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Generation of spatially incoherent strong motion time histories

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ABSTRACT: A method for generating spatially incoherent ground motions given on a coherency function and a reference time history is developed. The method is based on the relation between the absolute value of the coherency and statistical properties of the Fourier phase angles. The difference in the Fourier phase between two sites is due to the wave-passage effect and stochastic variations in the phase. The percentage of the phase difference that is stochastic is related to the absolute value of the coherency.

An algorithm using an iterative procedure to estimate the suite of incoherent ground motions is presented. As an example, the method is used to generate spatially incoherent ground motions at 11 support locations for a 1.5 km long bridge.

1 INTRODUCTION

In this paper, a method for generating a set of spatially incoherent ground motions from a reference ground motion that match a specified coherency function is presented. In a previous study, a method for generating spatially incoherent time histories was developed by Hao et al. (1989). There are two main differences between the Hao et al. method and the method developed in this paper. First, they attributed the incoherence to both phase and amplitude variations, whereas, I attribute the incoherence to only phase variations and treat amplitude variations separately,. Second, they estimate the incoherent motions by considering one station at a time, whereas, I estimate the incoherent motion by considering all the stations simultaneously.

1.1 Definition of coherence

The coherency, $\gamma_{jk}(f)$, between the j^{th} and k^{th} stations is given by

$$\gamma_{jk}(f) = \frac{S_{jk}(f)}{[S_{jj}(f)\, S_{kk}(f)]^{1/2}} \qquad (1)$$

where $S_{jk}(f)$ is the smoothed cross-spectrum given by

$$S_{jk}(f) = \sum_{m=-M}^{M} a_m\, u_j(f)\, \bar{u}_k(f) \qquad (2)$$

$u_j(f)$ is the Fourier transform of the time history at the j^{th} station, and a_m are the weights for a frequency window. The cross-spectrum is smoothed over 2M+1 discrete frequencies. In this paper, a hamming window is used for the frequency weights with M=5. (See Abrahamson et al., 1991 for a discussion of optimal frequency smoothing for estimation of coherency for engineering applications.)

The coherency is a complex number. It is common to use the absolute value of the coherency to describe the similarity of the waveforms at two stations without regard to the difference in the arrival times of the waves. The absolute value of the coherency is called the "lagged" coherency. In contrast, the wave-passage effect depends only on the time delays in the arrival of the waves. The lagged coherency represents stochastic variations in the ground motions, whereas the wave-passage effect represents systematic variations in the ground motions.

In coherency studies, the selection of the frequency smoothing (Eq. 2) is very important.For small values of M, the estimated lagged coherency is affected by the number of frequencies smoothed. For values of $|\gamma|$ that are not small, $|\gamma|$ is biased (Abrahamson et al., 1991). The expected value of the computed coherency is given by

$$E[\tanh^{-1} |\hat{\gamma}|\,] \approx \tanh^{-1} |\gamma| + \frac{g^2}{2(1-g^2)} \qquad (3)$$

where

$$g^2 \approx \sum_{m=-M}^{M} a_m \qquad (4)$$

For small values of $|\gamma|$ the median noise level is given by

$$|\gamma_{noise}| = \sqrt{1 - 0.5g^2/(1-g^2)} \quad (5)$$

For the frequency smoothing used in this study (Hamming window with M=5), g^2=0.14, the bias of $\tanh^{-1}|\gamma|$ is 0.08, and the median noise level of $|\gamma|$ is 0.33.

2 METHOD

By definition, the coherency is not affected by scaling the ground motion at the different stations (Eq. 1). Therefore, the method developed to generate incoherent ground motions assumes that the Fourier amplitudes of the motions at the different sites are identical. Realistic variations in the Fourier amplitude (such as discussed by Abrahamson et al., 1990) can easily be added to the generated incoherent time series, but to simplify the discussion, such variations in the Fourier amplitude spectra will not be addressed in this paper.

