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Computer simulation of wave propagation characteristics near a source using a framework model

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ABSTRACT: From practical economical viewpoints, the wave caused in proximity to the source by a normal force applied to the surface of a multi layered media have been often used for the evaluation of the shallow subsurface structure. The framework model was employed as a simulation technique. The validity and accuracy of this method in the far field were at first confirmed by application to an elastic half-space medium and a two layered media. Consequently, it is shown that the results of the framework model are in good agreement with those of the analytical method. The wave propagation characteristics in the near field were secondarily investigated by both the framework model and analytical methods.

1 INTRODUCTION

From practical and economical viewpoints, it is desirable that subsurface structure can be estimated by the analysis of the waves obtained at distances of 1 and 2 meters from the source. However, detailed information of wave phenomena in the near filed is still lacking today, and the complicated solutions presented by Nojima et al. (1981) is the only approach available. This may be due to the difficulty in computing complex integrals. Therefore, there is a great demand for more realistic method of analysis to study the properties of ground vibration in proximity to the source. In general, the finite difference and finite element methods are two of the most useful numerical techniques to explain wave propagation. In the finite difference method, it is very difficult to treat complicated boundaries as a corner or a slope. While such difficulty can be removed in the finite element method, large storage area is required for the computation. Therefore, it is necessary to develop a practical simulation technique, which is based on the theory of elastic waves, to study ground vibration properties from the standpoint of engineering applications. We employed the framework model originally proposed by Sato (1978). This method, with the restriction on the value of Poisson's ratio, has been improved by Harumi et al. (1978) and Ohbo (1985). We applied this procedure to the near field after verification in the far field where exact theoretical solutions are available. This paper compares analytical results with the computer simulation of wave phenomena through use of the framework model, and shows that waves near the source can be used for the evaluation of subsurface structures.

2 FRAMEWORK MODEL

2.1 *Fundamental wave equations*

Let us write the equations of motion in the x-z plane elastic medium as :

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+\mu)\frac{\partial\Delta}{\partial x} + \mu\nabla^2 u \qquad (1)$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda+\mu)\frac{\partial\Delta}{\partial z} + \mu\nabla^2 w \qquad (2)$$

where u and w are the displacement in the x and z directions, respectively, ρ is the density, t is time, and λ and μ are Lamé's constants.

2.2 *Framework model scheme*

In Fig. 1, let x and z be the two dimensional Cartesian coordinates and let h denote the grid increment. The grid point (j,k) is connected with the eight neighboring points. Here j and k are the indices along the x- and z-directions, respectively. The mass is concentrated at every spring. A horizontal or vertical spring connecting the grid points has the spring constant C_1 and a diagonal spring has the spring constant C_2. When we use only spring constants C_1 and C_2, the framework model is applicable only for the case of $\lambda \leq \mu$. However, Poisson's ratio on the actual ground may be larger than 0.25. Based on the suggestions by Ohbo (1985), we introduced an additional spring with spring constant C_3 in each diagonal direction.

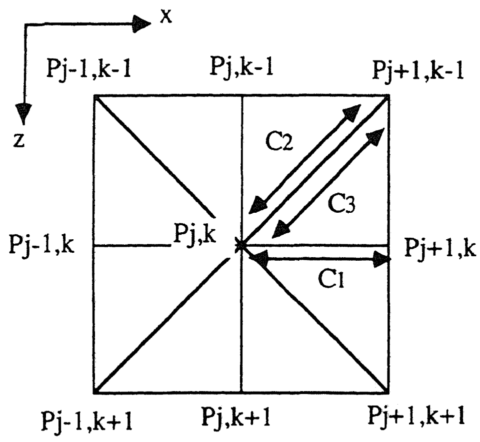


Fig. 1 Framework model.

