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Stochastic wave model of seismic ground motion

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ABSTRACT: The spatial variation of seismic ground motions is an important factor that should be carefully considered in the seismic design of buried lifelines such as tunnels and pipelines. The consideration of the spatial variation of ground motions may also have significant effects on the seismic response of structures with spatially extended foundations or multiple supports. The temporal and spatial variability of seismic ground motions has been inferred experimentally using data from closely spaced seismograph arrays. In this paper, the seismic ground motions varying in time and space domain are described analytically by the stochastic waves filtered by a surface soil layer, with random thickness, resting on rigid bedrock. The seismic wave, being assumed by a single plane wave, is transmitted to the soil layer from the rigid bedrock. The stochastic waves from the model possess the characteristics of both the coherent and incoherent components of ground motions. The analytic expressions of the frequency- wavenumber spectra of the stochastic waves at ground surface are then derived.

1 INTRODUCTION

The spatial variation of seismic ground motions is an important factor that should be carefully considered in the seismic design of buried lifelines such as tunnels and pipelines. The consideration of the spatial variation of ground motion may also have significant effects on the seismic response of structures with spatially extended foundations or multiple supports. In fact, for buried lifelines, the seismic deformation method was developed (Public Works Research Institute 1977) and is now in practical use in Japan. For the seismic design of the Akashi Kaikyo bridge foundations, a modified response spectrum was used taking account for the spatial variation of ground motions around the foundations (Kashima et al.1984,Kawaguchi et al.1987).

The temporal and spatial variability of earthquake ground motions has been inferred experimentally using data from closely spaced seismograph arrays. The SMART-1 array, for example, located at Lotung in the NE corner of Taiwan has provided valuable data for the analysis of ground motions in time and space domain. Numerous studies using the SMART-1 array data have been reported (Loh et al.1982, Harada 1984, Harada and Shinozuka 1986, Harichandran 1988, Abrahamson 1985, 1991). It is common in these studies that the accelerograms from each seismic event are described as samples from space-time stochastic processes or stochastic fields and eventually the spatial coherence functions or the frequency wavenumber spectra are estimated.

In this paper, the seismic ground motions varying in time and space domain are described by the stochastic waves filtered by a single surface soil layer, with random thickness caused by the irregular shape of free-surface, resting on rigid bedrock. The seismic wave, being assumed by a single plane wave, is transmitted to the soil layer from the rigid bedrock. The analytic expressions of the frequency wavenumber spectra of the stochastic waves at ground surface are then derived.

2 BASIC EQUATIONS OF EARTHQUAKE RESPONSE OF GROUND WITH IRREGULAR INTERFACE

For the wave propagation problem possessing cylindrical symmetry, the displacement wave field (u, v, w) in a Cartesian coordinate system (x, y, z) can be constructed from the solutions (u', v', w') of the two-dimensional problem associated with the P-SV wave propagation and the SH wave propagation in a new coordinate system (x', y', z) in frequency wave number domain such that (Buchon 1971),

$$u(\kappa_x,\kappa_y,z,\omega)=\frac{\kappa_x}{\kappa}u'(\kappa,z,\omega)-\frac{\kappa_y}{\kappa}v'(\kappa,z,\omega) \quad (1-a)$$

$$v(\kappa_x,\kappa_y,z,\omega)=\frac{\kappa_y}{\kappa}u'(\kappa,z,\omega)+\frac{\kappa_x}{\kappa}v'(\kappa,z,\omega) \quad (1-b)$$

$$w(\kappa_x,\kappa_y,z,\omega)=w'(\kappa,z,\omega) \quad (1-c)$$

where the Fourier transform of u with respect to x, t is defined such that,

$$u(\kappa,z,\omega)=\frac{1}{(2\pi)^3}\int\int\int u(\boldsymbol{x},z,t)e^{-i(\boldsymbol{\kappa}\cdot\boldsymbol{x}-\omega t)}d\boldsymbol{x}dt \quad (2-a)$$

with the inverse transform,

$$u(\boldsymbol{x},t)=\int\int\int u(\kappa,z,\omega)e^{-i(\boldsymbol{\kappa}\cdot\boldsymbol{x}-\omega t)}d\boldsymbol{\kappa}d\omega \quad (2-b)$$

and similarly for v, w, u', and v'. In Eq.(2),x is the spatial coordinate vector with components x and y, and κ is the wave number vector with components κ_x and κ_y, and ω the frequency. The wavenumber κ denotes:

$$\kappa=\sqrt{\kappa_x^2+\kappa_y^2}=\kappa_{x'} \quad (3)$$

and the angle θ between the two coordinate systems (x,y,z) and (x',y',z) is given by:

$$\cos\theta=\frac{\kappa_x}{\kappa}\ ;\ \sin\theta=\frac{\kappa_y}{\kappa} \quad (4)$$

It is noted here that the special symbols are avoided for simplicity of notations to denote that u (similarly for the other displacements) has been transformed.

