

## A numerical study on characteristics of Q values in two-dimensional inhomogeneous soil deposits

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**ABSTRACT:** The characteristics of Q values is investigated through the numerical analysis. Q values are separated into two components, frequency-independent component (internal damping) and frequency-dependent component (scattering damping), and modelled. The parameters of the model for Q values are identified with the transfer functions of the inhomogeneous ground calculated by the numerical analysis.

Main conclusions are as follows:

- 1) Q values in soil deposits can be evaluated as the sum of internal damping and scattering damping.
- 2) Frequency-dependent characteristics of the Q values is caused by the scattering in the multi-directional inhomogeneous deposits.
- 3) The value of the internal damping is equal to the hysteresis damping of soil materials.
- 4) Frequency-dependent component of the Q values caused by scattering becomes neglectively small compared to the internal damping during the strong ground motion.

### 1 INTRODUCTION

Both rigidity and damping are important factors to estimate the response of the ground during the earthquake. The latter is frequently represented by the Q values. Amplification characteristics of the ground motion is often estimated by assuming that the ground is horizontally layered deposits. Actually, the ground has the inhomogeneity due to spatial fluctuation of soil properties as well as the irregularity of ground such as a dipping basement and a cliff. Damping of the ground materials obtained in the laboratory tests is frequency-independent (Ishihara 1982). Q values derived from the observed seismic records, however, strongly depend on frequency (Aki 1980, Kinoshita 1983, Kobayashi et al. 1989), which is caused by scattering of the ground motion due to inhomogeneity of soil deposits (Sato and Kawase 1990). We showed based on the observed records during the 1987 Chibaken-toho-oki earthquake that Q values depend on frequency at the beginning of the earthquake but become independent during the strong ground motion (Suetomi et al. 1990). As mentioned, interpretation of Q values in soil deposits has not been understood clearly.

In this paper, we examine the effects of the inhomogeneity on transfer functions by

comparing the transfer functions obtained by 1-D analysis with those obtained by 2-D analysis. Then, the model of frequency-dependent characteristics of Q values is developed in relation to fluctuation of the shear modulus and the damping ratio.

### 2 EFFECTS OF INHOMOGENEITY ON TRANSFER FUNCTIONS

Soil properties are spatially fluctuated in the inhomogeneous ground. The coefficient of variation and the correlation distance in soil properties are examined by Matuo et al. (1977) and Horike et al. (1991). The former varies between 0.1 and 0.4 based on the laboratory tests using the undisturbed soil samples (Matuo et al. 1977) and varies between 0.1 and 0.3 based on logging data (Horike et al. 1991). The latter varies between 0.6 m and 1.4 m (Matuo et al. 1977) and varies between 1.0 m and 3.0 m (Horike et al. 1991). This kind of inhomogeneity is hardly taken into account in the dynamic response analysis. The effect for neglecting the inhomogeneity on the dynamic response is not investigated.

In the followings, the effect of inhomogeneity is investigated using the model shown in Figure 1 by one- and two-dimensional analysis. The multiple reflection theory is used in the 1-D

