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The use of ARMA models in strong motion modelling

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ABSTRACT: Parametric time series models of the ARMA type, are applied to strong motion accelerograms recorded in an Icelandic earthquake. Response spectra obtained from simulations using ARMA models, are compared to response spectra derived directly from recorded accelerograms. The sensitivity of the response spectra to changes in ARMA parameters and envelope functions is also investigated.

1 INTRODUCTION

This paper deals with the application of parametric time series models of the ARMA type in earthquake engineering, emphasizing modelling of strong motion acceleration.

The introduction of ARMA models into the field of earthquake engineering goes back some twenty years (see Liu, 1970, which appears to be one of the first attempts). Since that time ARMA models have been used by a number of research workers, both to describe ground motion as well as in response analysis and system identification (see for instance Polhemus and Cakmak, 1981; Chang et al., 1982; Kozin, 1988; Turkstra and Tallin, 1988; and Safak 1989).

A stationary stochastic process sampled at regularly spaced time intervals may be represented by an ARMA model as follows

$$x_k - a_1 x_{k-1} - .... - a_p x_{k-p} = w_k - b_1 w_{k-1} - .... - b_q w_{k-q} \qquad (1)$$

where, x_k denotes the process, w_k is a white noise sequence with zero mean and finite standard deviation $E[w^2]$, a_p and b_q are the model parameters, respectively, the AR and MA parameters. The left side of Eq.(1) is known as the autoregressive or AR part of the order p, while the right side part is known as the moving average or MA part of order q.

If the time series x_k has been obtained by measurements it is possible to determine an optimal set of parameters. Different computational approaches for the estimation of the model parameters and model order have been proposed (Marple, 1987). A widely used criterion for order determination is Akaike's Final Prediction Error (FPE) criterion. According to this criterion the optimum model is the one that gives the smallest FPE-value.

Calculating the autocorrelation of the residual sequence, provides another method to validate the model. If the model fits the data well, the residuals should be uncorrelated (white noise). Statistical procedures such as the chi-square test are applied to evaluate the whiteness of the residual sequence. It is preferable to keep the number of parameters as small as possible. Although the model becomes more flexible with added parameters, and its bias is reduced, the variance increases with greater number of parameters. It is also desirable to keep the number of parameters small, making it easier to establish a link with physical parameters of the earthquakes.

2 SIMULATING RESPONSE WITH ARMA MODELS

This study is based on the assumption that an earthquake acceleration record may be treated as a sample from an evolutionary stochastic process. A commonly applied approach is to treat the acceleration process $\alpha(t)$, as an amplitude modulated stationary Gaussian process (Jennings et al., 1968). That is

$$\alpha(t) = A(t)x(t) \qquad (2)$$

where $x(t)$ is the stationary zero mean Gaussian process, and $A(t)$ is a deterministic envelope or amplitude modulation function.

Variety of functions have been suggested for the amplitude modulation function. Most of them seem to follow broadly the envelope suggested by Jennings et al. (1968), which consists of a parabolic start-up phase, followed by a constant strong shaking phase, and finished with an exponentially decaying phase. It seems reasonable, however, to make the transition between these three phases smooth, which may be accomplished by a function of the following type

$$A(t) = c_1 t^{c_2} exp(-c_3 t) \tag{3}$$

where, c_1, c_2, and c_3 are parameters. Alternatively a polynomial has been suggested (Polhemus et al., 1981)

$$A(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + ... + a_k t^k \tag{4}$$

where simultaneous estimates of the parameters $a_0, a_1, ..., a_k$ can be obtained by a LS approach.

For a given set of ARMA parameters a stationary time series is readily obtained using Eq.(1) and a computer generated sample of white noise. An accelerogram, α_k, is obtained by multiplying the ARMA series x_k with a suitable amplitude modulation function (see Eq.(2)).

