Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Nonstationary models of earthquake accelerograms

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ABSTRACT: Two frequency nonstationary stochastic models of earthquake excitation are compared in this paper. The guidelines of the theoretical foundation of these recent models are introduced and commented. The first approach is based on the definition of a frequency modulated random process by means of an istantaneous power spectral density function in time and frequency. The second model is of seismological derivation and includes the source mechanism and the propagation of the waves to the site. It leads to the definition of an evolutionary power spectral density function allowing a complete description of the time-variant characteristics of strong ground motions. In the numerical example, the two stochastic models are characterized on the basis of significative Italian accelerograms. The comparison of time-frequency spectral funcions with the original spectra points out analogies and discrepancies between the two earthquake models.

1. INTRODUCTION

The use of the theory of stochastic models has been of common use for the simulation of strong ground motion time histories since long ago and is continuously updating. In literature one can find many different points of view for approaching the problem, see for example the works by Sargoni & Hart (1974), Priestley (1981), Lin (1986), Mark (1986), Kameda (1987), Kozin (1988), Yeh & Wen (1989), Faravelli (1988). Among them the most reliable are of pure nonstationary type, because able to follow the energy distribution in time and the contemporary variation of the frequency content with the accuracy required by the modern methods of nonlinear dynamics analysis. The energy release and the frequency content can strongly affect the response of a structural system expecially in the post-elastic range, Carli (1988) and Carli et al. (1989).

Two different nonstationary stochastic models of particular interest for practical engineering applications are considered in this work. The first approach, described in the works by Grigoriu et al. (1988) and by Yeh & Wen (1989), is based essentially on signal theory procedures. In particular it refers to the definition of the istantaneous spectrum of a nonstationary time process according the definition given by Mark (1986). The second model herein considered takes into account seismological interpretations of the local seismic condition and of the wave propagation from the source to the site by means of a simplified geomechanical formulation explicitly given in Carli & Faravelli (1990 b). The seismological approach is then coupled with the classical methods for the treatement of time processes Bendat & Piersol (1971) to account for a more precise time-frequency characterization. A short description of the two models will be followed by a numerical example taylored to give a visual comparison of the computational implications and to suggest comments about similarities, discrepances and capabilities of the described theoretical nonstationary methods. The numerical application is referred to two of the most significant records of the Italian data bank. One was recorded in the Friuli region during the North-Italian sequence in 1976, while the other one refers to the South-Italian event in 1980. The records have been selected with particular regard to the very different predominant frequency evolution in time that is developped in the two accelerograms. In this work particular attention is devoted to find out the main features and the implications underlaying the two different theoretical foundations for the approximate description of real seimological records. The comparison is performed on the basis of the estimated amplitude spectra for both the models and the real ones, as functions of time and frequency. In particular the spectrograms of the theoretical approaces and of the recorded signals allow a comfortable and clear representation of the estimated and the expected evolution of the predominant frequency content.

2. AN ISTANTANEOUS SPECTRUM MODEL

Let us introduce a stationary stochastic process $y(u)$ to be defined by the following spectral representation:

$$y(u) = \int_{-\infty}^{+\infty} e^{i\omega u} \, dz(\omega) \tag{1}$$

where ω is the angular frequency and $z(\omega)$ is a random process having orthogonals increments :

$$E[dz(\omega_1) \; dz(\omega_2)] = 0 \quad , \quad \forall \omega_1 \neq \omega_2 \tag{2}$$

The autocorrelation function $R_y(\tau)$ of the stationary process can be written as:

$$R_y(\tau) = \int_{-\infty}^{+\infty} e^{i\omega\tau} S_y(\omega)\, d\omega \quad (3)$$

being $S_y(\omega)$ the power spectral density function.

If one performs a scaling of the u axis according a nonlinear expression $u = u(t)$ with respect to the new variable t , the just obtained process $x(t) = y(u(t))$ will not be frequency stationary. Following the approach given in Yeh & Wen (1989) with regard to the definition of the istantaneous spectrum of a stochastic process, it can be proved that the following holds:

