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A new algorithm for simulating strong motion records

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ABSTRACT: A method is developed for generating samples of zero-mean stationary Gaussian vector processes. The method is based on a generalization of the sampling theorem for real-valued random processes with band-limited spectra. The simulation method can be applied to generate all components of the ground motion at one or more sites.

1 INTRODUCTION

The generation of samples of a stationary Gaussian vector process is currently based on continuous time models consisting of a finite sum of harmonics with random phase and deterministic or random amplitude (Shinozuka 1971). They can be obtained by approximating the power spectral density of the process by a discrete spectrum with power at a finite number of frequencies. The models are conceptually simple but can be inefficient because of the excessive storage required for simulation.

This paper develops a new model for generating samples of stationary Gaussian vector processes. The model is based on a generalization of the sampling theorem for real-valued band-limited stationary processes (Wong and Hajek 1985). The model is conceptually less simple but can be used to develop an efficient simulation algorithm. The storage requirement of the algorithm is minimal because the sample generation can be performed on-line.

2 THE MODEL

Let $\underline{X}(t)$ be a zero-mean stationary Gaussian vector process with real-valued components $X_r(t)$, $r = 1, ..., p$. It is assumed that the Gaussian processes $X_r(t)$ have mean zero, bandwidth $(-\bar{f}_r, \bar{f}_r)$, $0 < \bar{f}_r < \infty$, covariance functions $c_{ru}(\tau) = E\, X_r(t+\tau)\, X_u(t)$, and mean power spectral densities $s_{ru}(f) = \int_{-\infty}^{\infty} e^{-i2\pi f\tau} c_{ru}(\tau)\, d\tau$, in which $r, u = 1, ..., p$.

Let

$$\underline{X}_{\underline{N}}(t) = \begin{bmatrix} X_{1,N_1}(t) \\ . \\ . \\ . \\ X_{p,N_p}(t) \end{bmatrix} \tag{1}$$

be a vector of parametric random processes, in which $\underline{N} = (N_1, ..., N_p)'$,

$$X_{r,N_r}(t) = \sum_{k=-N_r}^{N_r} X_{r,k}\, \alpha_k(t;\, T_r), \tag{2}$$

$$\alpha_k(t;\, T) = \frac{\sin\,[\pi(t-kT)/T]}{\pi(t-kT)/T}, \tag{3}$$

$X_{r,k} = X_r(kT_r)$, and $T_r = 1/(2\bar{f}_r)$. It can be shown that (i) $X_{r,N_r}(t)$ has the same first two moments as $X_r(t)$ asymptotically as $N_r \to \infty$, $r = 1, ..., p$; (ii) $\underline{X}_{\underline{N}}(t)$ approaches $\underline{X}(t)$ in the mean square sense as $N_r \to \infty$, $r = 1, ..., p$; and (iii) $\underline{X}_{\underline{N}}(t)$ is a version of $\underline{X}(t)$ asymptotically as $N_r \to \infty$, $r = 1, ..., p$, because $\underline{X}(t)$ and $\underline{X}_{\underline{N}}(t)$ are Gaussian processes.

3 SIMULATION ALGORITHM

The use of the parametric model $\underline{X}_{\underline{N}}(t)$ in simulation can be inefficient because it depends on a large number of sample values $\{X_{r,k}\}$ of the process for large values of t. An alternative model based on a local representation is considered.

Let

$$\underline{Y}_{\underline{n}}(t) = \begin{bmatrix} Y_{1,n_1}(t) \\ . \\ . \\ . \\ Y_{p,n_p}(t) \end{bmatrix} \tag{4}$$

be a local model of $\underline{X}(t)$, in which

$$Y_{r,n_r}(t) = \sum_{k=n_{r,t}-n_r}^{n_{r,t}+n_r+1} X_{r,k}\,\alpha_k(t;\,T_r) \qquad (5)$$

