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Band limited duration and spectral energy: Empirical dependence on frequency, magnitude, hypocentral distance and site conditions

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ABSTRACT : A new definition of the strong motion duration which takes into account the dependence on frequency is proposed. The variations of duration and spectral energy with magnitude, hypocentral distance and site conditions are quantified for ten frequency bands, through the computation of empirical relations based on the Italian accelerometric data set.

1- INTRODUCTION

Duration is recognized as an important characteristic of strong motion which influences significantly the degree of damage to both structure and soils. Systematic works on this parameter are relatively recent and have first shown its major role in soil liquefaction. Records from the Parkfield earthquake in 1966 pointed out that high acceleration levels may produce little damage to structures if the duration of the strong motion is short (Housner, 1975). On the contrary, the exceptionally long duration shaking in the Mexico city basin which was associated with relatively low acceleration level (0.2 g), was one of the major cause of building collapse during the Michoacan earthquake in 1985. The objective of this study is to quantify the variations of duration with some relevant parameters and to provide a basis for a simple method of simulation of non stationnary synthetic accelerograms. We thus propose an improved definition of the duration which takes into account its frequency dependence. We consider ten frequency bands (0.6 to 19.2 Hz) and compute the duration and the spectral energy in each of them from Italian accelerometric records. We then use the weighted multilinear regression technique to derive empirical relations between both duration and spectral energy on one side, and magnitude, hypocentral distance and site conditions on the other.

2- FREQUENCY DEPENDENT DURATION AND SPECTRAL ENERGY

2.1 Definitions

The effective duration for short period oscillations is obviously less than for long period oscillations. We therefore propose a frequency dependent duration based on the duration definition given by Trifunac and Brady (1975) and used by Dobry et al. (1978) to study the duration of records from western US : for a given recorded acceleration a(t), the mean square integral of motion is :

$$A(t)=\int_0^t a^2(\tau)\, d\tau$$

and tends asymptotically toward a maximum Amax. The duration is defined as $D = t_{95\%} - t_{5\%}$ where $t_{95\%}$ and $t_{5\%}$ are the times where A(t) reachs respectively 95% and 5% of Amax.

We define the strong motion duration as a function of frequency as follow : the original accelerogram a(t) is filtered using a 40 dB/oct Butterworth filter in ten frequency bands (f_i, f_{i+1}) with :

$$f_0 = 0.6 \text{ Hz} \quad \text{and} \quad f_{i+1} = \sqrt{2}\, f_i$$

We choose logarithmic intervals in order to better dissociate low frequency from high frequencies since low frequencies are more affected by site effects. The duration D_i is then calculated for each filtered signal $a_i(t)$, using the Trifunac and Brady procedure.

We define the spectral energy SE_i in the same frequency bands :

$$SE_i = \int_{f_i}^{f_{i+1}} |A(f)|^2 \frac{df}{f}$$

where A(f) is the Fourier transform of a(t).

2.2 Data

In this study, we use accelerometric records from the

Italian accelerometric network. It consists of 260 SMA-1 accelerometers installed by the ENEA-ENEL throughout the whole country. 236 records corresponding to 11 earthquakes and their aftershocks were recorded between 1972 and 1984 and are available. 115 of them which can be related with specific geotechnical conditions were choosen with magnitude between 3.2 and 6.8, epicentral distance less then 60 km and depth less than 30 km. Geotechnical conditions are classified in 3 site categories (Sabetta and Pugliese (1987) :

- Site 0 : rock, 49 records
- Site 1 : shallow soil deposits with thickness between 5 and 20 m, 34 records
- Site 2 : shallow soil deposits with thickness greater than 20 m, 33 records

For each of these records, the frequency dependent duration and the spectral energy are calculated for the NS and WE components ; both values are then averaged.

3- EMPIRICAL RELATIONS FOR DURATION AND SPECTRAL ENERGY.

The objective of this analysis is to derive empirical attenuation laws by multilinear regression between duration and spectral energy on one side and magnitude, hypocentral distance and site condition on the other. The choosen form of the multiple regression equation is, for each frequency band i :

$$\ln(y_i)=\beta_{1,i}+\beta_{2,i}M+\beta_{3,i}\ln(HYPO)+\beta_{4,i}S_1+\beta_{5,i}S_2+\varepsilon \quad \textbf{(1)}$$

where y_i, M, HYPO, S_1, S_2, ε are vectors of dimension n and :

- y_i is duration or spectral energy in the frequency band i
- M is the magnitude
- HYPO is the hypocentral distance
- $S_1=0$ and $S_2=0$ for site 0
  $S_1=1$ and $S_2=0$ for site 1
  $S_1=0$ and $S_2=1$ for site 2
- $\beta_{k,i}$ k=1,5 are the regression coefficients
- ε_i is the error term ; the ε_j, j=1,n should have a mean of zero, and should be normally distributed.