$$S_x(t,\omega) = \frac{1}{u'(t)} S_y(\frac{\omega}{u'(t)}) \quad (4)$$

whith $S_x(t,\omega)$ the istantaneous power spectral density function of $x(t)$, being the $'$ symbol the first derivative of the function with respect to the time variable t. Equation (4) relates the power spectral density of the nonstationary process $x(t)$ with the one of a stationary process $y(u)$ 'associated' to $x(t)$. Infact $y(u)$ can be derived form $x(t)$ when the nonlinear scale function $u(t)$ is given. Moreover $u'(t)$ can be proved to be responsible of only a frequency modulation effect over the random process $x(t)$. The condition of nonstationarity for this frequency modulated process does not imply an evolutionary spectral representation for $x(t)$. The stationary spectrum $S_y(\omega)$ can be assumed to have different analytical representations such as the K-T model, the Clough-Penzien (1975), or the Boore (1986) model, obtaining in all the cases a nonstationary approach by frequency modulation in time of the selected spectral model.

It must be noticed that the integral frequency relation given in equation (4) leaves the variance of the process $y(u)$ unchanged. This means that when $y(u)$ is a filtered white noise the superposition of an intensity function $i(t)$ has the same effect of an amplitude modulation of the resulting nonstationary process $x(t)$ by means of the same time function $i(t)$. The modelling of a seismic excitation $a_r(t)$ by both frequency and amplitude modulation can be written as:

$$a_r(t) = i(t)\; y(u(t)) \quad (5)$$

The characterization procedure of the above described model is necessarely based on the availability of at least one record $r(t)$ to be assumed having most of the characteristics of the considered site. Under this condition $r(t)$ can be analysed to obtain the amplitude modulation function $i(t)$, the time scaling law $u(t)$ and the shape of the power spectrum $S_y(\omega)$ of the associated stationary process. The scaling function $u(t)$ is commonly assumed to obey the following relation:

$$u(t) = v(t) \;/\; v'(t_0) \quad (6)$$

where $v(t)$ is a relation giving the mean value of the points with zero ordinates in the accelerogram as function of time t . The parameter t_0 is the starting time for the most significant part (strong ground motion part) of the accelerogram. It can be estimated by visual inspection on the plot of the time history. The required conditions on $v(t)$ are to be differentiable and a not decreasing function in time. A polynomial function is usuallv assumed:

$$v(t) = c_1 t + c_2 t^2 + c_3 t^3 \quad (7)$$

where the three parameters c_1, c_2, c_3 are calculated by numerical approximation of $v(t)$ to the cumulate of the zero passages in the considered record $r(t)$.

Let us now consider the possiblity of decomposing the original record in a number n of signals such that:

$$r(t) = \sum_{i=1}^{n} r_i(t) \quad (8)$$

where each $r_i(t)$ is obtained by filtering the Fourier amplitude spectum of $r(t)$ in specified frequency ranges and coming back to the time axis.

Given the described decomposition the described identification procedure can be easily repeated on the different $r_i(t)$. This allow a stepwise and more accurate time-frequency characterization of the accelerometric signal while requiring a proportionally increasing numerical effort. Usually, the desired accuracy can be reached by the application of a two terms series for equation (8).

3. AN EVOLUTIONARY MODEL

Among the many different models of seismological derivation that can be found in literature, the approach proposed by Boore (1986) is considered one of the most interesting for the wide possibilities of application in structural engineering. The Boore model relates the Fourier amplitude spectrum $A_a(\nu)$ of a strong ground motion record with different physical quantities of seismological and geomecanical derivation according the following formulation:

$$A_a(\nu) = C_{sc} A_s(\nu) A_c(\nu) A_m(\nu) A_f(\nu) \quad (9)$$

In the previous equation $\nu = \omega/2\pi$ is the frequency variable while C_{sc} is a scale factor relating different seismological quantities. The other factors on the left side of equation (9) can be espressed by the following analytical functions , Boore (1986) and Faravelli (1987) :

$$A_s(\nu) = M_0(2\pi\nu_s)^2/(1+(\nu_s/\nu)^2) \quad (10)$$

$$A_c(\nu) = \alpha/(\sqrt{1+(\nu/\nu_c)^2} \quad (11)$$

$$A_m(\nu) = 2/(1+(\nu_m/\nu)^2), \nu > \nu_m \quad (12)$$

$$A_f(\nu) = C_f\sqrt{1+(\nu/\nu_f)^8} \quad (13)$$

$A_s(\nu)$ gives the source spectum at the fault with M_0 the seismic moment and ν_s the corner frequency at the source. $A_c(\nu)$ is a correcting factor for the source spectrum $A_s(\nu)$ where ν_c is a frequency parameter of sismological derivation, Faccioli et al. (1984), and α is a normalization coefficient. $A_m(\nu)$ acts as an amplification factor having the relative reference frequency ν_m , Faravelli (1987). The last factor $A_f(\nu)$ takes account, in simplified manner, of the wave attenuation, Boore (1986), where ν_f is the characteristic frequency of the filter and C_f is an attenuation parameter, Rovelli (1983).