$$r = 1, \ldots, p$$
$$n_{r,t}T_r \le t \le (n_{r,t}+1)\,T_r,$$

$n_{r,t}$ = the largest integer smaller than t/T_r, n_r= specified positive integers, and $\underline{n} = (n_1, \ldots, n_p)'$. Any component of the model depends on $2(n_t+1)$ values of $X_r(t)$ that are located symmetrically about the cell $[n_{r,t}T_r, (n_{n_{r,t}}+1)T_r]$ containing the current time t. The processes $Y_{r,n_r}(t)$ have the same asymptotic properties as $X_{r,N_r}(t)$ as $n_r \to \infty$, r = 1, ..., p. Therefore, $\underline{Y}_{\underline{n}}(t)$ can be used to approximate $\underline{X}(t)$.

To simplify the illustration of the simulation algorithm based on Eq. 5 consider that $T_r = T$ and $n_r = n$ so that $n_{r,t} = n_t$, r = 1, ..., p. Let $t \in [n_tT, (n_t+1)\,T]$ and $\underline{y}_{\underline{n}}(t)$ be a realization of $\underline{Y}_{\underline{n}}(t)$ in this cell depending on $\underline{X}((n_t+n+1)\,T) = \underline{x}_1$, ... $\underline{X}((n_t-n)T) = \underline{x}_{2(n+1)}$. The extension of $\underline{y}_{\underline{n}}(t)$ in the next cell $[(n_t+1)\,T, (n_t+2)T]$ involves a sample of random vector $\underline{X}((n_t+n+2)T)$ that depends on the previously generated nodal values of the process. This condition has to be accounted for in simulation and can be dealt with efficiently. This brief description of the algorithm shows that one single random vector has to be generated as t enters a new cell. The generation of this random vector can be performed on–line.

4 APPLICATIONS

Exact and approximate covariance functions are presented for two processes $\underline{X}(t)$ with dimension p = 1 and 2. Results show that the proposed model is efficient and accurate even for small values of parameters n_r, r = 1, ..., p, defining the size of the window about the current cell.

4.1 *Example 1*

Consider a band–limited white–noise process X(t) with spectrum $s(f) = s_0$ for $f \in (-\tilde{f}, \tilde{f})$ and zero otherwise. Figure 1 shows exact and approximate covariances of X(t) for $t/T = k, k + 1/4, k + 1/2$, and $k + 3/4$, in which k is an integer. The approximate covariances are based on Eq. 5 and approach rapidly $E\,X(t+\tau)\,X(t)$ as the window size increases.

4.2 *Example 2*

Let $\underline{X}(t)$ be a zero–mean bivariate Gaussian process with components

$$X_r(t) = \sqrt{1-\rho}\;Z_r(t) + \sqrt{\rho}\;Z(t), \quad 0 < \rho < 1,\ r = 1, 2 \qquad (6)$$

in which $Z_r(t)$, r = 1, 2, and Z(t) are independent zero–mean, unit–variance stationary Gaussian processes. The power spectral densities $s_{Z_r}(f)$, r = 1, 2, and $s_Z(f)$ of these processes are constant and equal to $s_r = 1/(4\pi f_r^*)$, r = 1, 2, and $s = 1/(4\pi f^*)$ in $(-f_r^*, f_r^*)$ and $(-f^*, f^*)$, respectively, and zero outside these frequency bands. Figures 2 and 3 show exact and approximate covariance functions of the vector process $(X_1(t), X_2(t))$ for $\rho = 0.5$ and $t/T_r = k_r, k_r + 1/2$, in which k_r are integers. The approximate covariances can be obtained from Eq. 5 and approach rapidly the exact values with increasing window size.

5. CONCLUSIONS

A new method was developed for generating samples of stationary Gaussian vector processes. The method is based on a generalization of the sampling theorem for real–valued processes with finite bandwidth. The proposed algorithm is efficient and requires little storage.