The ground motions are first generated to match the specified lagged coherency function, then they are lagged appropriately to account for the wave-passage effect. Let $\phi_j(f)$ be the Fourier phase at the j^{th} site. Ignoring the wave-passage effect, the phase at site j can be written in terms of the phase at site k:

$$\phi_j(f) = \phi_k(f) + (1-\alpha_{jk}(f))\, \varepsilon_{jk}(f) \quad (6)$$

where $\varepsilon_{jk}(f)$ is assumed to be uniformly distributed over the range $[-\pi,\pi]$ and $\alpha_{jk}(f)$ is a number between 0 and 1 that partitions the phase into deterministic and stochastic parts. If $\alpha_{jk}(f)$=1, then there is no random component to the phase difference between stations j and k. If $\alpha_{jk}(f)$=0, then the phase difference between stations j and k is completely random. The α term has a simple physical meaning: it is the fraction of the phase that is deterministic. For example, it $\alpha_{jk}(f)$=0.8, then 80% of the phase is deterministic and 20% is stochastic.

The parameter $\alpha_{jk}(f)$ is related to the lagged coherency. Ignoring amplitude variations, the lagged coherency is given by

$$|\gamma_{jk}(f)| = \left| \sum_{m=-M}^{M} a_m \exp\{i\phi_j(f_m)\} \exp\{-i\phi_k(f_m)\} \right| \quad (7)$$

Substituting Eq (6) into Eq (7) gives

$$|\gamma_{jk}(f)| = \left| \sum_{m=-M}^{M} a_m \exp\{-i(1-\alpha_{jk}(f_m))\varepsilon_{jk}(f_m)\} \right| \quad (8)$$

For large M, the expected value of $|\gamma|$ is given by

$$E[\,|\gamma_{jk}(f)|\,] = \left| \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{-i(1-\alpha_{jk}(f_m))\varepsilon\}\, d\varepsilon \right| \quad (9)$$

which reduces to

$$E[\,|\gamma_{jk}(f)|\,] = \frac{\sin\{(1-\alpha_{jk}(f))\pi\}}{(1-\alpha_{jk}(f))\pi} \quad (10)$$

This relation between the expected value of the lagged coherency and α is plotted in Figure 1. Since $|\gamma|\leq 1$, $\alpha(|\gamma|)$ is a single valued function so for a given lagged coherency value, Eq. (10) can be solved for α numerically.

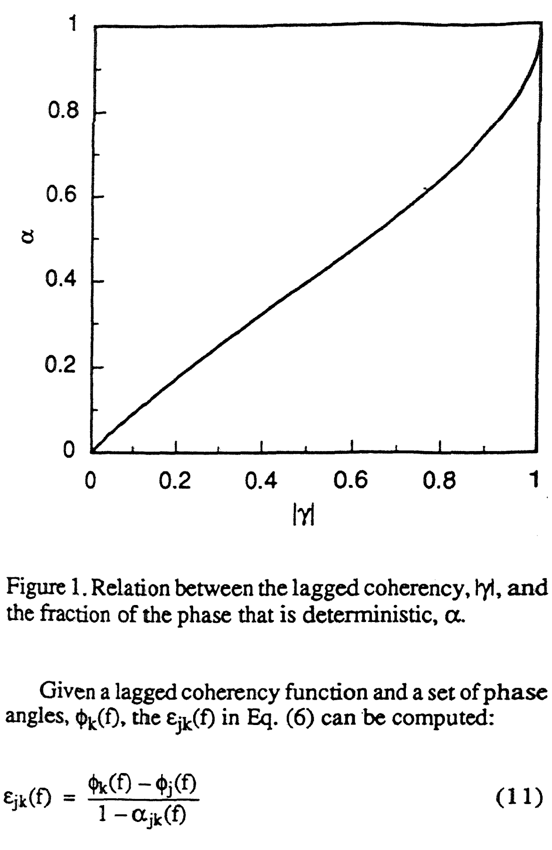

Figure 1. Relation between the lagged coherency, $|\gamma|$, and the fraction of the phase that is deterministic, α.