The above defined spectral function $A_a(\nu)$ can be retated to the power spectral density function $S_a(\nu)$ of a stationary random process $a(t)$ by means of the general relationship, Bendat & Piersol (1971):

$$S_a(\nu) = | A_a(\nu) |^2 / (\pi D) \qquad (14)$$

in which D is the duration of $a(t)$.

This stationary approach has been recently expanded (Carli&Faravelli(1990 b)) with the capability of a nonstationary description of both the frequency content and the energy in time. The resulting stochastic process $n(t)$ is associated to an evolutionary Fourier amplitude spectrum $A_n(t,\nu)$ by a frequency modulation procedure:

$$| A_n(t,\nu) |^2 = i(t) \mid A_a(\nu \mid \nu_s(t)) |^2 \qquad (15)$$

where $i(t)$ is the usual intensity function, Eq.(5), and $A_a(\nu \mid \nu_s(t))$ has the analytical formulation given in equation (9) with the assumption of a time dependent characteristic frequency ν_s:

$$A_a(\nu \mid \nu_s(t)) = C_{sc} \; \phi(\nu_s(t)) \; A_s(\nu \mid \nu_s(t)) \; A_c(\nu) \; A_m(\nu) \; A_f(\nu) \qquad (16)$$

In this equation appears the new frequency function $\phi(\nu_s)$ responsible of mantaining constant in time the energy released by the process. This condition is needed since the variation in shape of the amplitude spectrum, due to $\nu_s(t)$, reflects in significant variations of the energy for each instant t. By $\phi(\nu_s)$ the variance of the process can be controlled by the amplitude modulation function $i(t)$, in agreement with the istantaneous spectrum approach previously described. $\phi(\nu_s)$ is given in term of the single frequency parameter ν_s and can be evaluated only once for a predefined wide frequency range. In previous numerical investigations by Carli & Faravelli (1989-1991), a compound analytical form with two constant segments connected by a negative exponential has been suggested:

$$\nu_s(t) = \begin{cases} \nu_r = const. & ,0 \le t \le t_r \\ \nu_r \exp[-\ln \frac{\nu_r}{\nu_d} (\frac{t-t_r}{D_0})^k] & ,t_r \le t \le t_d \\ \nu_d = const. & ,t_d \le t \le D \end{cases} \qquad (17)$$

In this equation the parameter k rules the decay rate of the exponential segment, t_r represents the starting time of the strong ground motion part of the accelerogram while t_d give the starting time of the decay part i.e. the tail of the record and $D_0 = t_d - t_r$ can be seen as the equivalent stationary duration of the strong ground motion as reported in Vanmarke & Lai (1980) and Carli(1988).

The last two parameters to be defined ν_r and ν_d correspond to the corner frequencies ν_s, evaluated by a numerical approximation procedure (Carli et al. (1989) and Carli & Faravelli (1990 a)). The procedure is applied in terms of the Boore amplitude spectra related to the two parts $[0, t_r]$ and $[t_d, D]$ extracted from the record $r(t)$ by time window.

4. NUMERICAL EXAMPLE

In the numerical application of the discussed nonstationary stochastic models, a first part is devoted to the characterization of the two approaches. The values of all the parameters that take part in the description of the derived algorithms are identified on the basis of the selected record. Particular attention is paid to the adopted numerical procedures and to the effects on the most significant parameters. In the second part a comparison is given in terms of the density plots of the nonstationary spectra for both the two models and the recorded accelerogram. The identification of the needed parameters is based on one of the most significant record ($r(t)$ in the derivation of the equations) of the North Italian (Friuli) earthquake in 1976: N.38 (Tolmezzo recording station, N-S component) according the system of numbering in the national seismic catalog by ENEA-ENEL (1983).