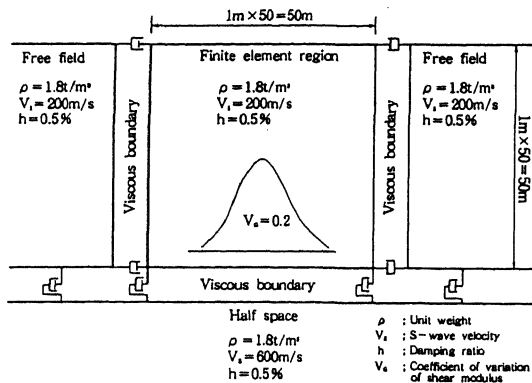


Figure 1. Analyzed model

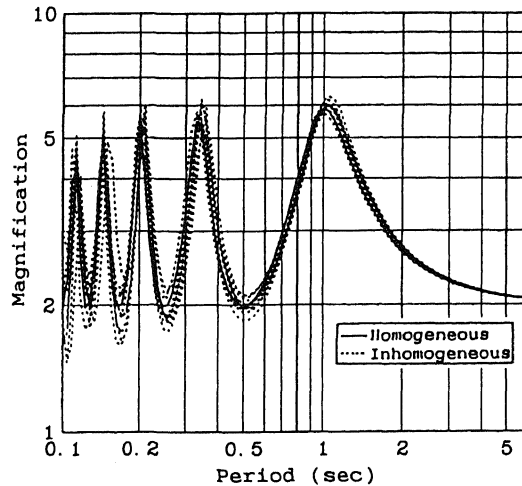
analysis. The finite element method is used in the 2-D analysis. The transfer function of the ground is calculated as the ratio of the Fourier spectrum of the synthetic wave to that of the incident wave.

The surface layer, with 50 m deep, is an inhomogeneous region and the basement is a homogeneous region. The width of finite element region in the 2-D analysis is 50 m. The elements have 1 m width and 1 m depth, hence the region consists of 50x50 elements in the 2-D analysis and the layer consists of 50 layers in the 1-D analysis. The viscous dashpots are adapted along the lateral and bottom boundaries. A pulse wave is used as vertically incident wave. Spatial fluctuation of the shear moduli is assumed to be normal distribution whose mean value is  $7350t/m^2$ . The coefficient of variation,  $V_G$ , and the damping ratio,  $h$ , are taken as analytical parameters, which are shown in Table 1. Ten models with different distribution of shear modulus to each other are analyzed under the constants in case 1 in Table 1.

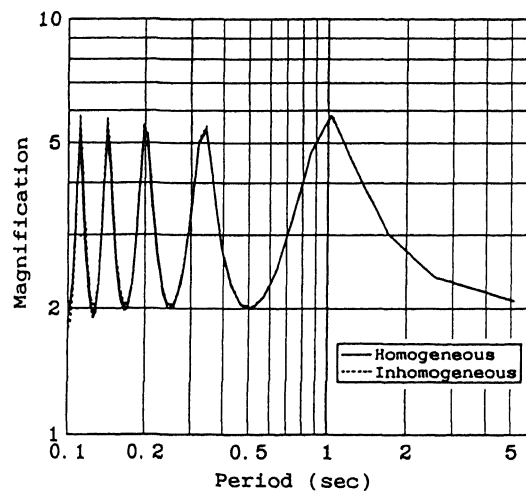
The transfer functions obtained by 1-D analysis and 2-D analysis are shown in Figure 2. Variation of magnifications by 1-D analysis is much larger than that by 2-D analysis. As for 2-D analysis, magnifications at natural periods at lower modes of inhomogeneous grounds of 10 models are almost equal to those of homogeneous ground, but those at the fifth mode are about 1.1 times larger in average. The mean

Table 1. Parameters of ground constants

Case	1	A2	A3	B2	B3
$V_G$	0.2	0.1	0.3	0.2	0.2
$h(\%)$	0.5	0.5	0.5	0.0	2.0



(a) 1-D analysis



(b) 2-D analysis

Figure 2. Comparison between the spectral ratio in homogeneous ground and that in inhomogeneous ground

value and the standard deviation of magnifications at each natural period are shown in Table 2. The variation of magnifications by 2-D analysis is clearly smaller than that by 1-D analysis. The variation of magnifications at natural period for higher mode is larger than that for lower mode.

The wavelength at the first mode is about 200 m and is larger by the factor of 200 to the size of the element, the wavelength at the fifth mode is 22 m and 22 times. It can be inferred that transfer functions are affected by the inhomogeneity when the wavelength is about 20~50 times to the scale length of the inhomogeneity.