A linear (or linearized) response of a second order structural system may be obtained as

$$y_k = a_{1s} y_{k-1} + a_{2s} y_{k-2} + \alpha_k \tag{5}$$

Here, a_{1s} and a_{2s} denote the AR parameters of the structural system, which are related to the natural frequency ω_s and critical damping ratio ξ_s of the structure by the following equations assuming that $\xi_s < 1$ (that is undercritically damped system)

$$\begin{aligned} a_{1s} &= 2exp(-\xi_s \omega_s \Delta t) cos(\omega_s \sqrt{1-\xi_s{}^2} \Delta t) \\ a_{2s} &= -exp(-2\xi_s \omega_s \Delta t) \end{aligned} \tag{6}$$

Here, Δt represents the sampling period. The relations for the AR parameters are obtained by equating the homogeneous solutions of the continuous and discrete systems.

The response spectrum can hence be obtained using the ARMA representation of ground motion. Applying the general definition of response spectrum the velocity spectrum for a linear (or linearized) system may be obtained formally as follows for a given accelerogram

$$S_v(\omega_s, \xi_s) = \omega_s E[max(y_k)|\omega_s, \xi_s] \tag{7}$$

In practical terms $E[max(y_k)|\omega_0, \xi_s]$ will be treated as an ensemble average of a finite ensemble of simulated response series.

3 ANALYSIS AND RESULTS

The earthquake which is analysed occurred in Vatnafjöll on 25th of May 1987, in a fracture zone on the Mid-Atlantic plate boundary, in South Iceland. This earthquake was the largest earthquake ($m_b = 5.8; M_S = 5.8; M_w = 5.9$) in South Iceland since the magnitude 7 earthquake in 1912, and was recorded on seven triaxial strong-motion accelerometers.

The objective of the analysis is to investigate the response spectra computed with Eq.(7) using accelerograms simulated with ARMA models, i.e. Eq.(1) and Eq.(2) as input into the second order system. In the simulation studies 20 runs are used and 2 percent of critical damping.

The 21 accelerograms obtained in the earthquake are pre-processed, by removing the dc-components and linear trends and bandpass filtering in the range 0.5-24 Hz. The accelerograms are also shortened, by discarding the segments containing the first 2 and last 4 percent of the cumulative energy.

Two accelerograms from this earthquake are given here as examples (see Figs.1 and 2). The results are similar for the other accelerograms obtained in this earthquake. They are both horizontal components obtained at two different stations, Station 105 which is located 31 km from the source of the earthquake and Station 108, which is located 26 km from the source.

A variety of envelope functions have been studied both piecewise and continuously differentiable. The maximums of the absolute values of the accelerogram are found, and smoothed with a moving average filter. The envelope function is then fitted to the smoothed curve, as shown in Fig.3. The polynomial envelope function of Eq.(4) is found to be both flexible and easy to estimate. A fourth degree polynomial envelope is found to give a reasonable fit to most of the accelerograms, and produce simulations which resemble the original time series. A quite adequate fit is also obtained with the exponential envelope function of Eq.(3). The response spectra are however, found to be insensitive to the type of envelope function used. This is indicated in Figs.14 and 15 showing ARMA response spectra computed with three different envelopes, that is one with the envelope function of Eq.(3) and two with Eq.(4), using 5 and 9 parameters.

The FPE-values are calculated for 49 models ranging from ARMA(1,1) to ARMA(7,7), in order to determine the optimum order. Table 1 and Fig.5 show the results for Station 108, where models are placed in order of increasing FPE-values. This seems to suggest that ARMA(4,1) has a fairly low FPE-value with respect to the number of parameters in the model. However, a somewhat higher model order has to be used to pass the 99 percent confidence limit test for whiteness of the residuals. This is indicated in

Fig.4 showing the correlation function for the residuals for Station 108, of an ARMA(7,7) model.