The modulating function $i(t)$ is evaluated by means of a convolution procedure (Carli & Faravelli (1990 b)). The polynomium $v(t)$, mean value of the zero crossings in time, is estimated by nonlinear least squares procedure over the cumulate of the zero crossings $v_r(t)$ counted in the record. The other parameter t_0 that comletes the definition of the scale function $u(t)$ for the time axis is evaluated to be equal to the t_r parameter used in the definition of $\nu_s(t)$, Eq.(17), as initial value of the strong ground motion part of the accelerogram. It is obtained jointly with the other parameter D_0 (equivalent stationary duration) used in $\nu_s(t)$, by trilinear approximation of the cumulate energy function (Carli (1988)). The stationary process $y(u)$, Eq.(5), associated to $a_r(t)$ is obtained by amplitude demodulation and hence the power spectrum $S_y(\nu)$ can be derived. The spectrum is approximated by the analytical function, Eq.(9), of the Boore' stationary model. With the previous assumptions, the calculated frequency parameters are: ν_c=2. Hz, ν_m=.32 Hz, ν_f=10. Hz, while the corner frequency of the source spectrum is ν_s=.64 Hz.

While mantaining valid some of the previously defined parameters, the characterization of the second model is strongly dependent on the definition of the frequency modulating function $\nu_s(t)$. In particular the following values are obtained for the other two frequency parameters: ν_r=4.44 Hz , ν_d= .68 Hz . The k exponent assumes a value (k=.34) that is very close to the ones obtained for records from different seismic events (Carli et al. (1989)) giving the confirmation of its low variability to relevant changes of the other parameters involved in the definition of $\nu_s(t)$.

Given the complete characterization of the models, the evolutionary amplitude spectrum (not smoothed, $A_d(t,\nu)$) of the demodulated record, can be compared with the parallel spectral functions of the seismological model $A_a(t,\nu)$, Eq.(16), and with the amplitude spectrum $A_x(t,\nu)$ derived from the istantaneous power spectrum $S_x(t,\omega)$ of Eq.(4). This three spectra, in terms of density plots, appear in

Fig.1a/1c with the just mentioned order. The representation emphasizes the meaning of the evolution of the predominant frequency content in time : it points out the influence of the frequency modulation procedures of the two models only, while the rate of energy release in time remains constant.

The results obtained when applying the modulating function $i(t)$ are shown in Fig.2a/2c where are plotted, in the order, the evolutionary spectrum of the recorded accelerogram $A_r(t,\nu)$, of the seismological model $A_n(t,\nu)$, Eq.(15), and of the istantaneous spectral model $A_i(t,\nu)$ obtained by modulation of $A_x(t,\nu)$.

For the demodulated spectra, Fig.1a/1c, the results point out the high scatter in the non-smoothed spectrum of the record when compared to the parallel spectra of analytical formulations. With this limitation $A_a(t,\nu)$ shows a better agreement with respect to $A_x(t,\nu)$ in approaching the rate of change of the predominant frequency content of the record from the initial high central frequency to the lower values reached within the first few seconds.

Similar considerations, with higher approximation, are valid for the modulated spectra. Infact $A_n(t,\nu)$ confirms its better fit to the central frequency of the record in time during the strong ground motion part, while showing a smoother shape with lower peaks.

5. CONCLUSIONS

In this work two non stationary models of recent formulation for the description of seismic excitations have been considered. The first model is based on the definiton of an istantaneous spectrum that in this paper is associated to a Boore type spectrum. This approach shows its strict dependence to the refinement level of the signal analysis to be performed on the reference record.

The evolutionary model recently developed by the author and by Faravelli points out a wider ductility of use especially when applied in presence of poor or not completely reliable records at the site. This result can be reached for the combianation of seismological hints with the classical methods of signal analysis that are used in the theoretical derivation of the evolutionary model. The comparison of the two approaches is performed in terms of the density plots of the underlaying nonstationary spectra, both modulated and demodulated in time for an unbiased evaluation of the performances relevant to the only frequency evolution. The method of seismological derivation proposed by the author shows a wider settlement in the range of frequency noticed in the analysis of the reference record, while preserving the order of magnitude of the numerical effort.

ACKNOWLEDGMENTS - The present work has been partially financed by grants of the Ministry of University and of Scientific and Technologic Research (MURST).

REFERENCES

Boore, D.M. 1986. Short Period P and S-Waves Radiation from Large Earthquakes: Implication for Spectral Scaling Relations. *B.S.S.A.* 76:43-64.

Carli, F. 1988. Aspetti della Caratterizzazione Temporale di una Registrazione Accelerometrica (in Italian). *Ingegneria Sismica.* 2:37-51.

