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Energy analysis for the non-linear earthquake responses of structures

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ABSTRACT: The non-linear energy responses of SDOF structures to eight artificial ground motions which are compatible with eight different design spectra in the Seismic Code of China (GBJ11-89) are analysed. Various structural parameters are considered. The effect of ground motion frequency characteristics and structural parameters on response energy quantities are discussed. Through statistic analysis, it is found that the total input energy spectrum is greatly influenced both in shape and in quantity by ground motion frequency characterisitcs. The input energy spectrum may be simplified to a bilinear form, with turning point period near the ground motion characteristic period T_g; larger damping ratio or smaller structural yield-strength reduce the peaks of an input spectrum. Except for structural periods less than 0.3 sec., the percentage of hysteretic energy in total input energy is independent of structural period and ground motion, but strongly dependent on damping ratio and yield-strength ratio.

1 INTRODUCTION

How to correctly estimate structural damage under strong earthquakes, that is, the structural damage criterion under earthquakes, has been one of the most important subject in earthquake-resistant engineering research. A lot of research results now have demonstrated that the dual damage criterion based on both the first excursion of maximum response and the accumulated fatigue damage is reasonable. According to this criterion, different damage-estimation expressions have been suggested, and nearly all related to the calculation of response energy quantities of the structure under earthquake. The structural response energy quantities and their transformations can explain well the influences of earthquake intensity, frequency characteristics and duration on structures, and easy to discribe accumulated fatigue damage of structures.

Therefore, the problem of the influence of ground motion characteristics and structural parametres on response energy indices, has attracted many researchers, and among them, Zahrah and Hall(1982), Hiroshi Akiyama(1985), Chin-hsiung Loh(1990) have done systematic work. With detailed numerical analysis, Zahrah fully demonstrated response energy is one of the most important index in estimating structure damage; Hiroshi Akiyama suggested simplified energy spectrum based on several representative earthquake records and presented corresponding limit-state design method; Chin-Hsiung Loh suggested two statistic relations between response energy quantities and the parameters of ground motion and structure.

From the energy equilibrium equation, it is easy to see, if the input ground motion is scaled by a factor s, the energy quantities will be multiplied by s^2; from the results of equivalent linearization analysis, structural response energy quantities are proportional to the duration of ground motion if the ground motion is stationary. So, this paper mainly discusses the influence of ground motion frequency characteristics and structural parameters on main energy quantities. Eight artificial ground motions are used as input ground motions. They are respectively compatible with eight different design spectra in the Seismic Code of China (GBJ11-89), but have the identical peak acceleration, duration and time history envelope. Simple structures with different restoring-force models and other parameters have been calculated, through statistic analysis some significant results have been obtained.

2 METHOD AND PARAMETERS IN THE ANALYSIS

2.1 Energy expressions

The equation of motion for a SDOF system subjected to an earthquake ground excitation can be written as follows

$$\ddot{X}(t)+\frac{4\pi\zeta}{T}\dot{X}(t)+R(t)=-\ddot{Y}(t) \qquad (1)$$

where $X(t)$ is the relative displacement of the mass with respect to the ground, and dots represent differentiation with respect to time; T is the primary period of the structure; ζ is the damping ratio; $R(t)$ is the resistance per unit mass of the structure; $\ddot{Y}(t)$ is the ground acceleration.

Integration of equation (1) with respect to $X(t)$ yields

$$\int_0^t \ddot{X}(\tau)\dot{X}(\tau)d\tau+\frac{4\pi\zeta}{T}\int_0^t\dot{X}^2(\tau)d\tau+\int_0^t R(\tau)\dot{X}(\tau)d\tau=-\int_0^t\ddot{Y}(\tau)\dot{X}(\tau)d\tau \quad (2)$$

where the three items on the left are the kinetic energy $E_K(t)$, damping energy $E_D(t)$ and hysteretic energy plus strain energy $E_H(t)+E_S(t)$ at time t, respectively; the right item represents the total input energy $E_I(t)$. Obviously, at any moment t exsists