The displacement wave field (u',v',w') corresponding to P-SV and SH wave propagations in a two-dimensional homogeneous isotropic medium is known to be expressed as matrix wave equation in the form (Kennet 1972):

$$\frac{\partial}{\partial z}\mathbf{B}(x',z)=\mathbf{A}_0(z)\mathbf{B}(x',z) \quad (5)$$

where $\mathbf{B}$ is the displacement-stress vector and $\mathbf{A}_0$ is the operator matrix defined such as

$$\mathbf{B}_{SH}=col\,[v',\ \tau_{y'z}],\quad \mathbf{B}_{PSV}=col\,[u',\ w,\ \tau_{x'z},\ \tau_{zz}] \quad (6)$$

$$\mathbf{A}_{0_{SH}}=\begin{pmatrix}0 & \frac{1}{\mu^*}\\ -\mu^*\partial_{x'x'}-\rho\omega^2 & 0\end{pmatrix} \quad (7-a)$$

$$\mathbf{A}_{0_{PSV}}=\begin{pmatrix}0 & -\partial_{x'} & \frac{1}{\mu^*} & 0\\ -b\partial_{x'} & 0 & 0 & b/\lambda^*\\ -a\partial_{x'x'}-\rho\omega^2 & 0 & 0 & -b\partial_{x'}\\ 0 & -\rho\omega^2 & -\partial_{x'} & 0\end{pmatrix} \quad (7-b)$$

with

$$a=\frac{4\mu^*(\lambda^*+\mu^*)}{\lambda^*+2\mu^*},\quad b=\frac{\lambda^*}{\lambda^*+2\mu^*} \quad (7-c)$$

$$\partial_{x'}=\partial/\partial x',\quad \partial_{x'x'}=\partial/\partial x'x' \quad (7-d)$$

where μ^* and λ^* are the complex Lame constants , and ρ the density of medium. It should be noted here that the Fourier transform has been done with respect to time and the frequency ω (time dependence is $exp(-i\omega t)$) is dropped out in Eq.(5) and will disappear in the following notations for brevity.

Welded boundary conditions at the irregular interface (see Fig.1) require continuity of displacement and traction at each point on the interface. Therefore, a new displacement-stress vector b, being measured with respect to the local tangent plane at each point on the interface, has to be introduced. The new displacement-stress vector takes the form (Kennet 1972):

$$\mathbf{b}(x',f)=(\mathbf{I}+\frac{\partial f}{\partial x'}\mathbf{Q}_0)\mathbf{B}(x',f) \quad (8)$$

where b and B are evaluated along the irregular interface located at the depth $z(x')$ defined by:

$$z(x')=z+f(x') \quad (9)$$

with z being the average depth of the interface and $f(x')$ being the lateral fluctuation of the interface (see Fig.1). In Eq.(8), I is the unit matrix and $\mathbf{Q}_0$ is given by:

$$\mathbf{Q}_{0_{SH}}=\begin{pmatrix}0 & 0\\ -\mu^*\partial_{x'} & 0\end{pmatrix} \quad (10-a)$$

$$\mathbf{Q}_{0_{PSV}}=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -a\partial_{x'} & 0 & 0 & -b\\ 0 & 0 & -1 & 0\end{pmatrix} \quad (10-b)$$

The irregular interface boundary condition can be expressed in the form:

$$\mathbf{b}_1(x',f)=\mathbf{b}_2(x',f) \quad (11)$$

where subscripts indicate the respective media.