Carli, F. 1988. Un Modello Stocastico di Azione Dinamica per Strutture Non Lineari con Degrado (in Italian). *Proc. IX A.I.M.E.T.A.:* 667-670. Bari.

Carli, F., Casciati, F. & Faravelli L. 1989. Evolutionary Response of Hysteretic Oscillators. *Proc. 1° Conv. Naz. Mecc. dei Materiali e delle Strutture:* 117-125. Roma.

Carli, F., Casciati, F. & Faravelli, L. 1989. Modeling the Nonstationarity of the Frequency Content in Strong Gro-und Motions. *Proc. Int. Conf. on Earth. Resist. Constr. and Design:* 63-72. Berlin: Balkema.

Carli, F. & Faravelli, L. 1989. Nonstazionarietà dell' Eccitazione Sismica con Modu-lazione in Frequenza (in Italian). *Proc. 4° Conv. Naz. Ing. Sismica in Italia:* 216-223. Milano: Patron.

Carli, F. & Faravelli, L. 1990 a. Modeling Nonstationary Ground Motions , *9-th Europ. Conf. on Earth. Eng.:* Vol.4-A: 63-72. Moscow.

Carli, F. & Faravelli, L. 1990 b. A Non Stationary Seismological Model for Strong Ground Motion. *Europ. Earth. Eng.* 3:29-42.

Carli, F. & Faravelli, L. 1991. Modelli non stazionari di eccitazione sismica (in Italian). *Proc. 5° Conv. Naz. Ing. Sismica in Italia:* 3-12. Palermo.

Casciati, F. & Faravelli, L. 1991 *Fragility Analysis of Complex Structural Systems.* Taunton: Research Studies Press.

Clough, R.W & Penzien, J. 1975. *Dynamics of Structures.* Tokyo: McGraw-Hill.

Der Kiureghian, A. & Crempien, J. 1989. An Evolutionary model for Earthquake Ground Motion. *Sructural Safety* 6:235-246.

ENEA-ENEL 1983. Uncorrected Strong Motion Data. *Rep. GT2.*. Roma.

Faccioli, E., Rovelli, A. & Fregonese, R. 1984. A Study of the Special Characteristics of Accelerogram from Recent Earthquakes in Italy and Near Regions. *Proc. 8-th W.C.E.E.:* 361-368. S.Francisco: ????.

Faravelli, L. 1987. Modeling the Seismic Input for a Stochastic Dynamic Structural Problem. *Proc. I.C. A.S.P. 5:* 230-237. Vancouver.

Faravelli, L. 1988. Source to Site Seismic Models in Structural Dynamics. *Proc. 3-rd Int. Conf. on Recent Advances in Str. Dynam.:* 1021-1027. Southampton.

Grigoriu, M., Ruiz, E. & Rosenblueth, E. 1985. The Mexico Earthquake of September, 19th, 1985 - Nonstationàry Models of Seismic Ground Acceleration. *Earthquake Spectra.* Vol.4, 3:551-568.

Kameda, H. 1975. Evolutionary Specra of Seismograms by Multifilter. *J. of Eng. Mech. A.S.C.E.* 101, 6:787-801.

Kameda, H. 1987. Engineering Application of Stochastic Earthquake Motion Models with Nonlinear Soil Amplification. *Proc. 9-th S.M.i.R.T.:* 327-336. Lausanne: Balkema.

Kozin, F. 1988. Autoregressive Moving Average Models of Earthquake Records. *J. of Prob. Eng. Mechanics* 3:58-63.

Lin, Y.K. 1986. On Random Pulse Train and its Evolutionary Spectral Representation. *J. of Eng. Prob. Mech.* 4:219-223.

Mark, W.D. 1986. Power spectrum representation for non stationary random vibrations. *Random Vibration - Status and Recent Developments.* Elishokoff, I. & Lyon, R.H. (Eds.). Elsevier.

Priestley, M.B. 1981. *Spectral Analysis and Time Series.* Academic Press.