The poles and zeros of the 49 models for both stations are plotted in Figs.6 and 7. These diagrams indicate one real zero near -1 in both cases. For Station 105 there is one strong pole-pair and another weaker one at higher frequencies. For Station 108 there is only one pole-pair cluster which is somewhat fuzzier than the main cluster for Station 105. These pole-zero diagrams seem to suggest ARMA(4,1) and ARMA(2,1) as underlying model orders.

Figs.8 and 9 show the PSD of the two accelerograms computed with FFT (dashed lines) and with ARMA(4,1) models (solid lines). This indicates a reasonable fit of the PSD computed with the ARMA models.

Figs.10 and 11 show response spectra computed directly from the measurements (dashed lines) and with ARMA(4,1) models (solid lines) using 4th degree polynomial envelope functions. The ARMA response spectra is seen to simulate the response spectra computed directly from the accelerogram reasonably well. Figs.12 and 13 shows a comparison of response spectra computed with three different ARMA models. It is interesting to note that in Fig.13 the response spectrum for the ARMA(2,1) case is closer to the spectrum obtained directly from the accelerogram than the ARMA(4,1) response spectrum, although that is not the case in general.

In the analysis it is assumed that the ARMA parameters are constant throughout the earthquake. Figs.16 and 17 show ARMA response spectra for three equal length segments of the accelerograms. For Station 105, the spectra is nearly identical (except for the difference in variance). For Station 108 the third segment has somewhat different parameters, but the two first segments which contain most of the energy, produce identical spectra, which justifies the stationary approximation used. However, if the accelerograms have great variation of frequency content between segments, a time varying ARMA model, such as in Safak (1989), would be preferable.

4 CONCLUSIONS

Findings of this study seem to indicate that time-invariant ARMA(4,1) models give reasonably accurate response spectra in most cases. This low model order makes simulation studies based on ARMA models computationally very efficient. Further it is found that the response spectra are insensitive to the envelope functions used, and only few parameters are needed.

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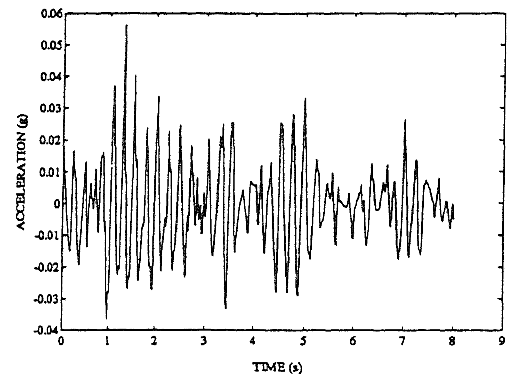

Fig.1. Accelerogram from Station 105.

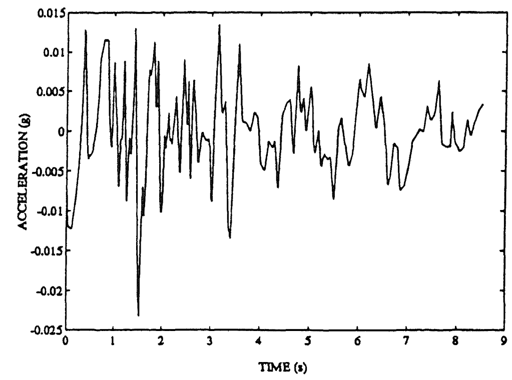

Fig.2. Accelerogram from Station 108.

Table 1. Models ordered according to FPE-value for Station 108.