$$E_K(t)+E_D(t)+(E_H(t)+E_S(t))=E_I(t) \quad (3)$$

If the structure gets into yield for some time, $E_H(t)$ will be much greater than $E_S(t)$, and at the end of the earthquake, $E_S(t)$ tends to be zero. So, for a yielded structure, approximately exists

$$\max(E_H(t))\doteq\max(E_H(t)+E_S(t)) \quad (4)$$

Newmark's Beta-method with Beta equal to 1/6 corresponding to a linear variation of the response acceleration $\ddot{X}(t)$ is employed herein, and also Dionisio Bernal's(1991) trinomial exact solution is used to deal with the event location problem of multi-linear model of restoring force in step-by-step integration.

2.2 Structural parameters

In the analysis, structure periods vary from 0.1 to 1.2 sec., with an increment of 0.1 sec.; damping ratios are given 0.01, 0.03, 0.05 and 0.1 respectively. The models of structural restoring-force are taken as bilinear and degrading trilinear type, with the definition of yield-strengh ratio $\beta=Q_y/Q_e$, where Q_y is the yield resistance of the structure, and Q_e is the maximum resistance of elastic analysis under the same input ground motion. β is in the range of 0.2 to 0.5, with a step of 0.1; the post-yield stiffness coefficient p is taken from 0.0 to 0.2(at p=0.0, the bilinear model becomes elastic-perfectly plastic pattern). As regard to trilinear model, crack-resistance is equal to $\frac{1}{3}Q_y$, post-crack stiffness equal to 85 percent of elastic stiffness.

2.3 Ground motion parameters

Artificial ground motions which are compatible with the design spectra of the Seismic Code of China (GBJ11-89) are taken as input. In order to simulate, the nonstationary characteristics of earthquake ground motion,

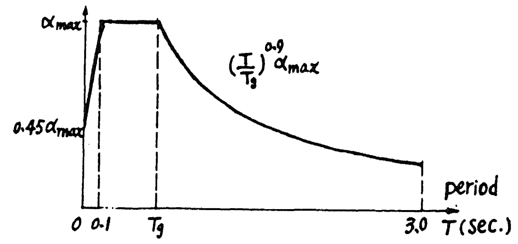

Figure 1. The design spectrum of GBJ11-89

Table 1. The values of T_g (sec.) in GBJ11-89

Field condition	1	2	3	4
Long epicentre distance	0.20	0.30	0.40	0.65
Short epicentre distance	0.25	0.40	0.55	0.85

Amin and Ang's (1968) time-history envelope is used, with t_1=2.5 sec., t_2=10sec., t_3=15sec., attenuation coefficient c=0.18 . Fig.1 shows the standard design spectra of GBJ11-89 represented by earthquake coefficient α , in which T_g is called the characteristic period of ground motion, and the value of which are shown in Table 1. They range from 0.20 to 0.85(sec.), and comprehensively reflect the influence of field condition and epicentre distance on spectrum characteristics. The Code regulates that α can not be less than $0.2\alpha_{max}$. In this paper, α_{max}=0.5 . Codes of A10, A20, A30, A40 are used to indicate the artificial ground motions compatible with field condition 1 to 4 in short epicentre distance respectively; similarly, codes of A11, A21, A31, A41 indicate artificial ground motions in long epicentre distance.

3 COMPUTATION RESULTS ANALYSIS

For convenience E_I, E_H and E_D are designated as the maximums of $E_I(t)$, $E_H(t)$ and $E_D(t)$ respectively, and hysteretic energy coefficient (HEC) are defined as

$$HEC=E_H/E_I\times100\% \quad (5)$$

Obviously, HEC stands for the percentage of hysteretic energy E_H in total input energy E_I. In fact, $E_H=E_H(t_f)$, $E_D=E_D(t_f)$, $E_I\doteq E_I(t_f)$, $E_I\doteq E_H+E_D$, where t_f indicates the time of the end of ground motion. Therefore , only the two indexes, E_I and HEC are necessary to discribe the main energy quantities concerned. In the following analysis the effects of ground motion frequency characteristics and structural parametres on E_I and HEC are discussed.