Rovelli, A. 1983. Frequency Relationship for Seismic Q_β of the Central Southern Italy from Accelerograms for

the Irpinia Earthquake 1980. *Physics of the Earth and Plan. Int.* 32:209-217.
Saragoni, G.R. & Hart, G.C. 1974. Simulation of Artificial Earthquakes. *Earth. Eng. and Struct. Dynam.* 2:249-267.
Vanmarcke, E.H. & Lai, S.-S.P. 1980. Strong-Motion Duration amd RMS Amplitude of Earthquake Records . *B.S.S.A.* 70:1293-1307.
Yeh, C.H. & Wen, W.K. 1989. Modeling of Nonstationary Ground Motion and Analysis of Inelastic Structural Response. *Structural Safety* 8: 281-298.

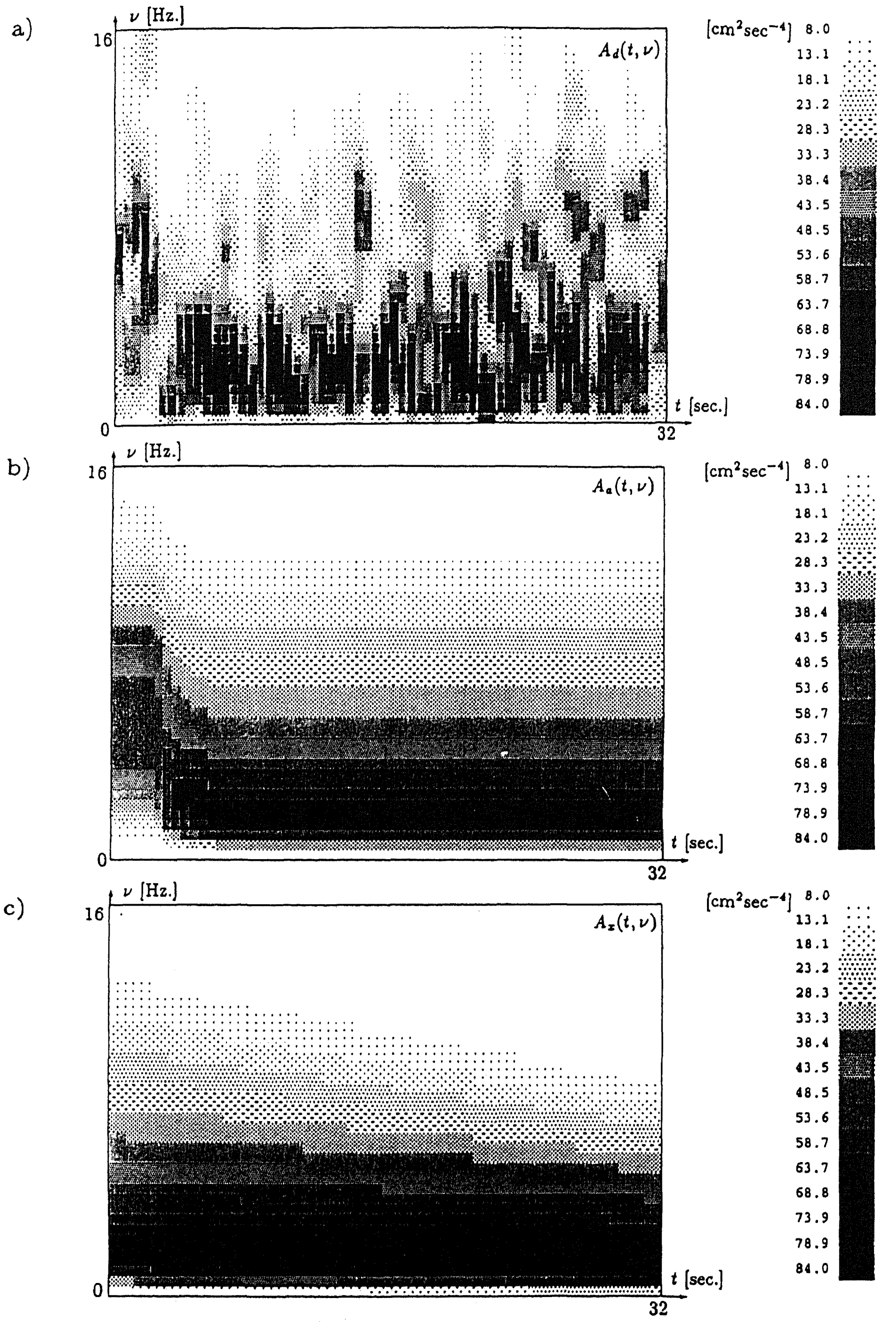

Fig. 1 - Density plots of the demodulated evolutionary spectra: a) $A_d(t,\nu)$ of the record , b) $A_a(t,\nu)$ of the seismological model, c) $A_x(t,\nu)$ of the istantaneous spectum model.