Given a lagged coherency function and a set of phase angles, $\phi_k(f)$, the $\varepsilon_{jk}(f)$ in Eq. (6) can be computed:

$$\varepsilon_{jk}(f) = \frac{\phi_k(f) - \phi_j(f)}{1 - \alpha_{jk}(f)} \quad (11)$$

The problem of generating a set of time histories to match a given coherency function reduces to finding a set of phase angles, $\phi_k(f)$, such that the corresponding $\varepsilon_{jk}(f)$ are uniformly distributed over the range $[-\pi,\pi]$. To measure how well the $\varepsilon_{jk}(f)$ approximate a uniform distribution requires a penalty function. In this paper, the penalty function is based on the variance between the sample density function and the desired density function.

The desired density function, $p(\varepsilon)$, is given by

$$p(\varepsilon) = \begin{cases} \frac{1}{2\pi} & \text{for } |\varepsilon| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The sample density function, $q(\varepsilon)$, is computed using continuous post-stratification

$$q(\varepsilon_{ij}) = \frac{2}{n^2-n} \sum_{k=2}^{n} \sum_{l=1}^{k-1} T(\varepsilon_{ij} - \varepsilon_{kl}) \tag{13}$$

where $T(x)$ is a taper function normalized such that

$$\int_{-\infty}^{\infty} T(x)\, dx = 1 \tag{14}$$

In this paper, a hanning window is used for the taper function, $T(x)$, with the width of the taper equal to $4\pi/(n^2-n)$. The normalized taper function is given by

$$T(x) = \begin{cases} \frac{n^2-n}{8\pi} \left[1 + \cos\left(\frac{x(n^2-n)}{4}\right) \right] & \text{for } |x| \leq \frac{4\pi}{n^2-n} \\ 0 & \text{for } |x| > \frac{4\pi}{n^2-n} \end{cases} \tag{15}$$

The variance between the sample density function and the desired density function is not affected by how far a sample is outside of the desired range$[-\pi,\pi]$. Therefore, an additional term is added to the penalty function to penalize solutions as $\varepsilon_{jk}(f)$ becomes further outside the range $[-\pi,\pi]$. The penalty function, χ^2, used in this study is given by

$$\chi^2 = \sum_{i=2}^{n} \sum_{j=1}^{i-1} [q(\varepsilon_{ij}) - p(\varepsilon_{ij})]^2 + \sum_{i=2}^{n} \sum_{j=1}^{i-1} c\,(|\varepsilon_{ij}|-\pi)^2 H(|\varepsilon_{ij}|-\pi) \tag{16}$$

where $H(x)$ is the Heaviside function, and c is a constant which controls the slope of the penalty function outside the range $[-\pi,\pi]$. The first term in Eq. (16) is proportional to the variance between the sample and desired density functions. The second term penalizes the solution for $|\varepsilon_{jk}(f)|>\pi$. In this study, a value of $c=5$ was found to work well.

With the penalty function defined in Eq. (16), we have a clearly defined optimization problem: at each frequency, find the set of phase angles, $\phi_k(f)$, that minimize χ^2. There are many solutions that will generate approximately uniform distributions for $\varepsilon_{jk}(f)$ so the solution is not unique. The approach is completely general in that it can consider any number of stations, and multiple components at each station simultaneously.

2.1 Incoherent Time History Generation Algorithm

The algorithm used to compute the phase angles at each frequency is given below.

1. Find an initial estimate of the phase angles ϕ_i,
2. Compute the penalty function χ^2,
3. Compute the partial derivatives of χ^2 with respect to the ϕ_i,
4. Modify the ϕ_i with the largest partial derivative by an amount $\Delta\phi$,
5. Compute the new penalty function, χ^2,
6. If there is a reduction in χ^2, then save the solution and go to step 3, otherwise reject the solution and reduce $\Delta\phi$,
7. If $\Delta\phi$ is less than the tolerance then stop, otherwise go to step 4.

The algorithm is applied to separately to each frequency. Since changing one phase angles effects multiple ε terms, only one phase angle is modified during each iteration (see step 4).

Given a reasonable set of starting values for ϕ_i, this algorithm converges rapidly. To insure a good fit to the desired coherency function, I recommend repeating the algorithm several times at each frequency with a different set of starting phase angles and selecting the solution that gives the smallest penalty function.

