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Stochastic characterization of ground motion and applications to structural response

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ABSTRACT: A parsimonious stochastic seismic ground-motion model is proposed for use in both ground-motion prediction studies and structural response studies. The model captures with at most nine parameters the features of the ground motion which are important for computing dynamic response, including the amplitude and frequency-content nonstationarities of the earthquake. Furthermore, the model can be efficiently used in both simulated and analytical random vibration response studies. An approximate analytical random vibration method is developed to provide insight into the effect of the temporal nonstationarity in both the amplitude and frequency content of ground motion on the response of simple structural models. It is found that the frequency content nonstationarity may significantly affect the response of both linear and softening non-linear structural models.

1 INTRODUCTION

In earthquake-resistant design of structures, a major problem to be resolved is the description of the ground motion, which should be complete enough for reliable prediction of the corresponding dynamic response of a structure. Current design techniques based on response spectra lead to difficulties, for example, in predicting non-linear response and in-structure equipment response. In addition, response spectra do not explicitly take into account the nonstationary features observed in real accelerograms, such as the changes of intensity and frequency content with time. Past models dealing with ground motion modeling, in order to simplify the random vibration analysis, have often neglected the temporal change of the frequency content. This is partly because it was difficult to incorporate this change in simple continuous ground motion models and identify it from earthquake records, and also partly because it was believed that it had no significant effect on linear structural response. Recently, Yeh and Wen (1989) and Papadimitriou and Beck (1990) have developed models which include the time variation of the frequency content and, at the same time, are efficient to use in analytical random vibration studies.

In this research, the new ground motion model is briefly described and then it is used to study the importance of ground motion nonstationarities on the response of simple structural models. In order to gain analytical insight into the response characteristics, an approximate random vibration methodology developed by Papadimitriou (1990) is applied to simple linear and softening non-linear structural models. Simple expressions for the mean-square response statistics are obtained which demonstrate analytically the effect of the temporal nonstationarity in the frequency content of the ground motion.

2 STOCHASTIC GROUND MOTION MODEL

The earthquake ground acceleration $a(t)$ at a site is treated as a specific realization of a stochastic process, described by the set of linear differential equations

$$\ddot{y} + 2\zeta_g(t)\omega_g(t)\dot{y} + \omega_g^2(t)y = f_g(t)e(t), \qquad (1)$$

$$\ddot{a} + 2\omega_c\dot{a} + \omega_c^2 a = \ddot{y}, \qquad (2)$$

where $\zeta_g(t)$, $\omega_g(t)$ and $f_g(t)$ are deterministic time-varying coefficients, and e(t) is a zero-mean Gaussian white-noise process. The model (2), which corresponds to Brune's (1970) source model, gives the correct behavior of the very low-frequency spectral amplitudes of the accelerograms, without substantially affecting the spectral amplitudes of the model (1) at frequencies greater than about $2\omega_c$. The corner frequency ω_c can be computed from source mechanism studies of the earthquake generating the ground motion.

For wide-band processes, usually the case in earthquake engineering, $f_g(t) = 2\omega_g(t)\sqrt{\alpha_g(t)}I_g(t)$ where $\alpha_g = \zeta_g(t)\omega_g(t)$, and $I_g(t)$ is the intensity of the ground motion process. Based on detailed analyses of earthquake accelerograms (Papadimitriou and Beck, 1990), the intensity $I_g(t)$ is adequately modeled by the envelope function

$$I_g(t) = I_{max}\tau^{\beta}\exp\left[\beta(1-\tau)\right] \qquad (3)$$

where $\tau = (t+t_0)/(t_{max}+t_0)$. The parameters I_{max} and t_{max} are the maximum intensity and the time of the maximum intensity of the ground acceleration, respectively. The variable t_0 is the time of the first non-zero acceleration before the triggering time,

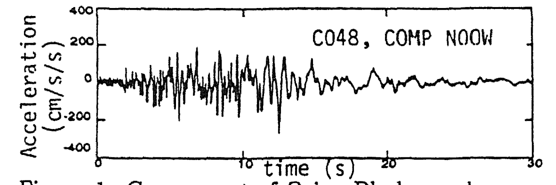

Figure 1. Component of Orion Blvd. accelerogram (1971 San Fernando earthquake).