Thus, the equations of motion without the restriction on the value of Poisson's ratio can be written as:

$$m_{j,k}\frac{d^2u_{j,k}}{dt^2} = C_1(u_{j+1,k} - 2u_{j,k} + u_{j-1,k}) + \frac{(C_2+C_3)}{2}(u_{j+1,k+1} + u_{j-1,k+1} + u_{j+1,k-1} + u_{j-1,k-1} - 4u_{j,k}) + \frac{C_2}{2}(w_{j+1,k+1} - w_{j-1,k+1} - w_{j+1,k-1} + w_{j-1,k-1}) \quad (3)$$

$$m_{j,k}\frac{d^2w_{j,k}}{dt^2} = C_1(w_{j,k+1} - 2w_{j,k} + w_{j,k-1}) + \frac{(C_2+C_3)}{2}(w_{j+1,k+1} + w_{j-1,k+1} + w_{j+1,k-1} + w_{j-1,k-1} - 4w_{j,k}) + \frac{C_2}{2}(u_{j+1,k+1} - u_{j-1,k+1} - u_{j+1,k-1} + u_{j-1,k-1}) \quad (4)$$

When $C_1=\lambda+\mu$, $C_2=C_1/2$, and $C_3=(\mu-\lambda)/2$, the equations can be reduced to Eqs. (1) and (2), respectively.

2.3 *Boundary conditions*

Figure 2 illustrates the condition of the free surface, which corresponds to the k-th level. There are no points with the second index smaller than k. Therefore, it is assumed that the mass m and the spring constant C_1 become 1/2, respectively. Consequently, the equations

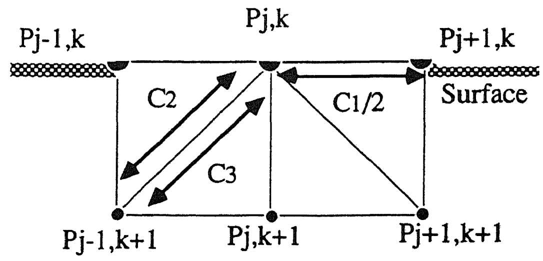


Fig. 2 Framework model at a free surface

of motion of the mass m/2 in the x- and z-directions at the point (j,k) can be given by the following expressions (Sato and Nitabaru, 1980).

$$\frac{1}{2}m_{j,k}\frac{d^2u_{j,k}}{dt^2} = \frac{1}{2}C_1(u_{j+1,k} - 2u_{j,k} + u_{j-1,k}) + \frac{(C_2+C_3)}{2}(u_{j+1,k+1} + u_{j-1,k+1} - 2u_{j,k}) + \frac{C_2}{2}(w_{j+1,k+1} - w_{j-1,k+1}) \quad (5)$$

$$\frac{1}{2}m_{j,k}\frac{d^2w_{j,k}}{dt^2} = C_1(w_{j,k+1} - w_{j,k}) + \frac{(C_2+C_3)}{2}(w_{j+1,k+1} + w_{j-1,k+1} - 2w_{j,k}) + \frac{C_2}{2}(u_{j+1,k+1} - u_{j-1,k+1}) \quad (6)$$

As the actual subsurface is generally multi-layered, we need to formulate conditions satisfied at an interface. Figure 3 shows the interface referred to as the k-th level.

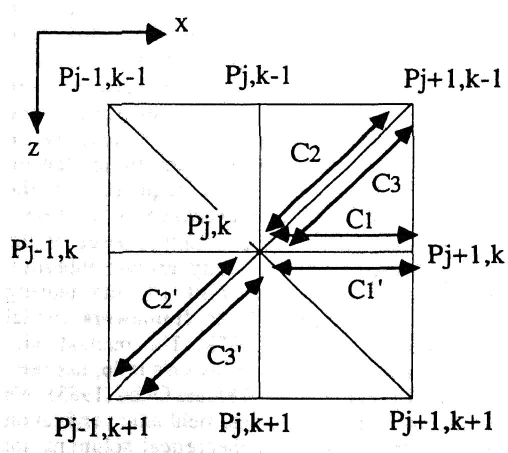


Fig. 3 Framework model at an interface

According to the expressions proposed by Ohbo and Katayama (1982), we can write the equations for the boundary as follows:

$$m'_{j,k}\frac{d^2u_{j,k}}{dt^2} = C'_1(u^{1,2}_{j+1,k} - 2u^{1,2}_{j,k} + u^{1,2}_{j-1,k})$$
$$+\frac{(C'_2 + C'_3)}{2}(u^2_{j+1,k+1} + u^2_{j-1,k+1} - 2u^{1,2}_{j,k})$$
$$+\frac{(C_2 + C_3)}{2}(u^1_{j+1,k-1} + u^1_{j-1,k-1} - 2u^{1,2}_{j,k})$$
$$+\frac{C'_2}{2}(w^2_{j+1,k+1} - w^2_{j-1,k+1})$$
$$+\frac{C_2}{2}(w^1_{j-1,k-1} - w^1_{j+1,k-1}) \quad (7)$$