Table 2. Mean value and standard deviation of the magnification at each mode

Mode	Mean value		Standard deviation	
	1-D	2-D	1-D	2-D
1	5.99	5.84	0.150	0.004
2	5.71	5.46	0.327	0.024
3	5.40	5.42	0.445	0.087
4	4.91	5.56	0.540	0.148
5	4.56	5.48	0.369	0.238

### 3 MODELLING OF THE CHARACTERISTICS OF Q VALUES

As seen in Figure 2, although natural periods of 10 models are the same, the magnifications differ to each other. It suggests that, when analyzing the inhomogeneous ground by modelling a homogeneous ground, the shear moduli can be set to be equal to average value (Aki and Richards 1980) but Q value is to be estimated considering the inhomogeneity. To ensure this assumption and to evaluate the characteristics of Q values, the shear wave velocity  $V_s$  and Q values are determined by minimizing residual sum of squares between the transfer functions of the equivalent homogeneous layer by S-wave multiple reflection theory and the ratios of the Fourier spectra in the inhomogeneous region by 1-D and 2-D analysis (Tsujihara et al. 1988). The range of period for identification is from 0.1 second to 6.0 second. Other parameters such as the unit weight, the thickness of the surface layer and the shear wave velocity of bedrock are shown in Figure 1.

In the nonlinear analysis, hysteresis damping is automatically considered, which is expressed as

$$Q_1 = a_1 \quad (1)$$

On the other hand, frequency-dependent characteristics of Q values estimated from the earthquake observation is modelled as

$$Q_2 = b_1 \cdot f^{b_2} \quad (2)$$

where  $f$  is frequency. Sato et al. (1990) indicate that Q values are controlled by two primary factors. The one is frequency-independent internal damping which is a unique characteristics of materials. The other is scattering damping which is frequency-dependent and is caused by the scattering in the inhomogeneous deposits. Therefore, we express Q values as the sum of

internal damping and scattering damping:

$$Q_3 = c_1 + c_2 \cdot f^{c_3} \quad (3)$$

The mean value and the standard deviation of identified parameters by 2-D analysis are shown in Table 3. The relationships between Q values and period derived from Equations (1), (2) and (3) are shown in Figure 3. Transfer functions are shown in Figure 4. Q values at the long period at which a surface layer theoretically behaves like a homogeneous ground should be equal to the Q value when there is no scattering,  $Q_h (=1/2h=100)$ . The value of  $Q_1$  is, however, always much larger than  $Q_h$  and  $Q_2$  is much smaller than  $Q_h$  at the period longer than 2 seconds. On the other hand,  $Q_3$  is close to  $Q_h$  at the period longer than 1 second. As seen, Equation (3) is the best model among three models. Therefore, it is concluded that Q values in soil deposits are the sum of internal damping and scattering damping.

The mean value, the standard deviation and the coefficient of variation of identified values of parameters in Equation (3) by 1-D analysis are shown in Table 4. It is shown that the coefficient of variation is larger than that in 2-D analysis shown in Table 3.

Table 3. Mean value, standard deviation and coefficient of variation of identified parameters by 2-D analysis

(a) Equation (1)			
	Mean value	Standard deviation	Coefficient of variation
$V_s$	199.6	0.14	0.00
$a_1$	253.8	96.0	0.38
(b) Equation (2)			
	Mean value	Standard deviation	Coefficient of variation
$V_s$	199.6	0.15	0.00
$b_1$	90.2	8.6	0.10
$b_2$	0.58	0.26	0.45
(c) Equation (3)			
	Mean value	Standard deviation	Coefficient of variation
$V_s$	199.6	0.14	0.00
$c_1$	77.1	7.7	0.10
$c_2$	38.5	3.8	0.10
$c_3$	0.77	0.46	0.60