In order to obtain the approximation of Eq.(8), the scattered wave field $\mathbf{B}(x',f)$ (displacement-stress vector in the medium with irregular interface) is approximately related to the background field $\mathbf{B}(x',z)$ (displacement-stress vector in the medium with horizontal plane interface) by using a Taylor expansion around the average interface depth $z(x')=z$, and then substituting into Eq.(8), and omitting terms of order higher than f and $\partial f/\partial x'$, one obtains:

$$\mathbf{b}(x',f)=\{\mathbf{I}+f\mathbf{A}_0+\frac{\partial f}{\partial x'}\mathbf{Q}_0\}\mathbf{B}(x',z) \quad (12)$$

Furthermore, introducing the new notations, $\mathbf{b}^0$ and $\mathbf{B}^0$, which represent the background wave fields, and denoting the first-order approximations of $\mathbf{b}$ and $\mathbf{B}$ by $\mathbf{b}^I$ and $\mathbf{B}^I$, respectively, one obtains the first-order approximations of Eq.(12) as:

$$\mathbf{b}^I(x', f) = \mathbf{B}^I(x', z) + \{f\mathbf{A}_0 + \frac{\partial f}{\partial x'}\mathbf{Q}_0\}\mathbf{B}^0(x', z) \quad (13)$$

with the following condition because $\mathbf{b}$ equals to $\mathbf{B}$ at the interface $z(x') = z$ in the case of horizontal plane interface ($f = 0$):

$$\mathbf{b}^0(x', z) = \mathbf{B}^0(x', z) \quad (14)$$

The Fourier transform of Eq.(13) with respect to x', using the result that the Fourier transform of a product is the convolution of the Fourier transform, yields:

$$\mathbf{b}^I(\kappa, f) = \mathbf{B}^I(\kappa, z) + \int f(\kappa - \kappa')\mathbf{J}(\kappa, \kappa')\mathbf{B}^0(\kappa', z)d\kappa' \quad (15)$$

where

$$\mathbf{J}(\kappa, \kappa') = \mathbf{A}_0(\kappa') + i(\kappa - \kappa')\mathbf{Q}_0(\kappa') \quad (16)$$

with

$$\mathbf{J}_{SH} = \begin{pmatrix} 0 & \frac{1}{\rho Cs^2} \\ \rho Cs^2\kappa\kappa' - \rho\omega^2 & 0 \end{pmatrix} \quad (17-a)$$

$$\mathbf{J}_{PSV} = \begin{pmatrix} 0 & -i\kappa' & \frac{1}{\rho Cs^2} & 0 \\ \{2(\frac{Cs}{Cp})^2 - 1\}i\kappa' & 0 & 0 & \frac{1}{\rho Cp^2} \\ 4\rho Cs^2\{1 - (\frac{Cs}{Cp})^2\}\kappa\kappa' - \rho\omega^2 & 0 & 0 & \{2(\frac{Cs}{Cp})^2 - 1\}i\kappa \\ 0 & -\rho\omega^2 & -i\kappa & 0 \end{pmatrix} \quad (17-b)$$

In Eq.(17), C_s and C_p are the complex P-wave and S-wave velocities, respectively, given by:

$$C_s = C_s^0(1 - iD_s), \quad C_p = C_p^0(1 - iD_p) \quad (18)$$

with C_s^0 and C_p^0 being the elastic P-wave and S-wave velocities respectively and D_s, D_p being the ratios of the linear hysteretic damping for P- and S-waves.

The irregular interface boundary condition given by Eq.(11) can be written for the first-order approximations in frequency-wavenumber domain as:

$$\mathbf{B}_1^I(\kappa, z) = \mathbf{B}_2^I(\kappa, z) + \mathbf{S}(\kappa, z) \quad (19-a)$$

where

$$\mathbf{S}(\kappa, z) = \int_{-\infty}^{\infty} f(\kappa - \kappa')\mathbf{L}_{21}(\kappa, \kappa')\mathbf{B}_2^0(\kappa', z)d\kappa' \quad (19-b)$$

$$\mathbf{L}_{21} = \mathbf{J}_2 - \mathbf{J}_1 = -\mathbf{L}_{12} \quad (19-c)$$

Equation (19-a) indicates the presence of irregular interface results in discontinuity in the scattered wave field $\mathbf{B}(x', z)$ at $z(x') = z$. This discontinuity acts like a seismic source $\mathbf{S}$ which can be evaluated directly from the background wave field.