$$m'_{j,k}\frac{d^2w_{j,k}}{dt^2} = C'_1(w^{1,2}_{j+1,k} - w^{1,2}_{j,k})$$
$$+C_1(w^{1,2}_{j,k} - w^1_{j,k-1})$$
$$+\frac{(C'_2 + C'_3)}{2}(w^2_{j+1,k+1} + w^2_{j-1,k+1} - 2w^{1,2}_{j,k})$$
$$+\frac{(C_2 + C_3)}{2}(w^1_{j+1,k-1} + u^1_{j-1,k-1} - 2w^{1,2}_{j,k})$$
$$+\frac{C'_2}{2}(u^2_{j+1,k+1} - u^2_{j-1,k+1})$$
$$+\frac{C_2}{2}(u^1_{j-1,k-1} - w^1_{j+1,k-1}) \quad (8)$$

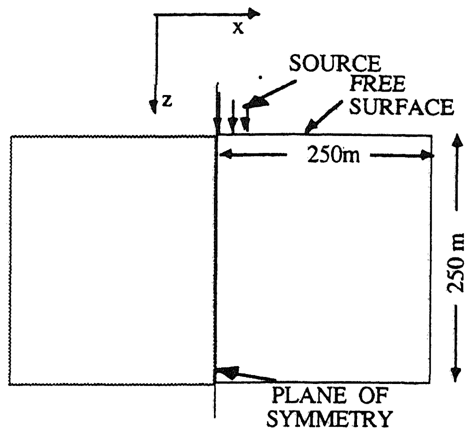
where the subscripts 1, 2, and 1,2 correspond to the first, second, and boundary layers, respectively.

3 CALIBRATION OF FRAMEWORK MODEL

In order to confirm the validity and accuracy of the numerical technique used in the framework model, we carried out illustrative simulations for several simple cases related to wave radiation in two dimensional medium. Figure 4 shows the configurations of two models representing an elastic half-space and a two-layered medium with a surface layer over-lying a semi-infinite substratum. Both models are two-dimensional and have a plane symmetry with respect to the z-axis. The medium is divided into meshes. The parameters adopted for the simulation are given in Fig. 4. The grid interval and the time step were decided by the following expressions (Alford et al., 1974 ; Kelly et al., 1976 ; Stephen et al., 1985).

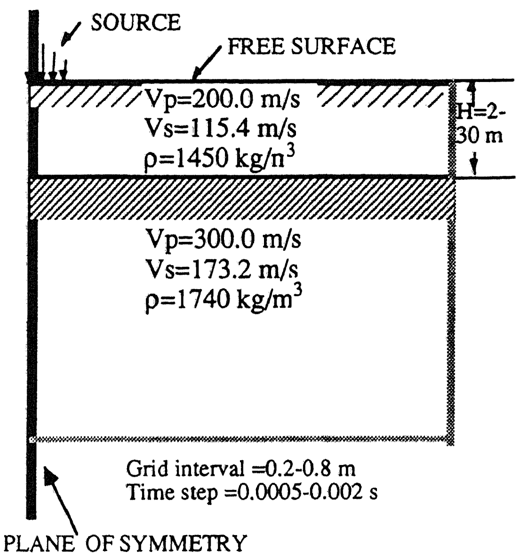
$$h = \frac{V_{min}}{10f_{max}} \rightarrow \frac{0.8V_s}{10f_{max}} \quad (9)$$

$$\Delta t \le \frac{\min(\Delta x, \Delta z)}{\sqrt{v_c^2 + v_s^2}} \quad (10)$$



Vp = 200.0 m/s Vs =115.4 m/s Poisson's ratio = 0.25
Grid interval =0.5 m Time step =0.002 s

(a) Half-space medium



(b) Two-layered medium

Fig.4 Geometry of the two-dimensional medium

where h and Δt indicate the grid interval and time step, respectively. A normal force P(t,x) is applied on the free surface z=0, and is given with the distribution shown in Fig. 5 :