To find a starting solution, the $\phi_i(f)$ are computed one station at a time. Given the reference phase $\phi_o(f)$, the phase at the i^{th} station is computed as follows. First, find station j that is closest to station i, for j = 0, 1, 2, ... i-1. Then, let

$$\phi_i(f) = \phi_j(f) + \eta\,(1-\alpha_{ij}(f)), \tag{17}$$

where η is a random number uniformly distributed over the range $[-\pi,\pi]$. This method for computing the starting solution preserves the high coherency aspects of the wave-field. The low coherency aspects are then accommodated by making small changes to the phase angles using the algorithm described above.

The wave passage effect is included by applying systematic time shifts to the time histories generated by the method described above. The time shifts are computed from the expected apparent velocity of the wavefield between the each station and the reference station. Since the incoherent ground motion is generated in the frequency domain, it is convenient to apply the wave-passage effect in the frequency domain as well. A time shift corresponds to a linear phase shift. That is, for a time shift $\Delta\tau_k$, then

$$\psi_k = \phi_k + 2\pi f \Delta\tau_k \tag{18}$$

where ψ_k is the phase at station k including the wave-passage effect.

For this paper, the Fourier amplitude of the ground motion at each station is equal to the Fourier amplitude of the reference motion. Combining the reference motion amplitude spectrum with the generated phase spectrum, $\psi_k(f)$, the time history at station k is computed by an inverse Fourier transform.

There is an important implicit assumption of the method. The incoherent ground motion generated using this method assumes that the Fourier spectrum is constant over the entire time window. As the ground motion becomes more incoherent and the phase angles more random, the energy is smeared out over the entire time window. For complete strong motion time histories, this is not appropriate. This limitation can be avoided by using multiple short time windows for which the Fourier spectrum is approximately constant. The incoherent ground motion algorithm described above is applied to each time window independently and then the short time history segments are recombined to give the complete time history.

3 EXAMPLE APPLICATION

As an example, the ground motions at 11 piers of a 1.5 km long bridge are generated. The pier locations are shown in Figure 2. The coherency functions are based on the coherency model given by Somerville et al. (1988) which describes the total coherency at a site as the product of the ray-path coherency and the extended source effect coherency. For this example, the following models are used for the path and source lagged coherencies:

$$|\gamma_{path}(f,\xi)| = \tanh([5.39 - 0.622 \ln(\xi)]e^{-0.252f} + 0.35) \quad (19)$$

and

$$|\gamma_{source}(f,\xi)| = \tanh(4.5\, e^{(-0.12f - 0.0025\,\xi)} + 0.6) \quad (20)$$

where ξ is the separation distance in meters and f is the frequency in Hz. The path coherency is based on analyses of recordings from the SMART 1 array in Taiwan (Abrahamson et al., 1987). The source coherency is site specific and should not be used at other sites.

For this example, the reference time history is the Pacoima Dam record. Sliding 2.56 second time windows are used. The apparent wave velocities along the bridge axis from a nearby fault are 6.8, and 3.8 km/s for the P and S waves, respectively, based on 1-D Green's functions calculations.

The incoherent time histories for one of the horizontal components are shown in Figure 3. Each time history has the identical Fourier amplitude spectrum, but significant variations in the waveforms are evident.

To test the method, the coherencies from the suite of generated time histories are shown in Figure 4. The specified coherency functions are also shown in this figure. The coherency from the generated motions are consistent with the input coherency function.

The suite of ground motions generated using this method are for similar site conditions. If the site conditions change significantly along the length of the structure, then the suite of incoherent ground motion should be further modified to represent the changing site conditions by convolving the ground motion with the appropriate transfer function.

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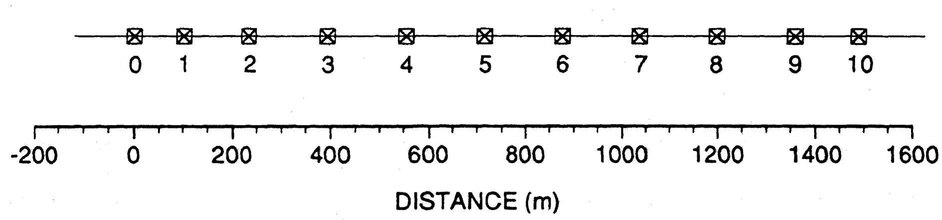

Figure 2. Locations of piers for example. The reference motion is at Peir 0.