3 RESPONSE OF A SINGLE SOIL LAYER WITH IRREGULAR FREE SURFACE RESTING ON RIGID BEDROCK

Response of a single soil layer with irregular free surface resting on rigid bedrock as shown in Fig.2 is considered. For a free surface the traction has to vanish, so that for an irregular free surface $z(x') = f(x')$, ($z = 0$), the scattered wave field takes the form:

$$\mathbf{b}(\kappa, f) = \mathbf{B}(\kappa, f) = col\ [\mathbf{U}(\kappa, f), \mathbf{0}] \quad (20)$$

Then, the first-order approximation of the interface boundary condition given by Eq.(15) can be expressed as:

$$\mathbf{B}^I(\kappa, f) = \mathbf{B}^I(\kappa, 0) + \int_{-\infty}^{\infty} f(\kappa - \kappa')\mathbf{J}(\kappa, \kappa')\mathbf{B}^0(\kappa', 0)d\kappa' \quad (21)$$

Making use of the propagator matrix $\mathbf{P}(\kappa, z, z_0)$ which satisfies (Kennet 1972),

$$\frac{\partial}{\partial z}\mathbf{P}(\kappa, z, z_0) = \mathbf{A}_0(\kappa, z)\mathbf{P}(\kappa, z, z_0) \quad (22-a)$$

$$\mathbf{P}^{-1}(\kappa, z, z_0) = \mathbf{P}(\kappa, z_0, z) \quad (22-b)$$

the displacement-stress vector at the bedrock $z(x') = H$ can be transformed to that at the free-surface $z(x') = 0$ such as:

$$\mathbf{B}(\kappa, 0) = \mathbf{P}(\kappa, 0, H)\mathbf{B}(\kappa, H) \quad (23)$$

It is assumed now that an input seismic motion is specified at the bedrock and propagates in the direction of x' axis with apparent wave speed c. Then, the input motion (represented by displacement-stress vector) at the bedrock is expressed in the form:

$$\mathbf{B}(\kappa', H) = \mathbf{B}(\kappa_0, H)\ \delta(\kappa' - \kappa_0) \quad (24-a)$$

where δ is the Delta function, and κ_0 is given by:

$$\kappa_0 = \frac{\omega}{c}. \quad (24-b)$$

Taking into account for Eqs.(23) and (24) in (21), one obtains:

$$\begin{aligned} \mathbf{B}^I(\kappa, f) &= \mathbf{P}(\kappa, 0, H)\mathbf{B}^I(\kappa, H)\delta(\kappa - \kappa_0) \\ &+ f(\kappa - \kappa_0)\mathbf{J}(\kappa, \kappa_0)\mathbf{P}(\kappa_0, 0, H)\mathbf{B}^0(\kappa_0, H) \end{aligned} \quad (25)$$

By considering the boundary conditions that the traction vanishes at the free-surface and the input motion is specified at the bedrock, Eq.(25) can be more explicitly expressed in the partitioned form:

$$\begin{pmatrix} \mathbf{U}^I(\kappa, f) \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{U}^0(\kappa, H) \\ \tau^I(\kappa, H) \end{pmatrix} \delta(\kappa - \kappa_0)$$

$$+ f(\kappa - \kappa_0) \begin{pmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{P}^0_{11} & \mathbf{P}^0_{12} \\ \mathbf{P}^0_{21} & \mathbf{P}^0_{22} \end{pmatrix} \begin{pmatrix} \mathbf{U}^0(\kappa_0, H) \\ \tau^0(\kappa_0, H) \end{pmatrix} \tag{26}$$

where $\mathbf{P}^0_{ij} = \mathbf{P}_{ij}(\kappa_0, 0, H)$. In Eq.(26), the relation, $\mathbf{U}^I(\kappa, H) = \mathbf{U}^0(\kappa, H)$, is used because the input motion displacement is specified at the bedrock. From Eq.(26), one can obtain the scattered wave field $\mathbf{U}^I(\kappa, f)$ and $\tau^I(\kappa, H)$.

4 EXAMPLES OF FREQUENCY WAVE NUMBER SPECTRA OF STOCHASTIC WAVES AT THE IRREGULAR FREE SURFACE

Closed form expressions are presented for the horizontal displacements of stochastic waves at the free-surface of single soil layer with irregular free-surface subjected to the horizontal seismic ground motions at the bedrock. The vertical components in both the input motions and the response of soil layer are not considered, primarily because the horizontal components are much more interested in earthquake engineering problems and also because the closed form expressions corresponding to P-SV wave propagation becomes relatively simple.