introduced to provide flexibility in fitting the data, and it is not needed when predicting response with the model. The parameter β is a nondimensional measure of the duration t_{dur} of the accelerogram. The parametric functions $\omega_g(t)$ and $\alpha_g(t)$ are adequately modeled by:

$$\omega_g(t,\underline{\theta}) = \omega_r + (\omega_p - \omega_r)\left(\frac{\omega_s - \omega_r}{\omega_p - \omega_r}\right)^{\tau} \quad (4)$$

where ω_p, ω_s, and ω_r may be interpreted approximately as the dominant frequencies of the P, S, and surface waves present in the ground motion, respectively, and

$$\alpha_g(t,\underline{\theta}) = \omega_p\zeta_p + (\omega_r\zeta_r - \omega_p\zeta_p)\ t/t_{dur} \quad (5)$$

where ζ_p and ζ_r measure the frequency range around ω_p and ω_r respectively that strongly contributes to the earthquake process. Papadimitriou and Beck (1990) have presented a Bayesian method for estimating the optimal model, i.e., the most probable parameter set $\hat{\underline{\theta}} = (I_{max}, t_{max}, t_{dur}, \omega_p, \omega_s, \omega_r, \zeta_p, \zeta_r)$ that provides, in a statistical sense, the best fit to a "target" accelerogram.

3 STRUCTURAL MODEL AND MEAN-SQUARE RESPONSE CHARACTERISTICS

The governing equation of motion of a SDOF nonlinear system subjected to the stochastic base excitation $a(t)$ is given by

$$\ddot{x}(t) + 2\zeta_0\omega_0\dot{x}(t) + \omega_0^2 R(t)/K_0 = a(t) \quad (6)$$

where ω_0 and ζ_0 are the initial (small-amplitude) structural frequency and viscous damping ratio respectively. The restoring force $R(t)$ is modeled by a non-linear elastic softening restoring force

$$R(t) = 2/\pi(K_0 x_y)\arctan[\pi x/(2x_y)] \quad (7)$$

where the nominal "yield" displacement $x_y = R_u/K_0$ is similar to the elastic limit displacement of a yielding system, with R_u and K_0 being the ultimate strength and the initial stiffness of the system, respectively.

The equivalent linearization method (Roberts and Spanos, 1990) is used to obtain the second-moment response statistics. The non-linear equation (6) is replaced by the equivalent linear one

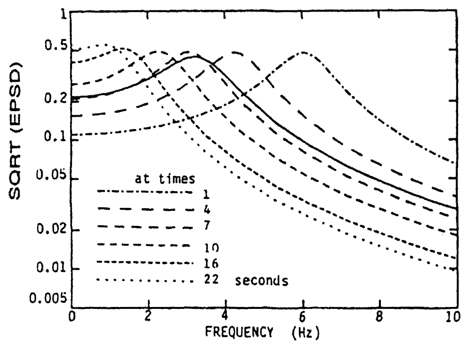

Figure 2. The normalized EPSD function computed by the (TV) (dashed curves) and the (TI) (solid curve) models for the accelerogram in Figure 1.

$$\ddot{x}(t) + 2\zeta(t)\omega(t)\dot{x}(t) + \omega^2(t)x(t) = a(t) \quad (8)$$

where $\omega(t)$ and $\zeta(t)$ are the equivalent linear frequency and damping ratio, respectively, given by

$$\frac{\omega(t)}{\omega_0} = \left[\sqrt{\pi}\gamma\exp\left(\gamma^2\right)\operatorname{erfc}(\gamma)\right]^{\frac{1}{2}}, \quad \zeta(t) = \frac{\zeta_0\omega_0}{\omega(t)} \quad (9)$$

with $\gamma = \sqrt{2}x_y/[\pi\sqrt{q_{11}(t)}]$ and $q_{11}(t) = E[x^2(t)]$. The mean-square statistics of the response are obtained by solving the non-linear moment equations corresponding to the system of equations (8), (1) and (2).

Papadimitriou (1990) has shown that for slowly-varying coefficients $I_g(t)$, $\omega_g(t)$ and $\alpha_g(t)$, the mean-square displacement $q_{11}(t)$ of the response of the equivalent linear system (8) can be obtained approximately by the simple first-order differential equation

$$\dot{q}_{11}(t) + 2\zeta_0\omega_0 q_{11}(t) = \frac{I_g^2(t)}{2\omega^2(t)} R_g\left(\omega(t), \zeta_0\omega_0; \underline{\theta}_g(t)\right) \quad (10)$$

where $\underline{\theta}_g(t) = (\omega_g(t), \alpha_g(t))$, and the form of $R_g(t)$ depends on the structure of the normalized evolutionary power spectral density (EPSD) of the ground acceleration defined by

$$S_N\left(\omega; \underline{\theta}_g(t)\right) = S\left(\omega; \underline{\theta}_g(t)\right)/I_g^2(t) \quad (11)$$

where $S\left(\omega; \underline{\theta}_g(t)\right)$ is the EPSD of the ground motion model. For lightly damped oscillators ($\zeta(t) \longrightarrow 0$),

$$R_g\left(\omega(t), \zeta_0\omega_0; \underline{\theta}_g(t)\right) \longrightarrow S_N\left(\omega(t); \underline{\theta}_g(t)\right) \quad (12)$$