No.	FPE	AR	MA	No.	FPE	AR	MA
1	1.4e-10	7	7	26	9.9e-08	4	4
2	1.6e-10	7	6	27	2.6e-07	4	1
3	1.9e-10	7	5	28	2.9e-07	4	2
4	2.4e-10	6	7	29	4.2e-07	4	3
5	2.5e-10	7	4	30	1.2e-06	2	7
6	3.1e-10	6	6	31	1.6e-06	3	1
7	3.3e-10	7	3	32	1.8e-06	3	2
8	4.1e-10	6	5	33	2.6e-06	3	7
9	5.2e-10	7	2	34	3.9e-06	3	5
10	5.9e-10	6	4	35	6.4e-06	2	6
11	9.4e-10	5	7	36	1.0e-05	2	1
12	9.7e-10	7	1	37	1.2e-05	3	4
13	9.8e-10	6	3	38	1.6e-05	1	7
14	1.3e-09	5	5	39	2.4e-05	3	3
15	1.8e-09	5	6	40	2.6e-05	2	2
16	1.9e-09	6	2	41	2.6e-05	1	3
17	2.2e-09	5	4	42	2.6e-05	1	6
18	4.7e-09	5	3	43	5.6e-05	1	2
19	4.9e-09	6	1	44	1.2e-4	2	5
20	1.3e-08	4	7	45	1.3e-04	2	4
21	1.3e-08	5	2	46	1.6e-04	1	1
22	2.7e-08	4	6	47	2.5e-04	2	3
23	3.5e-08	5	1	48	4.3e-03	1	5
24	8.7e-08	4	5	49	1.1e-02	1	4
25	9.6e-08	3	6				

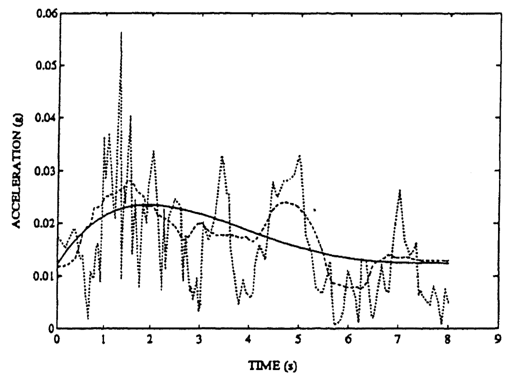

Fig.3. An example of a fitted envelope for Station 105.

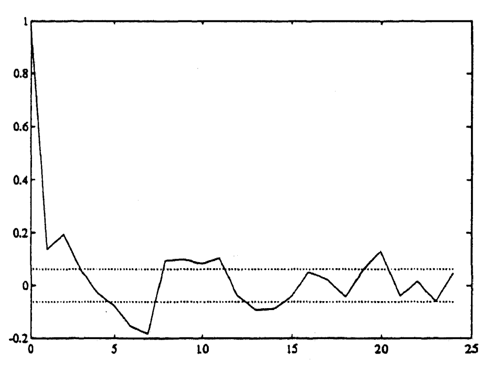

Fig.4. Correlation of residuals for Station 108, using ARMA(7,7).

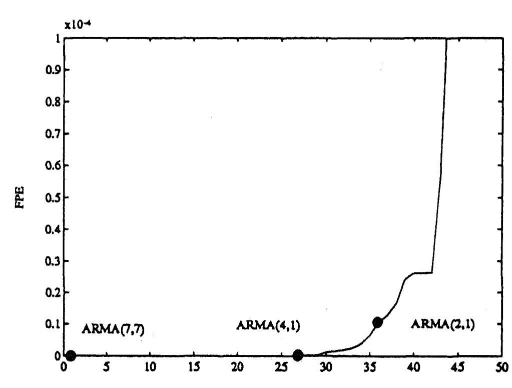

Fig.5. FPE-values from Table 1.

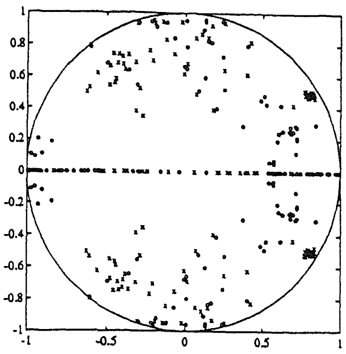

Fig.6. Pole-zero plot for Station 105, o denotes zeros, x denotes poles.