3.1 Effects on total input energy E_I

Four total input energy spectra which come from four different ground motions A10 ,A21,

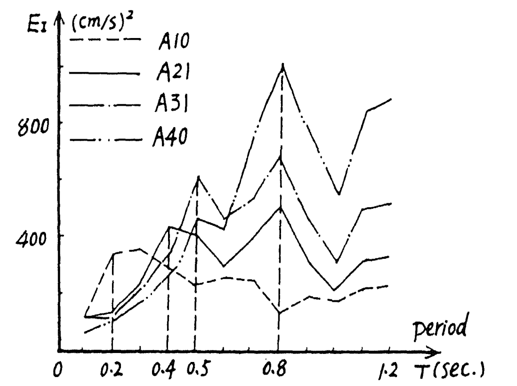

Figure 2. Total input energy spectra of different ground motions

A31, A40 with T_g equal to 0.2,0.4, 0.55,0.65 sec. respectively are presented in Fig.2 .It shows that ground motion frequency characteristics strongly effects both the shape and the intensity of E_I spectrum,and each E_I spectrum has a turning point, before which the values of E_I incease as structural peroids increse, after which they vary up and down about a line. Each turning point approximately corresponds to the period T_g of the related ground motion.Therefore an E_I spectrum may be simplified to a bilinear form. Fig.2 also shows that the wider of the frequency range of a ground motion, the stronger the corresponding E_I spectrum intensity after the turning point, although the ground motions' maximums of elastic response specta are the same.

E_I spectra with different damping ratioes are shown in Fig.3 to demonstrate the effect of damping ratio ζ on E_I. A larger damping ratio can remarkably reduce the peak value in an E_I spectrum and smooth it, especially when damping ratio is below the range of 0.05 . But an increment of damping ratio has little influence on non-peak range in an E_I spectrum.

Fig. 4 shows the effect of structural yield-strength ratio β on E_I spectrum. The effect are similar with that of damping ratio, except that a smaller values of β reduces the peak values in an E_I spectrum.

3.2 Effects on hysteretic energy coefficient HEC

To illustrate the effects of structural periods on HEC, the coefficient of variation COV of HEC with respect to the period range of 0.1 to 1.2 sec. are summarized in Table 2. Three different ground motions, damping ratioes and yield-strength ratioes are considered. It shows that damping ratio and ground motions have little effect on the values of COV,but yield-strength ratio β has strong effect on them.Values of COV increase

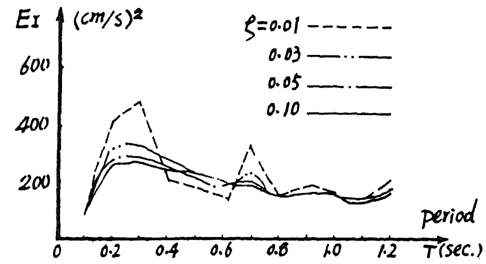

Figure 3. Total input energy spectra of different damping ratioes

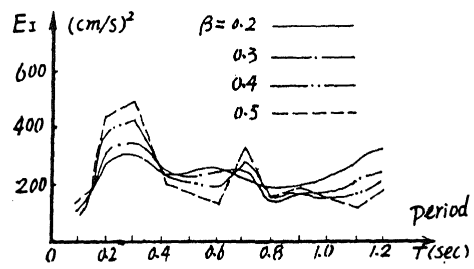

Figure 4. Total input energy spectra of different yield-strength ratioes

as β increase. When $\beta \leq 0.3$, nearly all the COVs below the range of 0.10; when $\beta > 0.3$, since the values of HEC are much smaller for high frequency structures ($T \leq 0.3$sec.) than for other structures discussed,values of COV are large till to the maximum of 0.294 . If the periods equal to or below 0.3 are not involved in statistics, the values of COV are as small as those when $\beta < 0.3$.Therefore, except for T less than 0.3 sec.,the hysteretic energy coefficients can be considered independent of structural periods and ground motion frequency characteristics. This means that hysteretic energy spectra are nearly parallel to corresponding total input energy spectra, they are similar in shape.