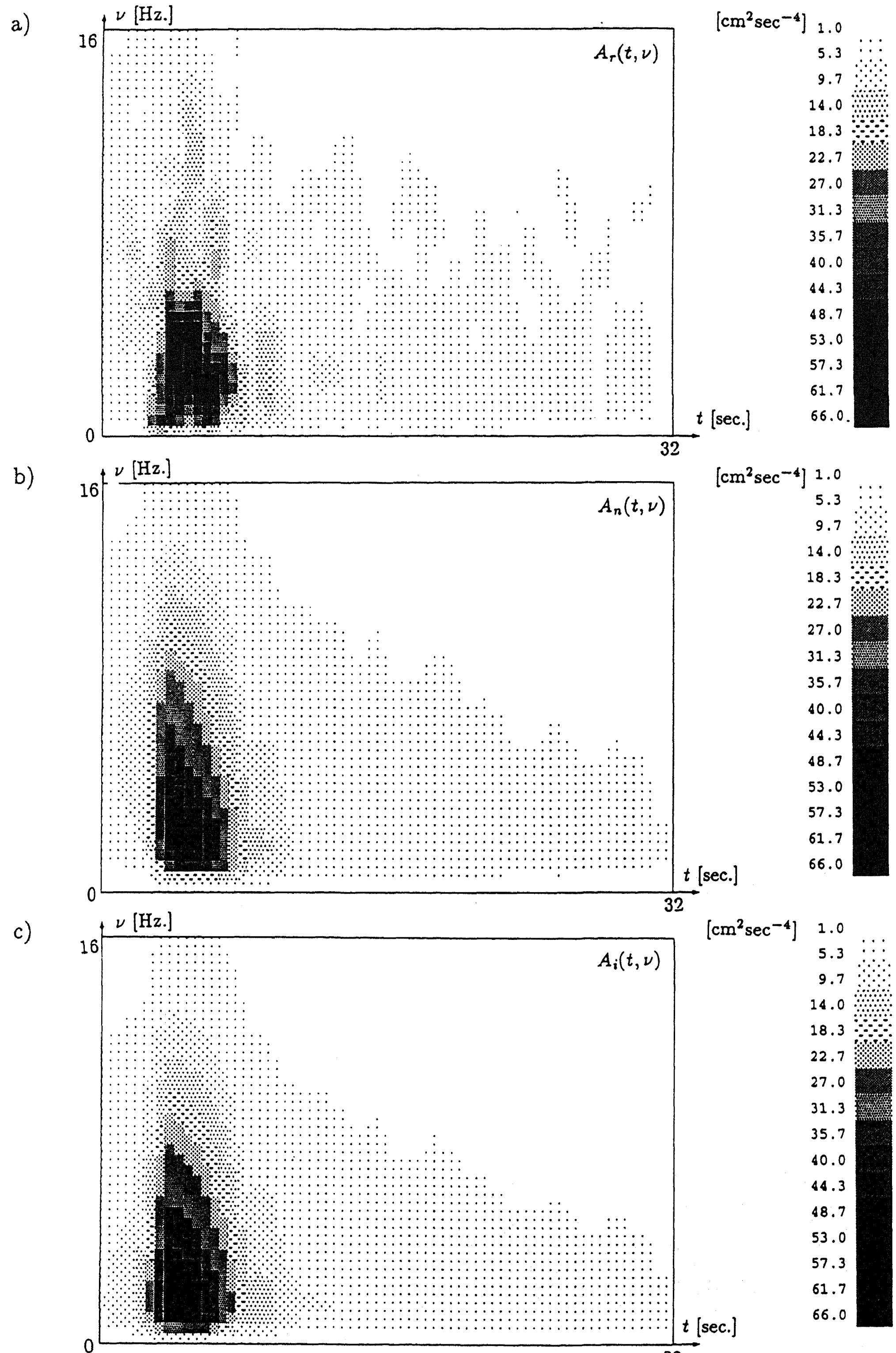

Fig.2 - Density plots of the modulated evolutionary spectra: a) $A_r(t,\nu)$ of the record , b) $A_n(t,\nu)$ of the seismological model, c) $A_i(t,\nu)$ of the istantaneous spectum model.