From the form given by Eq.(26), the horizontal displacement components (u', v') have the form:

$$u' = \{A\delta(\kappa - \kappa_0) + Bf(\kappa - \kappa_0)\}u'_g \qquad (27-a)$$

$$v' = \{C\delta(\kappa - \kappa_0) + Df(\kappa - \kappa_0)\}v'_g \qquad (27-b)$$

where $u' = u^I(\kappa, f), v' = v^I(\kappa, f)$, $u'_g = u^0(\kappa_0, H)$, $v'_g = v^0(\kappa_0, H)$. By making use of Eq.(1), the horizontal displacement components (u, v) in a Cartesian coordinate system (x, y, z) can be obtained in the form:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} H_{uu} & H_{uv} \\ H_{vu} & H_{vv} \end{bmatrix} \begin{Bmatrix} u_g \\ v_g \end{Bmatrix} \tag{28}$$

where

$$H_{uu} = [A(\frac{\kappa_x}{\kappa})^2 + C(\frac{\kappa_y}{\kappa})^2]\delta(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0)$$

$$+[B(\frac{\kappa_x}{\kappa})^2 + D(\frac{\kappa_y}{\kappa})^2]f(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0) \qquad (29-a)$$

$$H_{uv} = H_{vu} = (A - C)\frac{\kappa_x\kappa_y}{\kappa^2}\delta(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0)$$

$$+ (B - D)\frac{\kappa_x\kappa_y}{\kappa^2}f(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0) \qquad (29-b)$$

$$H_{vv} = [A(\frac{\kappa_y}{\kappa})^2 + C(\frac{\kappa_x}{\kappa})^2]\delta(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0)$$

$$+ [B(\frac{\kappa_y}{\kappa})^2 + D(\frac{\kappa_x}{\kappa})^2]f(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0) \qquad (29-c)$$

$$\kappa_x^0 = \frac{\omega\cos\theta}{c} \quad , \quad \kappa_y^0 = \frac{\omega\sin\theta}{c} \qquad (29-d)$$

By assuming that the input seismic motion is the stationary stochastic process and the fluctuation of irregular free-surface is the isotropic stochastic field, eventually the response displacement wave fields may be the stationary and homogeneous stochastic wave fields. Then, its frequency wavenumber spectra are obtained as:

$$S_{uu}(\kappa_x, \kappa_y, \omega) = |u(\kappa_x, \kappa_y, \omega)|^2 \qquad (30-a)$$

$$S_{vv}(\kappa_x, \kappa_y, \omega) = |v(\kappa_x, \kappa_y, \omega)|^2 \qquad (30-b)$$

If $v_g = 0$ in Eq.(30), then, Eqs.(28) to (30) yield:

$$S_{uu}(\kappa_x, \kappa_y, \omega) = [|A(\frac{\kappa_x}{\kappa})^2 + C(\frac{\kappa_y}{\kappa})^2|^2\delta(\kappa_x - \kappa_x^0, \kappa_y - \kappa_y^0)$$

$$+|B(\frac{\kappa_x}{\kappa})^2 + D(\frac{\kappa_y}{\kappa})^2|^2 S_{ff}(\kappa_x, \kappa_y)]S_{u_g u_g}(\omega) \tag{31}$$

where S_{ff} is the wavenumber spectrum of the fluctuation of irregular free-surface $f(x')$, being the isotropic stochastic field. The assumption of isotropic characteristics for $f(x')$ may be necessary to maintain cylindrical symmetry of the problem considered in this paper. The quantities A, B, C, and D appearing in Eqs.(27) to (31) are given by:

$$A = \frac{(\gamma^2 + \kappa^2)\nu\gamma}{R}[(\gamma^2 - \kappa^2)\cos\nu H + 2\kappa^2\cos\gamma H] \qquad (32-a)$$

$$B = A_0\frac{[4\kappa\kappa_0(\gamma^2 - \nu^2) - (\gamma^2 + \kappa^2)^2]\gamma}{R}\times$$

$$[\nu\gamma\cos\nu H\sin\gamma H + \kappa^2\cos\gamma_0 H\sin\nu_0 H] \qquad (32-b)$$