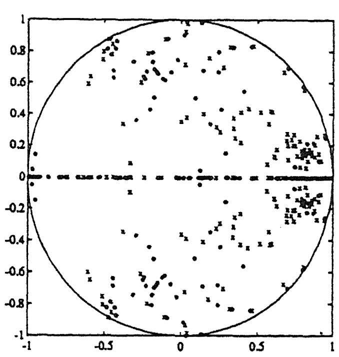

Fig.7. Pole-zero plot for Station 108, o denotes zeros, x denotes poles.

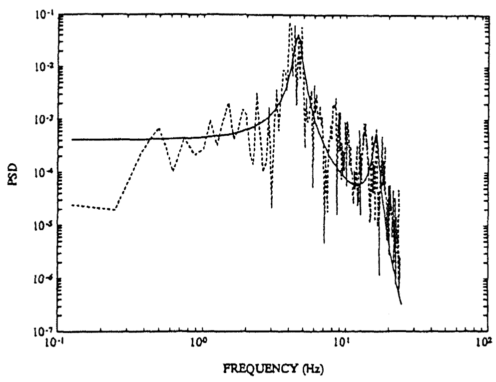

Fig.8. PSD for Station 105. Units are m^2/s^3.

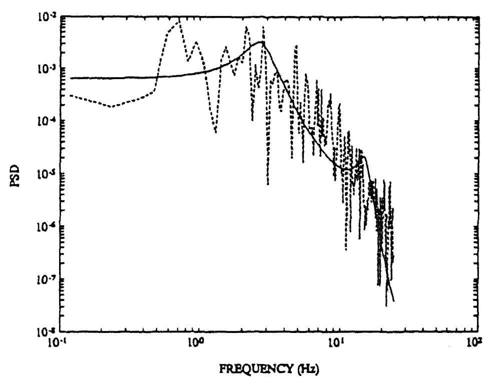

Fig.9. PSD for Station 108. Units are m^2/s^3.

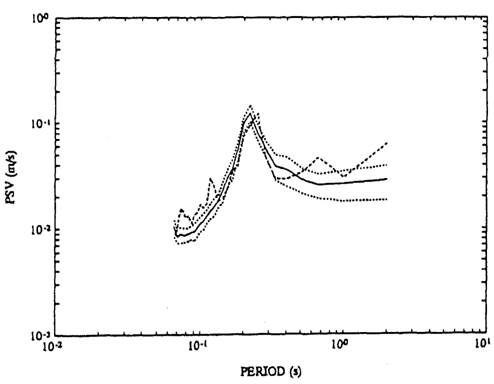

Fig.10. ARMA(4,1) response spectrum for Station 105. Dotted lines indicate +/- one standard deviation.

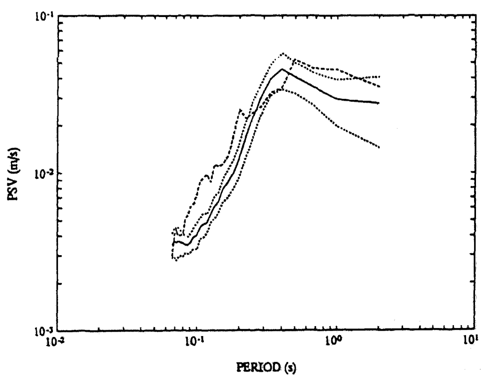

Fig.11. ARMA(4,1) response spectrum for Station 108. Dotted lines indicate +/- one standard deviation.

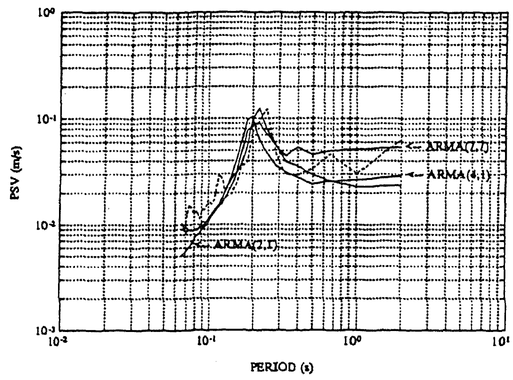

Fig.12. ARMA(2,1), (4,1) and (7,7) response spectra for Station 105.