Their standard deviation σ_i is called the standard error of regression.

3.1 Method

Figure 1 shows the distribution of the data with respect to magnitude M and hypocentral distance HYPO for each site category. In order to obtain reliable results, the data have to be distributed evenly among M and HYPO for each site. We therefore affect a weight w_j to each data using M and HYPO intervals which are drawn on figure 1. Their values are in the range 0.41 to 4.6 (Caillot, 1992).

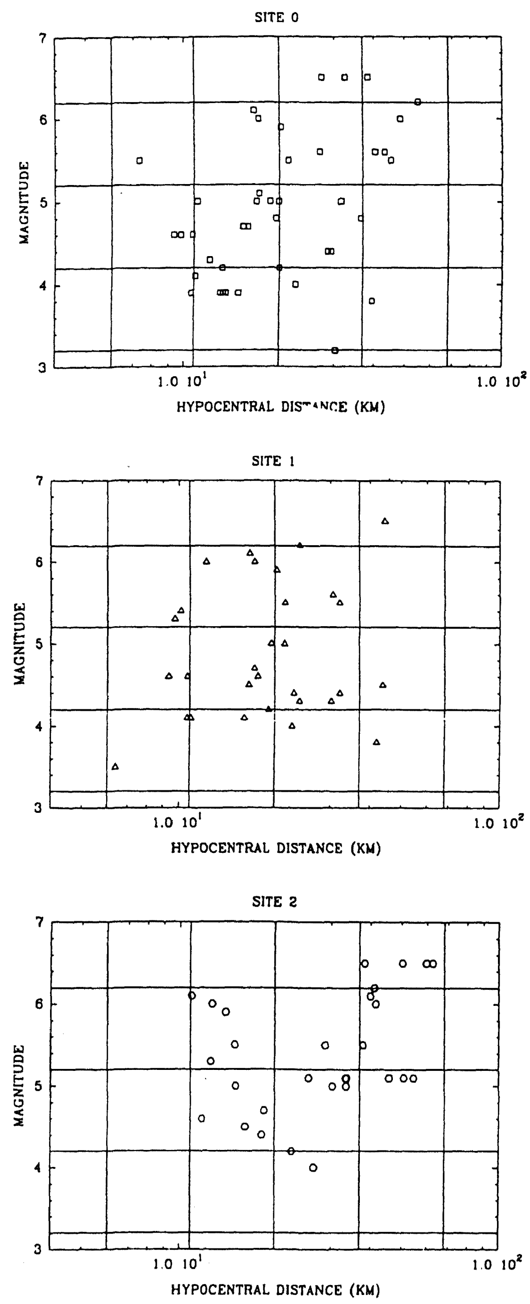


Figure 1 : Distribution of the data in magnitude and hypocentral distance for each site category ; intervals used to calculate the weights.

The coefficients $\beta_{k,i}$ are then calculated in minimizing :

$$\chi^2 = \sum_{j=1}^{n} w_j \varepsilon_j^2 \qquad \sum_{j=1}^{n} w_j = n$$

The goodness of the fit is represented by the multiple correlation coefficient Rc. It is necessary to check if the residuals ε_i have a mean of zero and are normally distributed. We evaluate the adequacy of the model by an analysis of the residuals ; it consists in plotting the residuals versus the independent variable (M, HYPO, S_1, S_2). If these plots do not show apparent trend, the model can be considered adequate.

3.2 Duration

Using the previously defined weighted multilinear regression, we compute the $\beta_{k,i}$ for the duration. The obtained multiple correlation coefficients Rc are in the range 0.54 to 0.75. The stability of this result is then tested in computing $\beta_{1,i}$, $\beta_{2,i}$ and $\beta_{3,i}$ separately for each site category using the relation, in the frequency band i :

$$\ln(D_i) = \beta_{1,i} + \beta_{2,i}\,M + \beta_{3,i}\ln(HYPO) + \varepsilon_i \quad (2)$$

The correlation is correct for Site 0 and Site 2: 0.43<Rc<0.88 and 0.71<Rc<0.85 respectively. For Site 1, we obtained 0.27<Rc<0.56. From a theoretical point of view, the amplification of high frequency motion by thin alluvium deposits may explain the low correlation between the frequency dependent duration for Site 1 data and the magnitude and the hypocentral distance. We therefore eliminate these data from the regression analysis. Consequently, the form of the regression equation becomes :

$$\ln(D_i) = \beta_{1,i} + \beta_{2,i}\,M + \beta_{3,i}\ln(HYPO) + \beta_{4,i}\,S_2 + \varepsilon_i \quad (3)$$

The value of the multiple correlation Rc are in the range 0.57 to 0.82. The regression coefficients $\beta_{k,i}$ and the standard error of regression σ_i are listed in Table 1. Residuals do not show any trend with magnitude, hypocentral distance and site conditions.