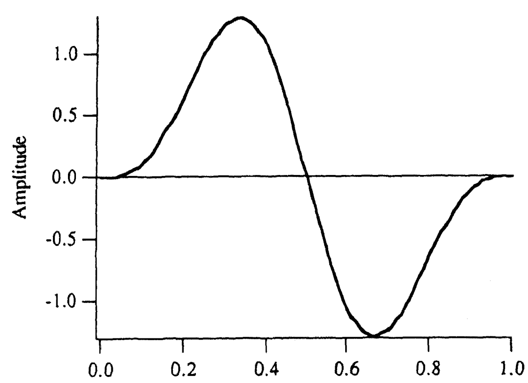
$$P(t,x) = F(t)\,G(x) \quad (11)$$

where

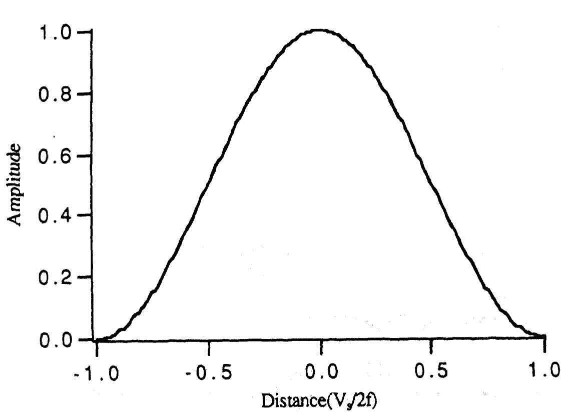
$$F(t) = \begin{cases} 0 & (t < 0, t > \frac{1}{f}) \\ \sin(2\pi ft) - \frac{\sin(4\pi ft)}{2} & (0 \le t \le \frac{1}{f}) \end{cases} \quad (12)$$

$$G(x) = \begin{cases} 0 & \left(|x| > \dfrac{V_s}{2f}\right) \\ \dfrac{1 + \cos\left(\dfrac{2\pi f x}{V_s}\right)}{2} & \left(|x| \le \dfrac{V_s}{2f}\right) \end{cases} \tag{13}$$

F(t) and G(x) are the time and spatial source functions, Vs is the S-wave velocity of the medium directly beneath the free surface, and f is the frequency. The frequency is assumed to be 10 Hz.



(a) F(t)



(b) G(x)

Fig. 5 Source function for applied force. F(t), time source function; G(x), spatial source function.

The general validity of the numerical technique in the far field was at first examined in the case of a semi-infinite medium. Namely, the features of the wave propagation, amplitudes and arrivals of various types of waves were compared with those obtained from the different numerical approaches including the finite difference method. The waveforms observed at several points along the ground surface are shown in Fig. 6. The left side is the result of the finite difference method and the right that of the framework model. The amplitude scale in both cases is arbitrary but the ratio between the horizontal and vertical amplitudes is correct. The upward and downward arrow in Fig. 6 indicate the P and Rayleigh pulse arrivals, respectively. The velocities of P and Rayleigh waves determined from the observed waveforms are about 200 m/s and 104 m/s, respectively. These values coincide with the velocities adopted for the numerical model. Figure 7 shows the distribution of the vertical displacement in the wave field at t=0.2 sec. In this wave propagation pattern, we can easily recognize the propagations of P-, S-, and Rayleigh waves with different velocities. Therefore, it may be said that the visualization of wave phenomena is one of the most effective approaches to study wave propagation characteristics.

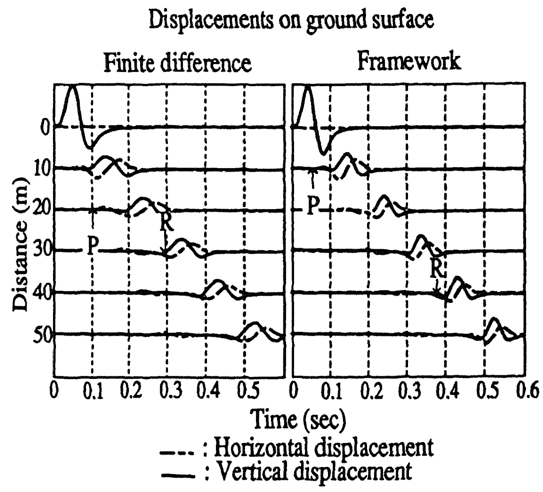


Fig.6 Vertical and horizontal displacement waveforms on the free surface of the elastic half-space.