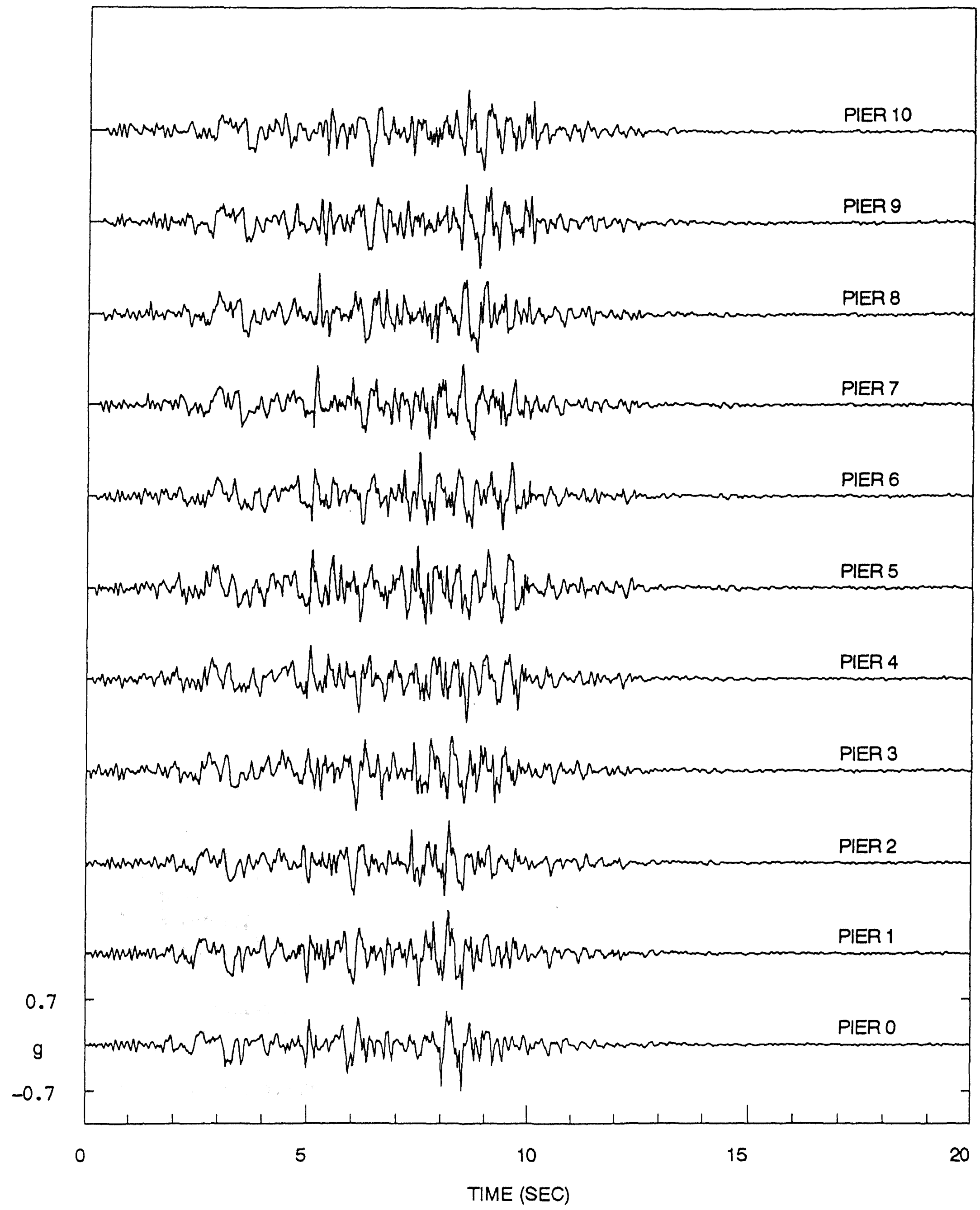

Figure 3. Example suite of incoherent time histories. The motion at Pier 0 is the reference motion. The reference time history is a modified version of the Pacoima Dam recording during the 1971 San Fernando Earthquake.