REFERENCES

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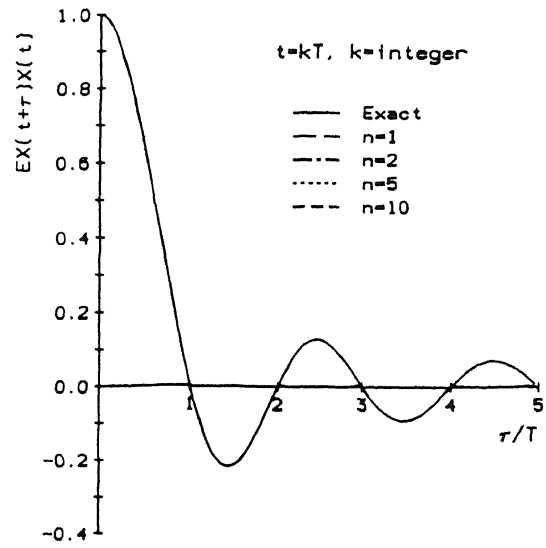
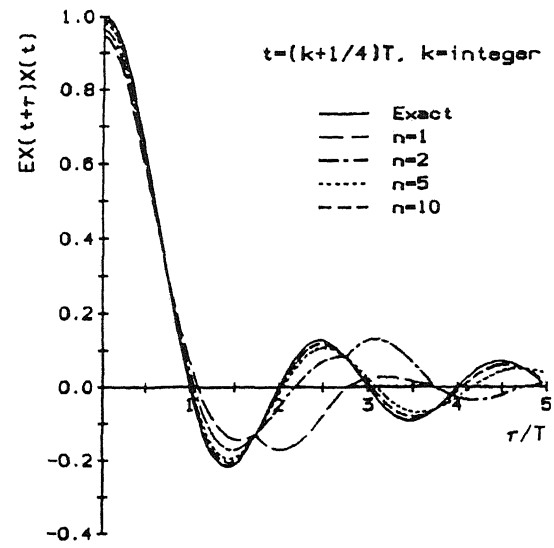
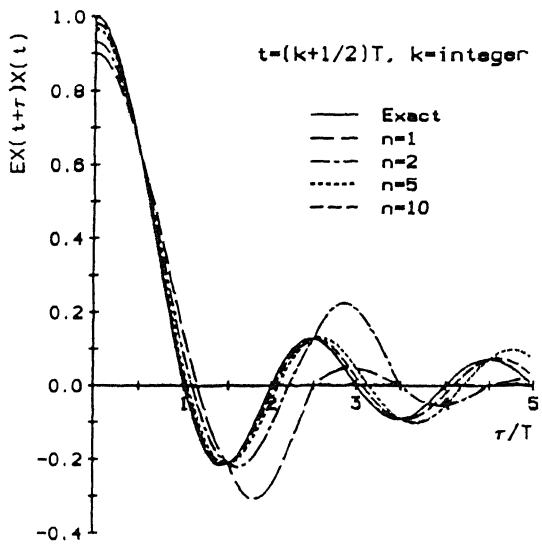
Wong, E. and B. Hajek. 1985. *Stochastic Processes in Engineering Systems*. New York: Springer Verlag.







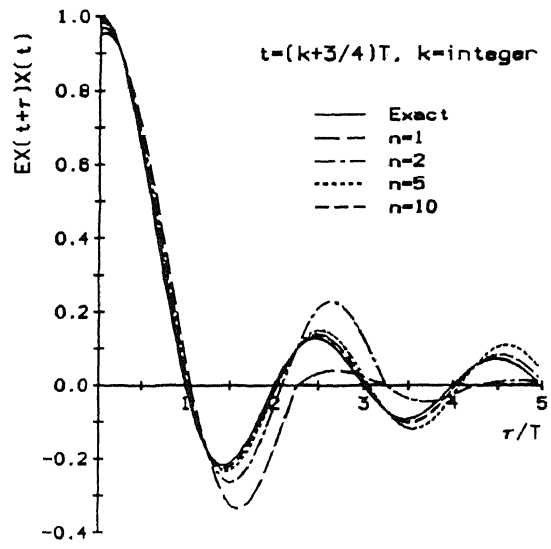


Figure 1. Exact and approximate covariance functions for a band-limited white-noise process X(t).