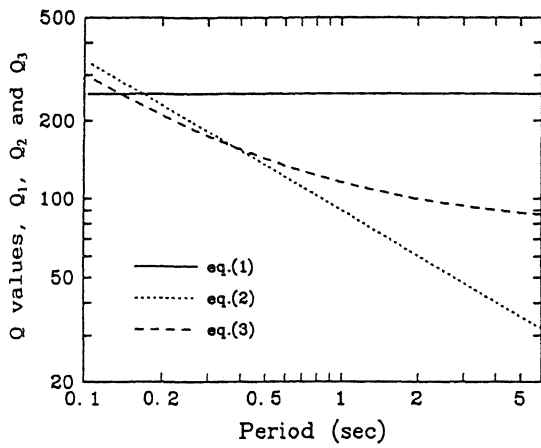


Figure 3. Calculated Q values

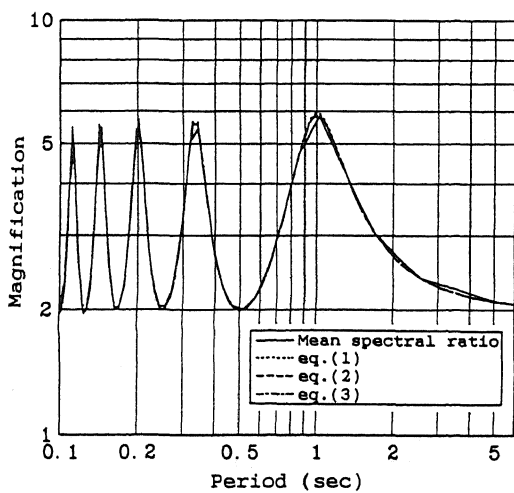


Figure 4. Comparison between the mean spectral ratio and transfer functions

Table 4. Mean value, standard deviation and coefficient of variation of identified parameters using Equation (3) by 1-D analysis

	Mean value	Standard deviation	Coefficient of variation
$V_s$	196.3	1.19	0.01
$c_1$	72.3	17.0	0.24
$c_2$	35.3	7.3	0.21
$c_3$	0.40	0.40	1.00

The value  $c_3$ , the measure of frequency-dependence, is smaller than that in 2-D analysis. Hence, effects of the inhomogeneity in soil deposits should be examined by transfer functions considering not only vertical fluctuation but also lateral fluctuation.

#### 4 EFFECTS OF FLUCTUATION OF SHEAR MODULUS AND INTERNAL DAMPING ON THE FREQUENCY-DEPENDENT CHARACTERISTICS OF Q VALUES

For 3 values of  $V_G$ , transfer functions calculated by the 2-D analysis are shown in Figure 5. Identified parameters are shown in Table 5. When  $V_G=0.3$ , the case that the variation of the shear moduli is the largest, the magnification at higher mode is much larger than that of homogeneous ground. It is also shown that, the larger  $V_G$  becomes, the more strongly Q values become dependent on frequency.

Transfer functions calculated by 2-D analysis are shown for three values of  $h$  in Figure 6. Identified parameters are shown in Table 6. It is shown that, the smaller  $h$  becomes, the more strongly Q values become

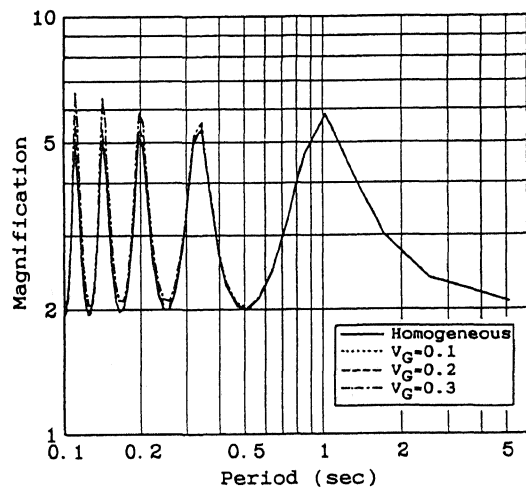


Figure 5. Comparison of transfer functions

Table 5. Identified values for specified coefficients of variation of shear modulus  $V_G$

$V_G$	$V_s$	$c_1$	$c_2$	$c_3$
0.1	199.9	71.3	35.6	0.12
0.2	199.6	77.1	38.5	0.77
0.3	199.2	100.0	50.0	4.32