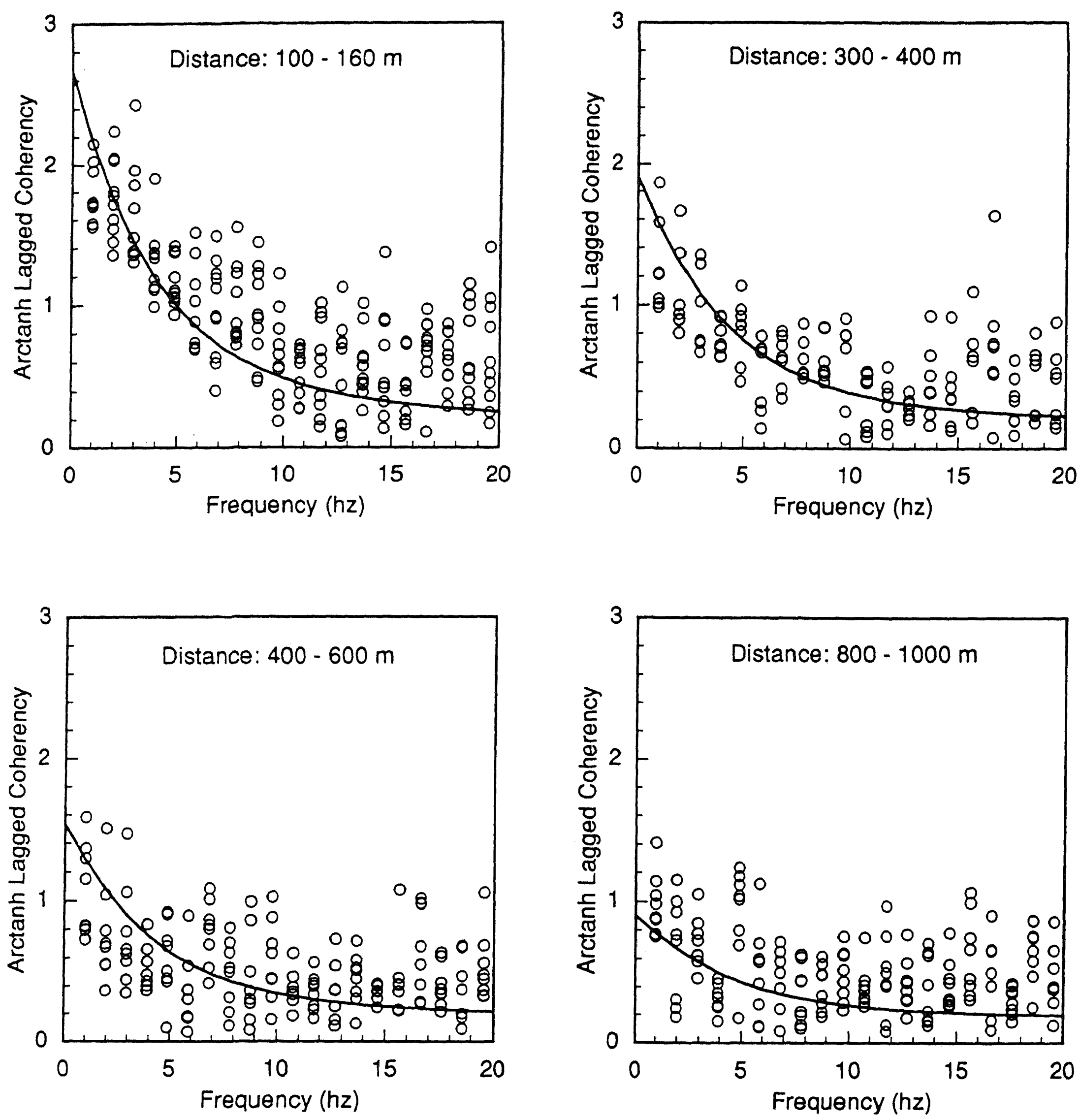

Figure 4. Comparison of the computed lagged coherency with the specified coherency function for four separation distances.