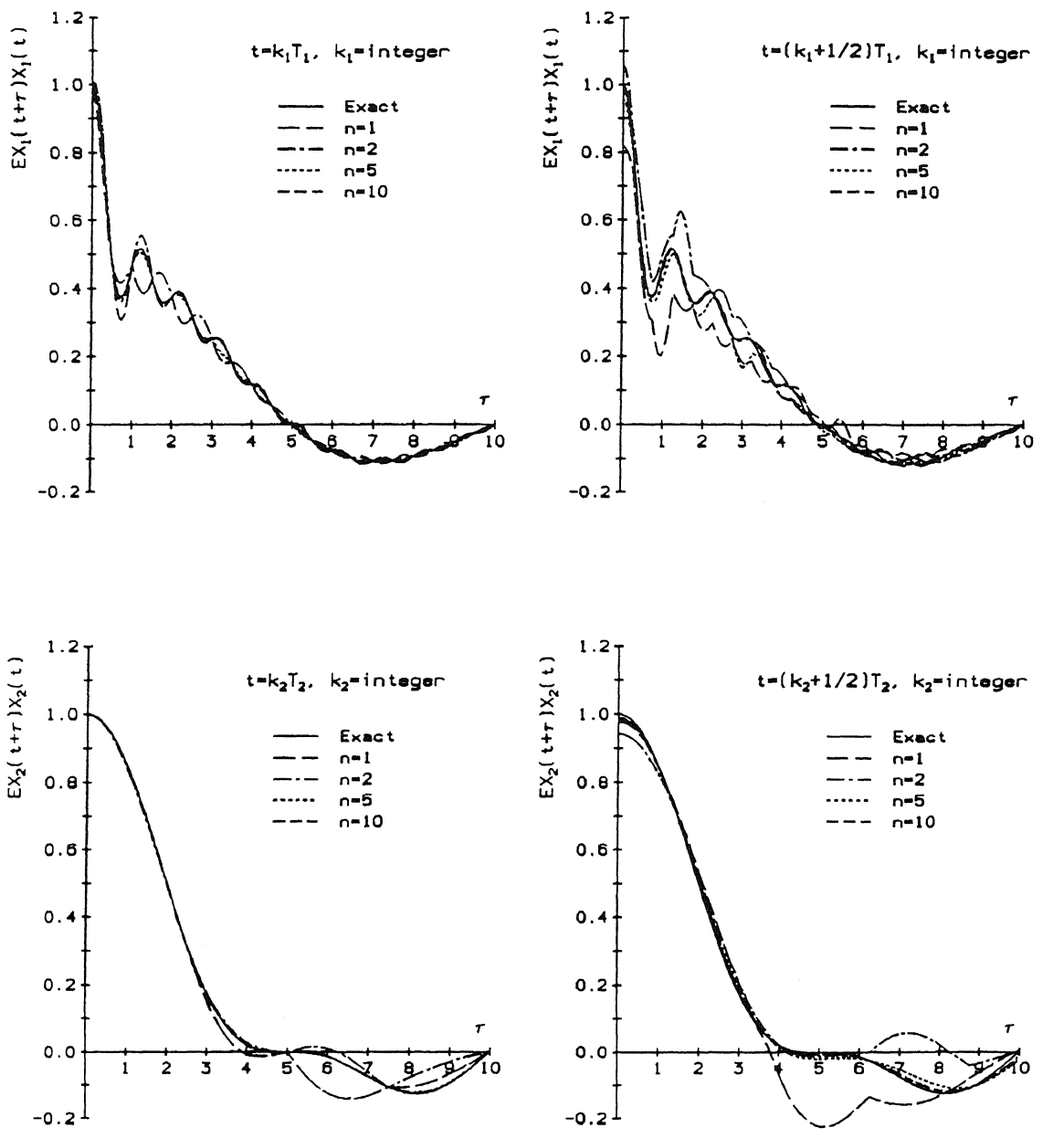


Figure 2. Exact and approximate covariance functions for the bivariate Gaussian process $\underline{X}(t)$ in Eq. 6.