4 APPLICATIONS; MOVING RESONANCE EFFECT

The previous formulation provides valuable analytical insight into the mean-square response characteristics and their dependence on the nonstation-

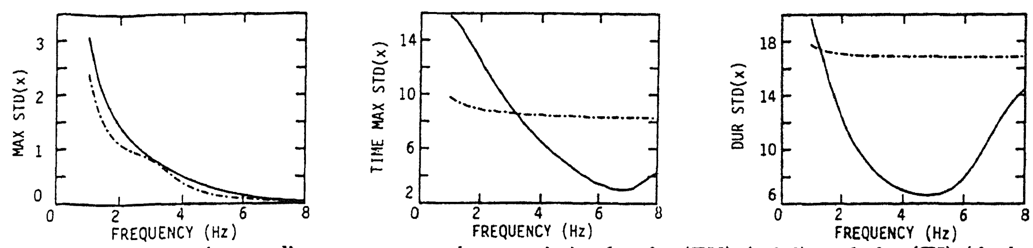

Figure 3. Nonstationary linear response characteristics for the (TV) (solid) and the (TI) (dashed-dotted) excitation for various structural frequencies ω_0 ($\zeta_0 = 0.05$).

ary intensity (via $I_g(t)$) and the frequency content (via $R_g(t)$). For illustration purposes, consider the response of the system to two types of excitation which differ only in the way they model the frequency content of the ground motion. The first excitation, denoted by (TV), has time-varying frequency content, while the second excitation, denoted by (TI), has time-invariant frequency content throughout the duration of the excitation. The parameters of the (TV) and (TI) models are estimated by applying our Bayesian method to the accelerogram shown in Figure 1. The frequency content of the (TI) model is chosen to fit the segment of the accelerogram with the stronger intensity, that is, from 2 to 12 seconds. The time variation of the intensity is the same for both models ($I_{max} = 71$, $\beta = 1.34$, $t_{max} = 7.4$ and $t_0 = 0.66$). The normalized EPSD are compared in Figure 2 for both excitations. It is clear that the predominant frequencies of the recorded ground motion shift to lower frequencies with increasing time, as reflected by the EPSD for the (TV) model. This is often observed in earthquake ground motions.

For linear oscillators ($x_y \longrightarrow \infty$) and for the (TI) ground motion, S_N is constant and the shape of the forcing term in (10) is controlled only by the amplitude nonstationarity $I_g(t)$ of the ground motion. However, for the time-varying frequency content, the spectral component $S(\omega_0; \underline{\theta}_g(t))$ of the ground motion varies with time, altering the shape of the forcing term in (10). Therefore, the characteristics of the response in the time-varying case are expected to be different from those in the time-invariant one. For non-linear oscillators, it may happen that the decrease of the structural frequency of the softening structure tracks the decrease of the predominant frequency of the ground motion, resulting in significant amplification of the response. This "locking" of the structural frequency with the predominant frequency of the ground motion will be referred to as the "moving resonance" effect. This effect is less likely to occur for the (TI) excitation since the softening at resonance will cause a decrease in the structural frequency which will move the structure out of resonance with the ground motion.

4.1 Linear oscillators

Define the STD response as $\sqrt{q_{11}(t)}$. Its maximum, the corresponding time that the maximum occurs and the duration of the STD response are compared in Figure 3 for the two types of excitation and for a linear oscillator with fundamental frequencies ranging from 1 to 8Hz. The duration is defined as the difference between the two times that the STD of the response upcrosses and downcrosses 50% of its maximum value. All numerical results correspond to 5% initial damping ratio. For the (TI) excitation, the time of the maximum STD and the STD duration, which control the shape of the nonstationary STD response, do not vary significantly over the linear oscillator frequencies examined. This is because the time variation of the forcing term in (10) depends only on $I_g(t)$ in this case. The values of the oscillator parameters ω_0 and ζ_0 control only the overall amplitude of the forcing term. Therefore, the resulting shape for the STD response is controlled mainly by the product $\zeta_0\omega_0$. However, as $\zeta_0\omega_0$ increases the shape becomes independent of this product (see equation (10)). For the (TV) excitation, however, the shape of the forcing term in (10) is sensitive to the individual values of the oscillator parameters ω_0 and ζ_0, and so, therefore, is the resulting shape of the STD response. The time of the maximum STD responses as well as the duration of the responses corresponding to the (TI) and (TV) excitations can differ by a factor of 2 or more.