3.3 Spectral energy.

A similar analysis is employed for the spectral energy using equation (1). We obtain a mutiple correlation coefficient 0.50<Rc<0.83. The regression coefficient $\beta_{k,i}$ (i=1,5) and the standard errors of regression σ_i are listed in Table 2.

4- RESULTS

We present the results for both duration (Site 0 and 2) and spectral energy (Site 0, 1 and 2) for HYPO=30 km and M=4.0, 5.0, 6.0, 7.0 (Figure 2-a and 3-a) and for M=6.0 and HYPO=10, 20, 30, 40, 50 km (Figure 2-b and 3-b). The values of durations and spectral energies are calculated with a 80% prediction interval (see Weisberg, 1985 for the calculation of the prediction interval).

Table 1 : Regression coefficients $\beta_{k,i}$ (k=1,4) and standard error of regression s in each frequency band for duration.

ΔF	σ	β_1	β_2	β_3	β_4
0.6 - 0.9 Hz	0.449	0.534	0.300	0.041	0.251
0.9 - 1.2 Hz	0.456	0.739	0.283	0.022	0.132
1.2 - 1.7 Hz	0.389	0.699	0.177	0.199	0.196
1.7 - 2.4 Hz	0.418	0.251	0.195	0.287	0.239
2.4 - 3.4 Hz	0.433	-0.399	0.257	0.334	0.250
3.4 - 4.8 Hz	0.487	-0.962	0.323	0.357	0.185
4.8 - 6.8 Hz	0.497	-1.648	0.458	0.329	0.022
6.8 - 9.6 Hz	0.456	-2.129	0.490	0.405	0.066
9.6 - 13.6 Hz	0.441	-2.118	0.458	0.458	0.054
13.6 - 19.2 Hz	0.429	-2.127	0.430	0.508	0.035
19.2 - 27.2 Hz	0.430	-2.048	0.388	0.559	0.065

Table 2 : Regression coefficients $\beta_{k,i}$ (k=1,5) and standard error of regression s in each frequency band for spectral energy.

ΔF	σ	β_1	β_2	β_3	β_4	β_5
0.6 - 0.9 Hz	1.56	-18.16	2.19	0.45	-0.11	0.60
0.9 - 1.2 Hz	1.53	-16.19	2.16	0.55	-0.43	0.64
1.2 - 1.7 Hz	1.49	-13.94	1.96	0.77	-0.12	1.09
1.7 - 2.4 Hz	1.45	-11.18	1.57	0.91	0.64	0.71
2.4 - 3.4 Hz	1.48	-8.92	1.17	0.83	0.80	0.36
3.4 - 4.8 Hz	1.45	-8.8	0.82	0.28	0.59	0.27
4.8 - 6.8 Hz	1.48	-9.29	0.87	0.29	0.32	0.37
6.8 - 9.6 Hz	1.44	-9.62	0.79	0.31	0.26	-0.44
9.6 - 13.6 Hz	1.45	-11.34	0.93	0.56	0.38	-0.46
13.6-19.2 Hz	1.45	-14.72	1.35	0.91	0.36	-0.18

These figures display a number of significant results :

1- Duration is generally decreasing with increasing frequency : for example, it is reduced by 35% from 1 Hz to 10 Hz for a magnitude 6.0 event at 30 km on a rock site.

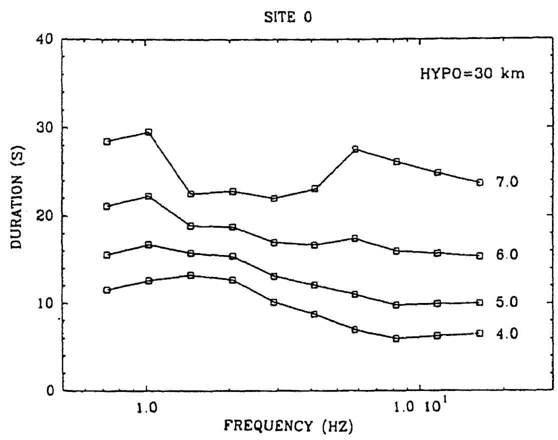
2- Another significant result is that duration is systematically longer for deep alluvium (Site 2) than for rock (Site 0) from about 20% for the range 0.6 Hz to 5 Hz, which is consistent with the results obtained by Dobry et al. (1978) on the total duration (non frequency dependent).