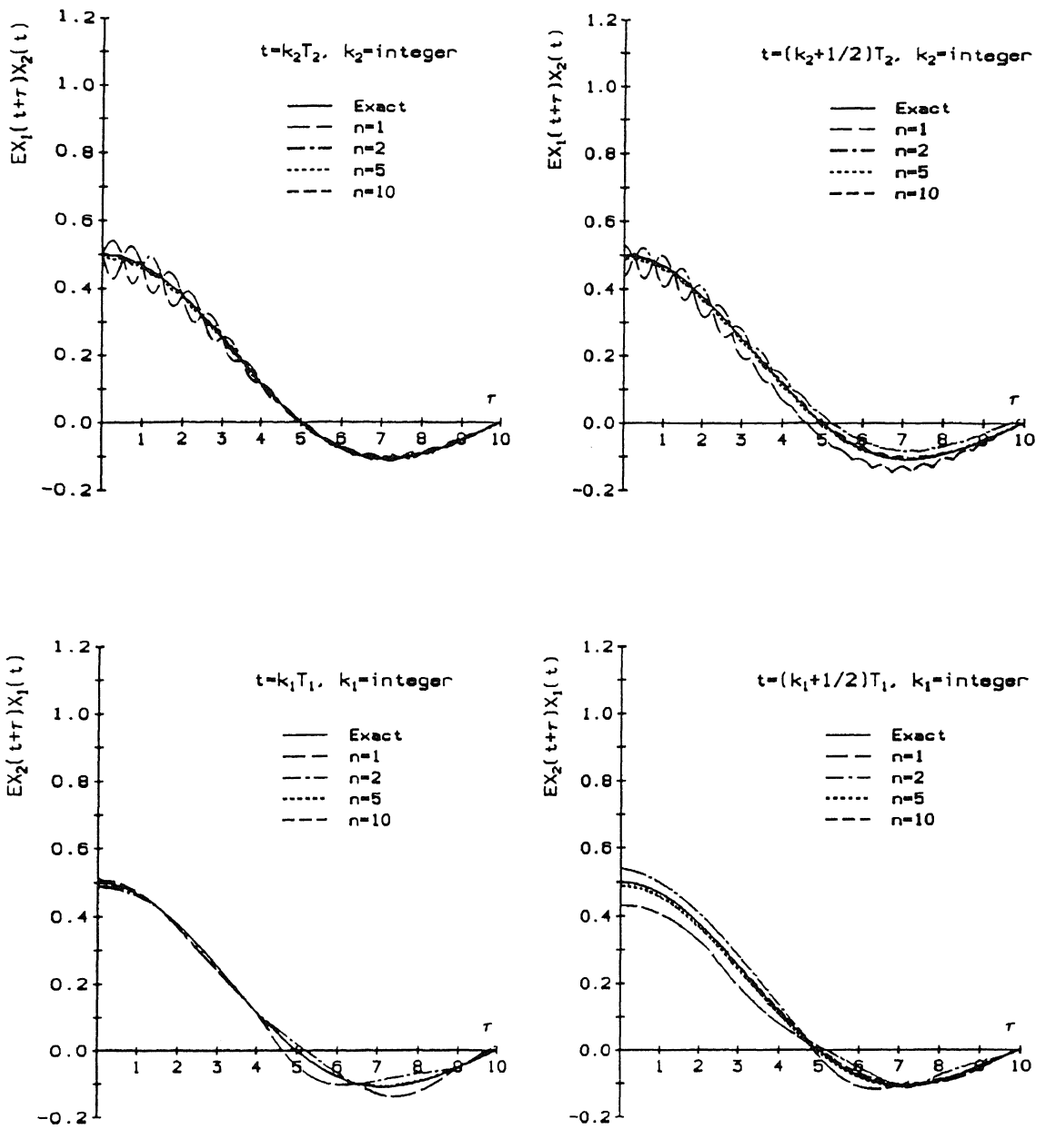


Figure 3. Exact and approximate cross-covariance functions for the bivariate Gaussian process $\underline{X}(t)$ in Eq. 6.