The next step in the confirmation is to investigate the amplitude distribution. Figure 8 illustrates the theoretical and calculated results of the vertical and horizontal amplitude distributions with depth at a point 40 m distant from the source.

The amplitude in this figure are normalized by the respective maximum amplitude at the surface. We can find from this figure that the framework method is superior to the finite difference method in the level of agreement.

As a next test of the framework model, we applied it to a two layered model with depth to substratum H = 4 m. The phase and group velocities determined from the waves at the distances of 30 m and 32 m are plotted in Fig. 9 against the frequency together with the theoretical dispersion curves obtained from the Haskell's method (1953). From this figure, we can clearly see that the observed velocities are in good agreement with the theoretical dispersion curves. Thus, the numerical technique is under all conditions applicable to the two layered media.

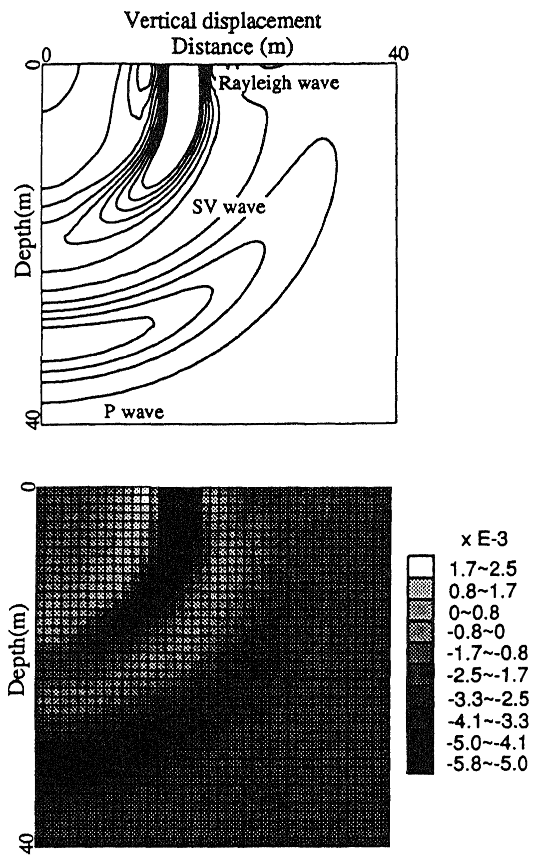


Fig. 7 Wave propagation pattern in elastic half-space at t = 0.2 sec.

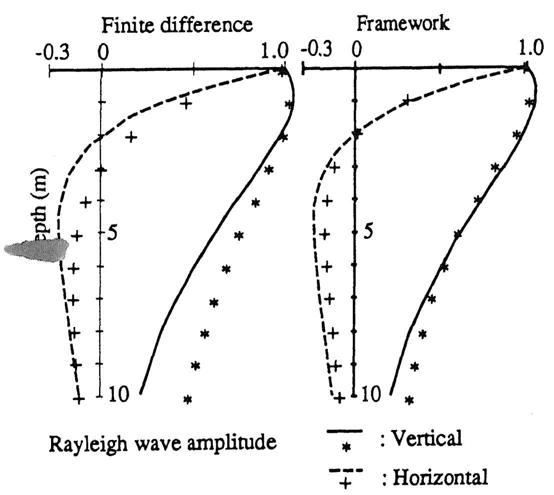


Fig.8 Amplitude distributions of the Rayleigh wave with depth at a point x = 40 m.

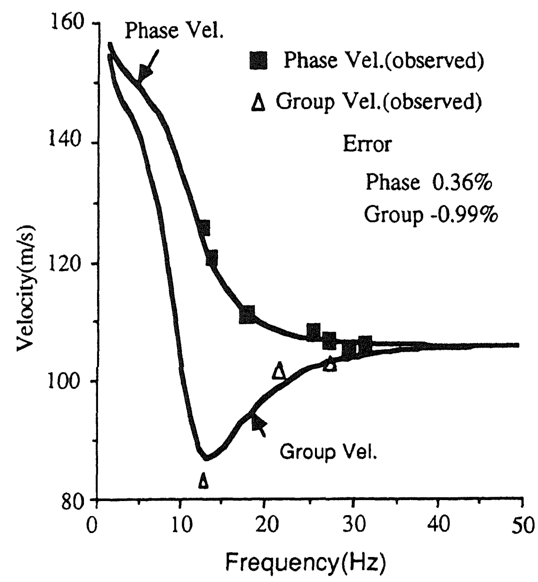


Fig. 9 Comparison of the observed phase and group velocities with the theoretical dispersion curves.