$$C = \frac{1}{\cos\gamma H} \qquad (32-c)$$

$$D = \frac{\gamma_0\sin\gamma H}{\cos\gamma H\cos\gamma_0 H}(\frac{\kappa\kappa_0 - \kappa_0^2}{\gamma\gamma_0} - \frac{\gamma_0}{\gamma}) \qquad (32-d)$$

$$R = 4\kappa^2\nu\gamma(\gamma^2 - \kappa^2) + \nu\gamma[4\kappa^4 + (\gamma^2 - \kappa^2)^2]\cos\nu H\cos\gamma H$$

$$-\kappa^2[4\nu^2\gamma^2 + (\gamma^2 - \kappa^2)^2]\sin\nu H\sin\gamma H \qquad (32-e)$$

where the vertical wavenumbers ν and γ satisfy:

$$\nu = \sqrt{(\frac{\omega}{Cp})^2 - \kappa^2} \;;\; Im\nu \geq 0 \qquad (33-a)$$

$$\gamma = \sqrt{(\frac{\omega}{Cs})^2 - \kappa^2} \;;\; Im\gamma \geq 0 \qquad (33-b)$$

and A_0, ν_0, and γ_0 correspond to the values of A, ν, γ when $\kappa = \kappa_0 = \omega/c$ (see Eqs.(3),(4), and (24-b)). If the vertical displacement w of the response is constrained to be zero for the solutions of P-SV wave propagation, the quantities A and B take the form:

$$A = \frac{1}{\cos \frac{C_p}{C_s} \nu H} \qquad (34-a)$$

$$B = \frac{\frac{C_p}{C_s}\nu_0 \sin \frac{C_p}{C_s}\nu H}{\cos \frac{C_p}{C_s}\nu H \cos \frac{C_p}{C_s}\nu_0 H}\left(\frac{\kappa\kappa_0 - \kappa_0^2}{\nu\nu_0} - \frac{\nu_0}{\nu}\right) \quad (34-b)$$

5 NUMERICAL EXAMPLES

Numerical examples are given to visualize the shape of the frequency wave number spectrum given in Eq.(31). In order to visualize the filtering effect of the soil layer with irregular free-surface, the power spectrum of input seismic motion displacement is assumed unity, and the direction and apparent wave speed of input motion is assumed as:

$$S_{u_g u_g}(\omega) = 1 \;;\; \theta = 45° \;;\; c = 523m/s \qquad (35)$$

The following values are used for the various constants necessary to evaluate Eq.(31):

$$Elastic\ P-wave\ velocity : C_p^0 = 573m/s \quad (36-a)$$

$$Elastic\ S-wave\ velocity : C_s^0 = 191m/s \quad (36-b)$$

$$Material\ damping\ ratio : D_p = D_s = 0.3 \quad (36-c)$$

$$Average\ thickness\ of\ layer : H = 100m \quad (36-d)$$

The following analytic expression is assumed for the wavenumber spectrum of $f(x')$:

$$S_{ff}(\kappa_x, \kappa_y) = \begin{cases} \frac{2\sigma_{ff}^2}{\pi\kappa_u^2}\cos^2(\frac{m\pi}{\kappa_u}\kappa) & 0 \le \kappa \le \kappa_u \\ 0 & otherwise \end{cases} \qquad (37)$$

where the parameters are assumed as $m = 4, \kappa_u = 0.01147\ rad/m, \sigma_{ff} = 10$.

According to the above values, the first natural frequency ω_1 of the soil layer with horizontal plane free-surface, obtained by the vertically propagating S-wave, is estimated as $\omega_1 = 3rad/s(2\pi C_s^0/4H = 2\pi \cdot 191/400)$. Eq.(37)is plotted in Fig.3. Figure 4 shows $S_{uu}(\kappa_x, \kappa_y, \omega)$ as a function of wavenumbers κ_x and κ_y for the same region of Fig.3 ($\kappa_{xmax} = \kappa_{ymax} = 0.0081rad/m$) at 6 different values of frequency . It is observed from Fig.4 that the relatively sharp mountain appears up to approximately 3 rad/s while widely spreading mountains and troughs over the wavenumber plane tend to appear in high frequency range. Such pattern as shown in Fig.4 is quite similar to the pattern estimated by Abrahamson (1985) using the SMART-1 array recordings for event 5.