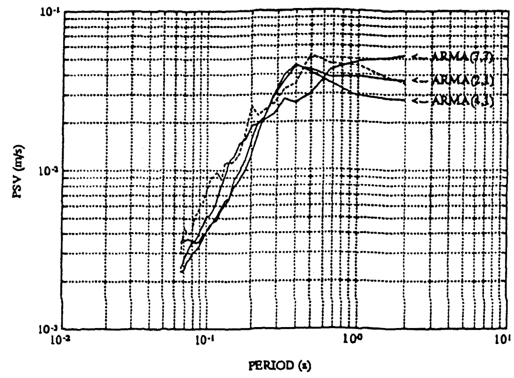

Fig.13. ARMA(2,1), (4,1) and (7,7) response spectra for Station 108.

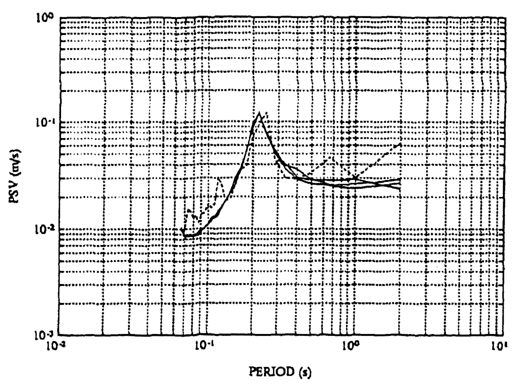

Fig.14. ARMA(4,1) response spectra with three different envelopes for Station 105.

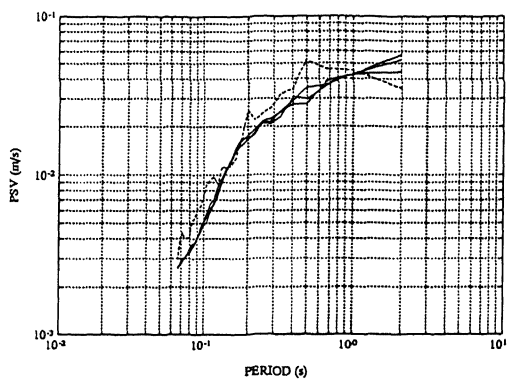

Fig.15. ARMA(7,7) response spectra with three different envelopes for Station 108.

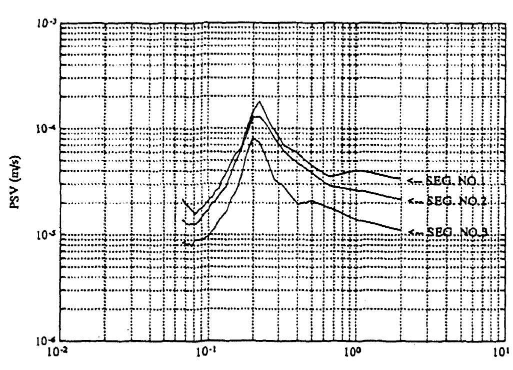

Fig.16. ARMA(4,1) response spectra for Station 105, using three segments.

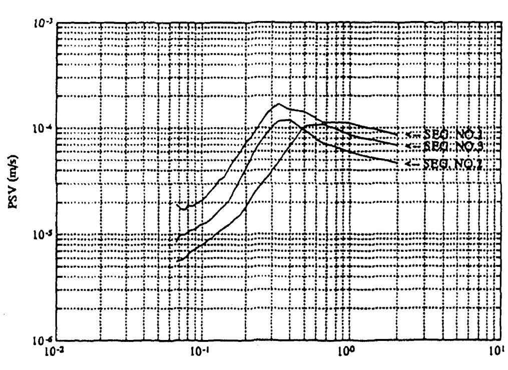

Fig.17. ARMA(4,1) response spectra for Station 108, using three segments.