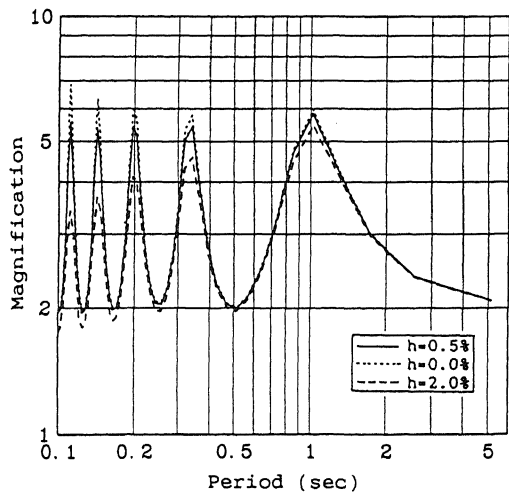


Figure 6. Comparison of transfer functions

Table 6. Identified values for specified damping ratio  $h$

$h(\%)$	$V_s$	$c_1$	$c_2$	$c_3$
0.0	199.6	506.0	50.6	4.28
0.5	199.6	77.1	38.5	0.77
2.0	199.6	9.13	18.3	0.03

dependent on frequency.  $Q$  values by the use of observed weak ground motions have frequency-dependence, because the small shear strain is associated with the small damping ratio. On the other hand,  $Q$  values by the use of observed strong ground motions become independent on frequency, because the damping ratio becomes large with the increasing shear strain. Therefore, Equation (1) may be sufficiently useful in the design during the strong ground motions, because the large shear strain is considered.

Frequency-dependent characteristics of  $Q$  values are formulated as follows based on the observed weak ground motions;

$$Q = V_s \cdot f / 20 \quad (\text{Kobayashi et al. 1989})$$

$$Q = 6.94 \cdot f^{0.856} \quad (\text{Sato et al. 1991})$$

$$Q = V_s \cdot f^{0.5} / 20 \quad (\text{Sugiyama et al. 1990})$$

They are compared with the calculated  $Q$  values ( $h=0.5, 2.0\%$ ) using Equation (3) in Figure 7.  $Q$  value for  $h=0.5\%$  is larger than that of the others and  $Q$  value for  $h=2\%$  is independent on frequency. Magnifications at higher modes of transfer functions are

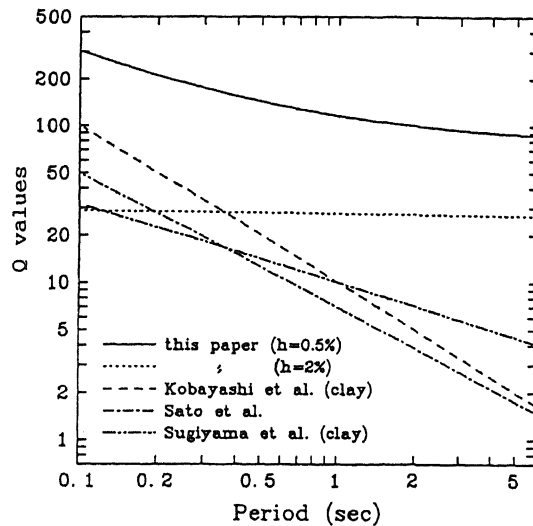


Figure 7. Comparison of  $Q$  values

affected by  $Q$  values. Transfer functions can be estimated by even Equation (1) and (2) with adequate parameters at higher modes. However,  $Q$  values at longer period than 1 second by the others are smaller than those obtained by laboratory tests, because of the lack of the physical meaning in Equation (2). Therefore, Equation (3) should be used in the identification of  $Q$  values from the observed seismic records.

## 5 CONCLUSIONS

In this paper, effects of the inhomogeneity in soil deposits on the amplification characteristics were discussed. Next, frequency-dependence of  $Q$  values was related to the inhomogeneity in soil deposits and damping ratio by the use of the optimization technique. Main conclusions are as follows.

- 1) Frequency-dependence of  $Q$  values is caused by the scattering because of the multi-directional inhomogeneity of soil deposits.
- 2)  $Q$  values in soil deposits are evaluated as the sum of the frequency-independent component (internal damping) and the frequency-dependent component (scattering damping). The value of internal damping is equal to the damping ratio of soil materials.
- 3) Frequency-dependence of  $Q$  values due to scattering becomes negligibly small during the strong earthquake ground motions.

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