In order to demonstrate an application of the frequency wave number spectrum, a sample stochastic wave form is simulated on the basis of the spectral representation method where the simulated wave consists of the superposition of a number of plane waves having amplitudes consistent with the frequency wave number spectrum (Shinozuka, et al. 1987). Figure 5 shows a sample wave form $u(x, y, t)$ of the irregular free surface area (6.6 km by 6.6 km) at two time instants (t=2,4 sec.). In the simulation, the cut off frequency and wavenumber are used as $\omega_u = 6$ rad/sec, κ_u =0.01147 rad/m, and the disrete numbers of frequency wave number spectrum are used as $N_x = N_y = N_\omega$ =18 (see Shinozuka, et al. 1987). It is seen from Fig.5 that the wave forms possess the characteristics of both the coherent (represented by plane wave) and incoherent (represented by scattering waves) components of wave motion.

6 CONCLUSIONS

In this paper, a closed form analytic expression of the frequency wave number spectrum is established. The corresponding seismic ground motion is produced by the seismic response of a single soil layer, with random thickness, resting on rigid bedrock. The seismic wave, assumed by a single plane wave, is transmitted to the soil layer from the rigid bedrock. The stochastic waves from the model possess the characteristics of both the coherent and incoherent components of ground motions. Although the present paper considers the inhomogeneity of local soil layer caused by the lateral variation of the thickness of soil layer only, the effect of the inhomogeneity caused by the spatial variation of soil properties on the ground motions is also studied (Harada,T. et al. 1990).

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Fig.1 An irregular interface between two homogeneous media

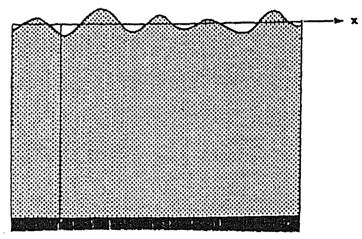


Fig.2 Single soil layer with irregular free-surface resting on rigid bedrock

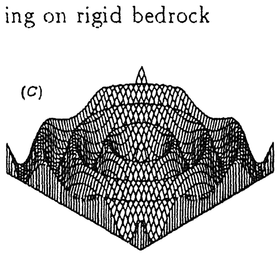
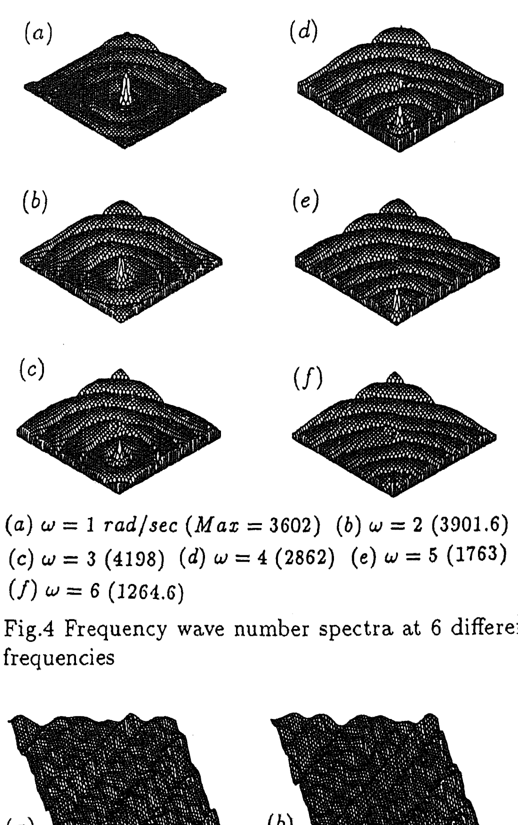


Fig.3 Wave number spectrum of irregularity of free-surface used in numerical examples



$(a)\ \omega = 1\ rad/sec\ (Max = 3602)$ $(b)\ \omega = 2\ (3901.6)$
$(c)\ \omega = 3\ (4198)$ $(d)\ \omega = 4\ (2862)$ $(e)\ \omega = 5\ (1763)$
$(f)\ \omega = 6\ (1264.6)$

Fig.4 Frequency wave number spectra at 6 different frequencies

$(a)\ t = 0\ sec\ (Max = 7.1803)$ $(b)\ t = 2\ (7.1314)$

Fig.5 Simulated stochastic wave at 2 time instants