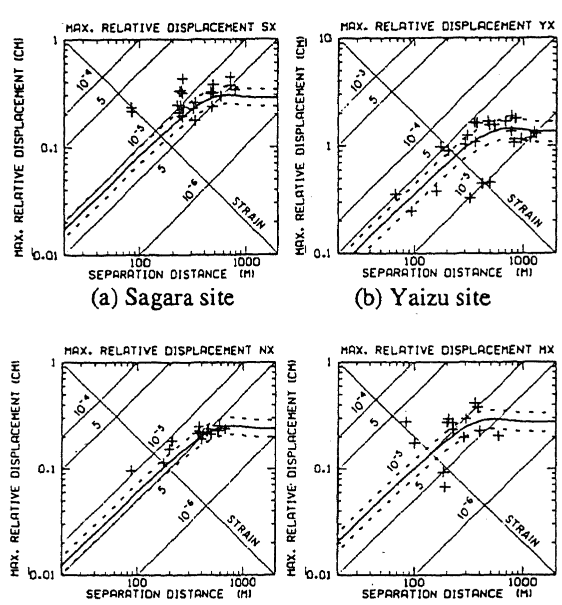


Figure 5. Temporal maximum relative displacement (Numazu, radial component, p=0.5)

Figure 6 shows the temporal maximum relative displacement estimated from TSSC model. The solid line and dashed lines correspond to p=0.5 and p=0.84, 0.16, respectively. Though certain scatter can be seen, the theoretical formula of maximum relative displacement agrees well with the trend of observed data. Note that the range between 0.16 and 0.84 fractiles appears too narrow. This is because they include only random process uncertainty in the



(a) Sagara site (b) Yaizu site

(c) Numazu site (d) Matsuzaki site

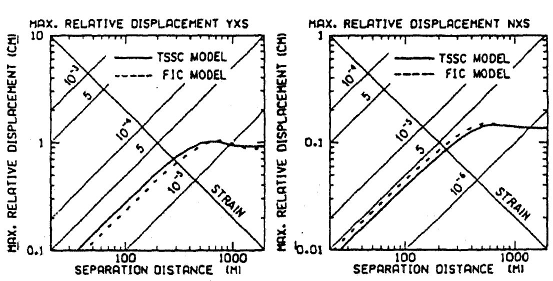
Figure 6. Temporal maximum relative displacement estimated from TSSC model (radial component, p=0.84, 0.5, 0.16)

Gaussian model, not gross uncertainty in the model and its parameters.

The maximum ground strain between two points can be described as

$$\varepsilon_{max}=d_{max}/\xi \tag{12}$$

where ξ is a separation distance between two points. From the definition by Eq.(12), the maximum ground strain can be read from axes which go up from left to right with 45 degrees in Figure 6. The maximum temporal ground strains over an any fixed distance are $(25\text{–}35)\times10^{-6}$ for Yaizu site and $(6\text{–}10)\times10^{-6}$ for other sites, when p is taken as 0.5.



(a) Yaizu site (b) Numazu site

Figure 7. Spatial maximum relative displacement (radial component, p=0.5)

Figure 7 shows the spatial maximum relative displacement estimated from TSSC model and FIC model, where the solid and dashed lines correspond to the former and latter model, and p is taken as 0.5. Results are shown for Yaizu and Numazu sites.

5. CONCLUSIONS

Two ground motion spectral models are proposed and applied to estimate the maximum relative displacement over a temporal or spatial interval between two points. Numerical results are calibrated with the array observation data. The following conclusions may be deduced from the present study:

1) Both TSSC and FIC models have been found to yield fairly accurate relative displacement statistics over separation distance up to 1-2 km.

2) Little difference exists between TSSC and FIC models, once the spatial correlation function in the former model and the spatial coherence in the latter model are suitably estimated.

An advantage of TSSC model is that conventional independent strong motion records can be incorporated into estimating the root mean square displacement and the temporal correlation function, because these quantities may be obtained by pooling ground motion records temporally. Taking account of this advantage, TSSC model has been further applied to estimate the attenuation characteristics of the maximum relative displacement, partly using the SMAC accelerograph records obtained in Japan (Tamura and Aizawa, 1992).

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