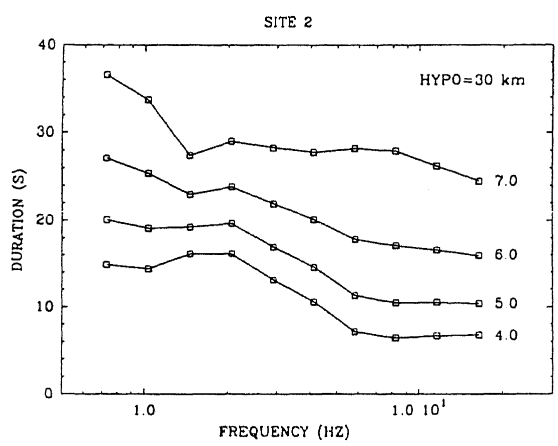
3- Duration is increasing with the increase of magnitude and hypocentral distance, as expected.

4- The spectral energy SE is not affected by site geology, excepted in the low frequency domain (0.6-2 Hz) where SE for Site 2 is higher than for Site 0 and Site 1.

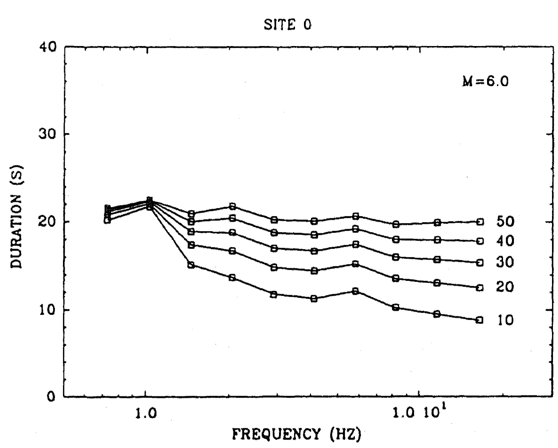
5- SE is shifted to low frequencies for increasing values of magnitude and is only slightly affected by the hypocentral distance (which suggests that anelastic attenuation depends linearly of the frequency).

a)





b)



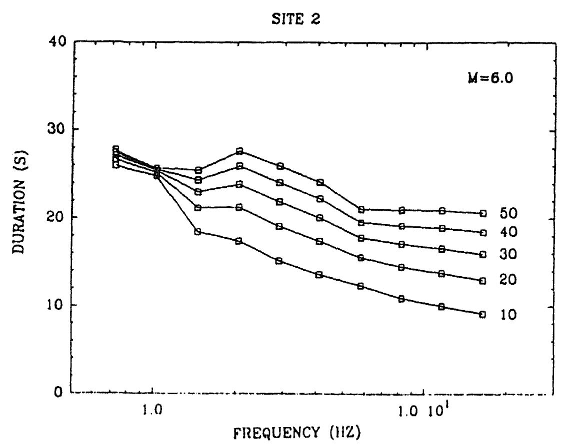
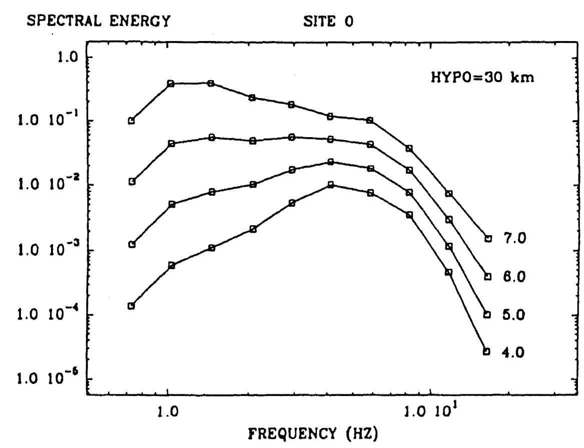
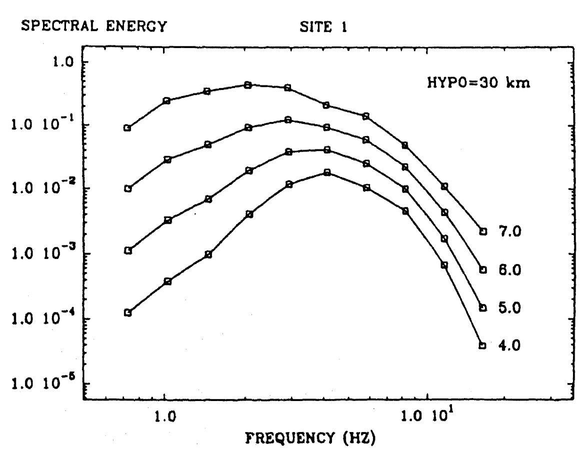
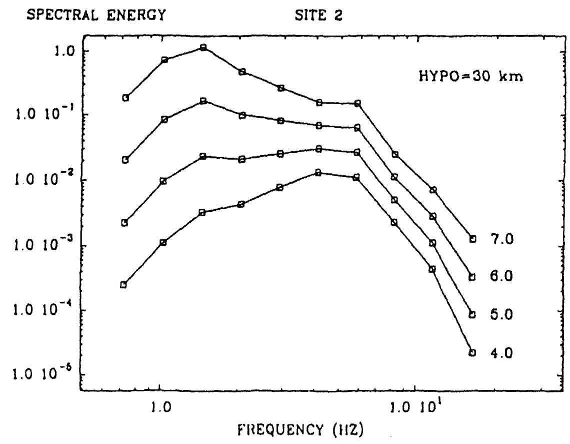