Table 2. Coefficients of variation COV of HEC in the period range of 0.1 to 1.2 sec. (p=0.0)

Wave	ζ	β=0.2	β=0.3	β=0.5
A10	0.01	0.054	0.091	0.157
A10	0.05	0.049	0.108	0.272
A10	0.10	0.037	0.105	0.294
A30	0.01	0.032	0.066	0.110
A30	0.05	0.030	0.037	0.195
A30	0.10	0.040	0.061	0.176
A40	0.01	0.040	0.077	0.146
A40	0.05	0.026	0.047	0.159
A40	0.10	0.029	0.064	0.121

In order to examine the effect of different ground motions, in Table 3, with fixed structural parameters (ζ , β and p) but different gruond motions, the mean value of HEC with regard to periods are listed. It is shown that different ground motions have little effect on HEC.

Table 3. Mean value μ of HEC with respect to structural periods (ζ=0.05, β=0.3, p=0.0)(%)

Wave	A10	A11	A20	A21	A30	A31	A40	A41
μ	67.0	67.1	64.2	65.0	64.5	66.4	70.4	69.6

The data in Table 4, illustrate the effect of yield-strength ratio on HEC.With ζ =0.05, p=0.0, first the average of HEC with respect to periods is taken, then the mean value of these avearges corresponding to each β are obtained with respect to eight different ground motions, together with the coefficients of variation COV. It is shown that HEC almost decrese linearly as β increases in the range of 0.2 to 0.5 .The small values of COV indicate again that as an average about periods, HEC is independent of ground motions.

Table 4. Mean value μ of HEC with respect to structural periods and ground motions,and corresponding COV (ζ=0.05, p=0.0)(%)

β	0.2	0.3	0.4	0.5
μ	75.78	66.74	54.47	42.35
COV	0.021	0.035	0.043	0.048

The same method is used to examine the effect of damping ratioes on HEC. Table 5 shows the relation between mean value of HEC and damping ratioes.

Table 5. Mean value μ of HEC with respect to structural periods and ground motions and corresponding COV (β=0.3, p=0.0)(%)

ζ	0.01	0.03	0.05	0.10
μ	82.24	71.72	66.74	57.31
COV	0.040	0.025	0.035	0.048

Table 6 reflects the effect of post-yield stiffness coefficient p on average HEC. In the range of 0.0 to 0.2,p has negligible effect on average HEC, with a larger p corresponding to a smaller value of HEC averagely. Although only the situation of β=0.3 are listed here,the results are similar for other values of β .

4 SUMMARY AND CONCLUSIONS

The effect of ground motion frequency characteristics and structural parameters on structural response energy quantities are

Table 6. Mean values of HEC with respect to structural periods (A21, β=0.30) (%)

ζ	p=0.0	p=0.1	p=0.2
0.01	79.41	78.67	77.58
0.05	65.03	63.18	60.32
0.10	55.50	52.81	50.47

analyzed. Total input energy and hysteretic energy coefficients HEC are chosen as representative indices.

The total input energy spectrum is greatly influenced both in shape and in quantity by ground motion frequency characteristics. It may be simplified to a bilinear form with turning point period near the T_g of the ground motion. Larger damping ratio or smaller yield-strength ratio reduce the peak of an E_I spectrum, and smooth it; except for structural periods T less than 0.3 sec., the percentage of hysteretic energy in total input energy is independent of the structural period and ground motion, but strongly dependent on damping ratio and yield-strength ratio, that is, a hysteretic energy spectrum is nearly parallel to corresponding total input energy spectrum. The mean values of hysteretic energy coefficients have been given corresponding to several discrete damping ratioes and yield-strength ratioes.

Further statistic work may be done to present a statistic formula of hysteretic energy coefficients as the function of damping ratio and yield-strength ratio, companied with a simplified total input energy spectrum, then the hysteretic energy can be calculated easily.

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