4.2 Non-linear oscillators

The full time history of the STD displacement response are compared in Figures 4(a) and 4(c) corresponding to different parameters of the non-linear system. The STD displacement response of the non-linear oscillator to the (TI) and (TV) excitations are denoted by NTI and NTV, respectively. The time variation of the corresponding equivalent structural frequencies $\omega(q_{11}(t))$ computed by (9) are plotted in Figures 4(b) and 4(d). For comparison purposes, each figure also includes the time variation of the dominant damped frequency $\omega_g'(t)$ of each ground motion model. Considerable amplification of the response is expected to occur in the resonance situation, that is, when the dominant frequency of the ground motion approximately coincides with the oscillator frequency.

In Figure 4(a), the maximum STD response corresponding to the (TV) excitation differs from the maximum response corresponding to the (TI)

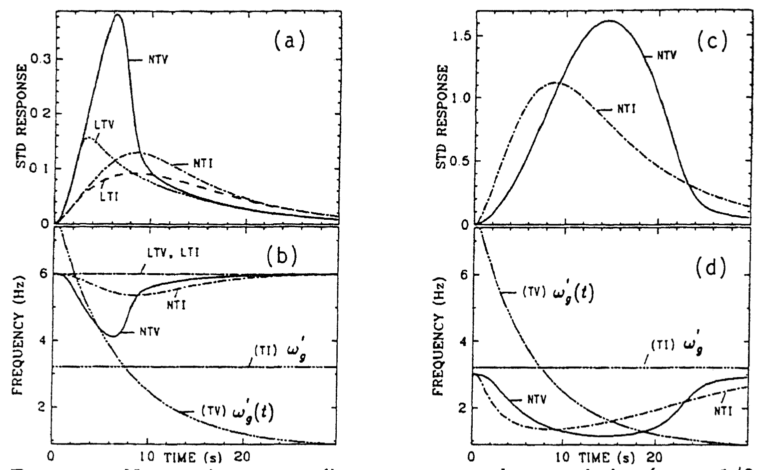

Figure 4. Nonstationary nonlinear response characteristics ($x_y = 1/3$, $\zeta_0 = 0.05$) for the (TV) and (TI) excitations for initial frequencies $\omega_0 = 6$ Hz (a,b) and $\omega_0 = 3$ Hz (c,d).

excitation by a factor as high as three. For comparison, the response to the (TI) and (TV) excitations of a linear oscillator with the small-amplitude parameters of the non-linear oscillator is plotted in Figure 4(a,b), labeled by LTI and LTV respectively. The linear STD responses which correspond to the (TI) and (TV) excitation differ only by a factor of two. For the (TI) excitation, the non-linear system is behaving almost linearly since the equivalent frequency (Figure 4(b)) does not vary significantly. The much larger STD response for the non-linear oscillator compared with the linear oscillator is attributed to the moving resonance effect occurring for the (TV) excitation. This is apparent from Figure 4(b) from approximately the first to the seventh second of the excitation, where the structural frequency tracks the changing predominant ground motion frequency over this time period.

In Figure 4(c,d), the initial structural frequency was chosen to be close to the dominant frequencies of the strong S-waves of the ground motion. For the (TI) excitation, the equivalent linear system (8) is never in resonance with the ground motion. Similarly, for the (TV) excitation, over the first 10 to 15 seconds of the highest ground intensity, the equivalent linear system is not in resonance with the ground motion. However, at later times when the weaker surface waves of the ground motion are arriving, the equivalent linear system resonates with the ground motion from approximately 15 to 22 seconds, causing an amplification of the response. Therefore, in this case, the maximum STD response is controlled primarily by surface waves rather than the S-waves. Modeling the ground motion by the (TI) excitation where the S-waves control the response results in an underestimation of the importance of the weaker intensity surface waves.

5 CONCLUSIONS

A "realistic" ground motion model was proposed which appears to be promising for seismic risk studies involving ground motion time histories rather than simplified representations such as peak ground quantities or response spectra. An approximate random vibration formulation was used to analyze the effect of the temporal nonstationarity in the frequency content of the ground motion on the response of simple linear and non-linear systems. The characteristics of both linear and non-linear response strongly depend on the time variation of the frequency content of the excitation, so time-invariant frequency content models are inappropriate to model ground motions with time-varying frequency content. In particular, the temporal nonstationarity in the frequency content of the ground motion can have a substantial effect on the response of non-linear structures of softening type, especially when the lengthening of the structural periods due to the softening of the structure tracks the shift of the dominant frequencies of the ground motion.

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