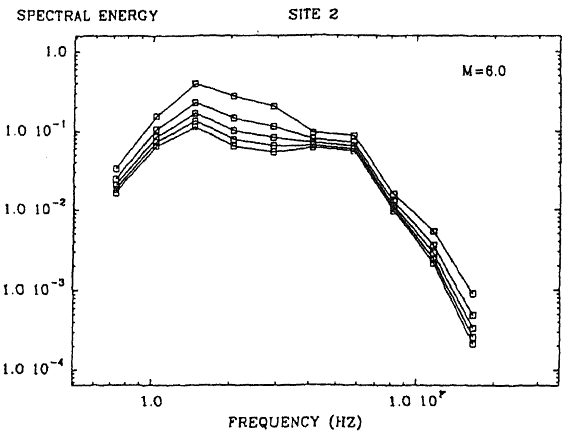
Figure 2 : Durations calculated with a 80% prediction interval and for :
a) HYPO= 30 km and M=4.0, 5.0, 6.0, 7.0
b) M=6.0 and HYPO=10, 20, 30, 40, 50 km

a)



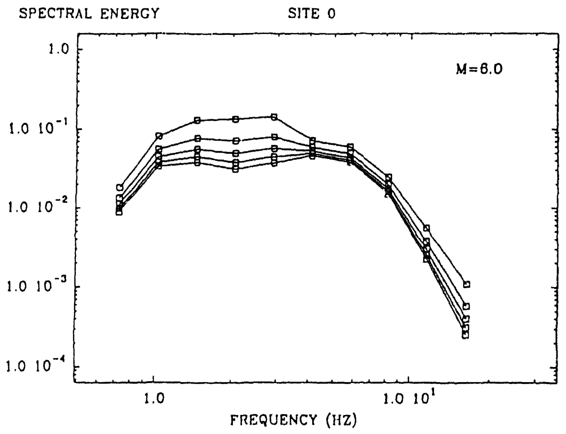


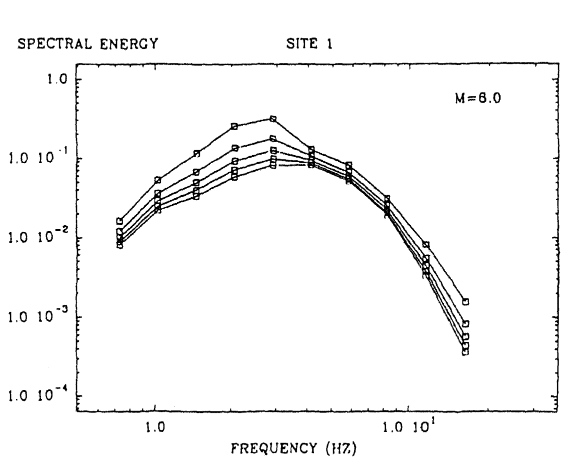




b)

Figure 3 : Spectral energies calculated with a 80% prediction interval and for :
a) HYPO= 30 km and M=4.0, 5.0, 6.0, 7.0
b) M=6.0 and HYPO=10, 20, 30, 40, 50 km





5- CONCLUSION

We proposed a new definition of the strong motion duration, based on the definition of the duration given by Trifunac and Brady (1975). The variations of duration and spectral energy were quantified in ten frequency bands according to magnitude, hypocentral distance and site conditions. The results show mainly a strong dependence of duration with frequency, magnitude, and site conditions. The empirical relationships obtained for duration and spectral energy can directly be used to develop a model of generation of non stationnary accelerograms which may be interesting, especially for studies of nonlinear systems.

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