4 CALCULATIONS IN THE NEAR FILED

4.1 *Analytical method*

Let r and z be the cylindrical coordinates, and let u and w denote components of displacement in the r and z directions, respectively. When a normal force is applied on the free surface, the displacements on it in a two layered media can be given by the following expressions (Nojima et al., 1981).

$$u = \int_0^\infty d\zeta\left[-\zeta J_1(\zeta a)\left(x_1 + \beta_1 \bar{y}_1\right)\right]\exp(iwt) \qquad (14)$$

$$w = \int_0^\infty d\zeta\left[\left(\alpha_1 \bar{x}_1 + \zeta^2 y_1\right) J_0(\zeta a)\right]\exp(iwt) \qquad (15)$$

where

$J_0(\zeta a)$, $J_1(\zeta a)$: Bessel function of the zero and first order

ζ : integral variable

x_1, $\bar{x}_1$, y_1, $\bar{y}_1$: integral constants

$a = \omega r/V_s$: dimensionless distance

V_p : P-wave velocity

V_s : S-wave velocity

$\alpha 1 = \mathrm{sqrt}\{\zeta^2-(vs/vp)^2\}$

$\beta 1 = \mathrm{sqrt}\{\zeta^2-1\}$

The integrals cannot be evaluated by direct integration, because the integral constants have poles. We therefore first investigated the loci of poles of the integral constants. On referring the results, we practise the calculations of the displacements. The phase velocities at each frequency are obtained from the phase difference between distances of 1 m and 2 m.

4.2 *Numerical results*

For the first time in the simulation the framework model is adapted to the near field. The grid spacing are 1 m, 0.5 m, 0.25 m, 0.2 m according to the source frequency and time increment is 0.0005 sec. Frequencies used for the calculations are 2, 3, 4, 5, 7, 10, 15, and 20 Hz. P-, and S-wave velocity in the upper layer is 86.6, and 50 m/s, respectively. The density is 1650 kg/m^3. Thickness of this layer is 10 m. P- and S-wave velocity of the substratum is 173.2 and 100 m/s, respectively. The density is 1750 kg/m^3. The phase velocities used are determined by the spectral analysis of the waveforms at the points 1 m and 2 m distant from the source. The results are shown in Fig. 10. The white circle symbol indicates the numerical results. This figure suggests that even waves extremely near the source have dispersive characteristics which can be used for the evaluation of subsurface velocity structures. Thus, it is also shown that the framework model is applicable to the near field. We can therefore study all kinds of dynamic properties, such as the wave propagation in the ground, using this technique.As the framework method uses simple concepts, involves arbitrary selection of time steps, and has no difficulty in satisfying boundary conditions, it can be applied to the study of more general problems.

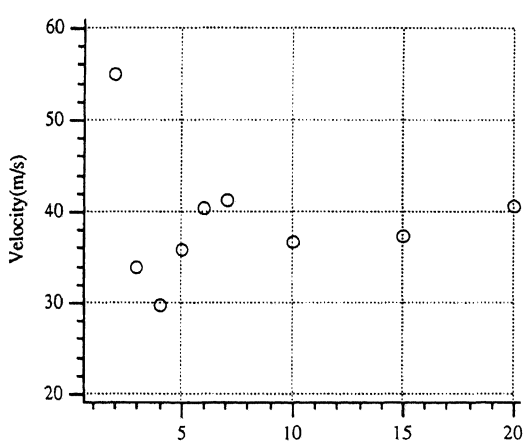


Fig 10. Phase velocities in the near field obtained from numerical simulations.

5 CONCLUSIONS

The framework model was used as a simple and convenient method for the estimation of subsurface structure by the analysis of waves in proximity to the source. After the validity of the method was precisely confirmed in the far field, the capability of the application in the near field was carefully examined. It is found that the waves near the source can be utilized for the estimation of subsurface structures. This method can also be used to investigate, in sufficient detail, the mechanism of wave propagations by means of computer animation.

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