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# Inelastic response spectra for narrow band earthquakes

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Abstract: The seismic design coefficients included in typical seismic codes are normally obtained dividing a smooth elastic design spectrum for 5% damping by a reduction factor, function primarily of the allowable ductility. The use of this reduction factor is based on the studies on inelastic response spectra conducted first by Newmark and Hall (1973) for elastic-perfectly plastic systems and by other later for more general nonlinear systems. All these studies considered however earthquake records corresponding to motions with a broad frequency content. Earthquake motions on soft soil deposits (such as the lake zone of Mexico City) have on the other hand narrow banded elastic response spectra with a very large peak at the natural frequency of the site. The applicability of Newmark's rules and the general characteristics of the inelastic response spectra for this type of earthquake are investigated and discussed in this paper.

## 1 INTRODUCTION

The usefulness of response spectra as a means to characterize an earthquake and its effects on structures was recognized very early. For some time, however, designers were using in every case the set of response spectra corresponding to the 1940 El Centro earthquake which had been extensively studied an documented. A first step to improve this situation was taken by Housner (1959) who considered the records of five strong motion earthquakes, scaled them to a same intensity and averaged the corresponding response spectra. The resulting set of smooth, average spectra was used extensively for some years. A second major step towards a better characterization of strong motions for design purposes was taken by Newmark (1967) who defined the earthquake by three parameters (maximum ground acceleration, velocity and displacement) and recognized seven distinct ranges in a response spectrum. Figure 1 shows the general shape of Newmark's smoothed spectra with the seven ranges. Of these the most important for typical structures are the intermediate ones (3, 4 and 5) and particularly the 4th and 5th. They correspond to the maximum spectral displacement, maximum spectral pseudo-velocity and maximum spectral pseudo-acceleration. They are often referred to as the displacement, velocity and acceleration ranges in short. Newmark (1967) suggested factors to construct smooth elastic response spectra from the peak ground displacement, velocity and acceleration as a function of the structural damping. Alternative factors were suggested by Garcia and Roesset (1970), Newmark, Hall and Mohraz (1973) and Riddell and Newmark (1979).

All these results are of value when designing structures which are assumed to remain elastic or nearly elastic under the design earthquake. Most regular buildings will undergo, however, inelastic deformations when subjected to strong seismic motions, a fact

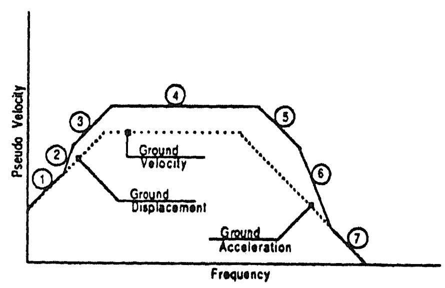


Figure 1. Newmark's smoothed spectrum.

implicitly recognized in design codes and accepted in present design philosophy. Inelastic response spectra for elasto-plastic systems with zero and ten percent damping were presented by Blume, Newmark and Corning (1960) for the NS component of the 1940 El Centro earthquake and ductility ratios of 1 (elastic system), 1.25, 2 and 4. Newmark and Hall (1973) suggested rules to construct smooth inelastic spectra for different levels of ductility and elasto-plastic systems with 5% damping. Studies by Sehayek (1976) showed that these rules apply very well to the average of nine earthquake records for elasto-plastic systems with 5% initial damping, although they do not provide a uniform degree of conservatism over the complete frequency range. Studies for other inelastic systems and different values of elastic (initial) damping have been conducted by Riddell and Newmark (1979) and Al Sulaimani (1982), who concluded that the details of the force deformation relation for the inelastic systems had very little effect on the ductility demands for a given structural resistance or on the required resistance for a given allowable ductility (force reduction factor). The systems considered by Al Sulaimani (1982) had all a

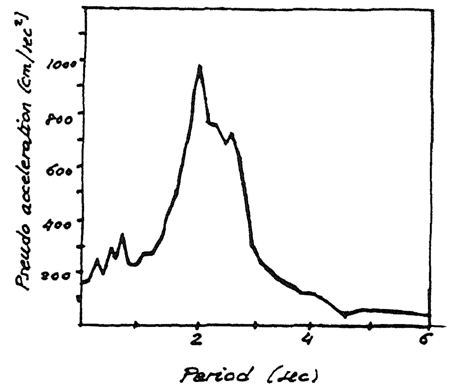


Figure 2. 5% elastic response spectrum SCT record EW.

perfectly plastic range and in this case the ductility demands were slightly larger when accounting for stiffness degradation and pinching effects than for the standard elasto-plastic systems. The systems considered by Riddell and Newmark (1979) had on the other hand a strain hardening second slope and in this case the ductility demands were slightly smaller. In all cases however the increases or decreases in ductility demand were practically negligible. The earthquake motions considered in these studies had all broad band response spectra.

## 2 OBJECTIVES AND FORMULATION

The effects of the soft clay on the amplitudes, frequency content and duration of the seismic motions experienced in the lake zone of Mexico City have long been recognized. Figure 2 shows the 5% elastic response spectrum corresponding to the EW component of the motion recorded at the SCT site during the 1985 earthquake. It can be noticed that there is a very large amplification at a period of about 2 seconds corresponding to the natural period of the soil deposit at the site (an amplification of nearly 6 instead of the 2.5 to 3 values normally encountered for broad band motions.) Similar results are possible at various sites in the San Francisco area underlain by the soft bay mud. The purpose of this study was to study the characteristics of the inelastic spectra for this type of motions with a pronounced predominant frequency, and to assess the applicability of Newmark's rules to these cases.

The EW component of the SCT record (the one with the largest acceleration) was used for the studies. The nonlinear systems considered are represented schematically in Figure 3. They are an elasto-plastic (elastic perfectly plastic) system, a general degrading system with stiffness degradation function of the maximum deformation reached and pinching, and a cyclic degrading model with additional stiffness degradation with each cycle of vibration and hysteretic loops that do not close exactly. In addition the well known Takeda (1970) model which is intermediate between the elasto-plastic and the general degrading models was used for some studies.

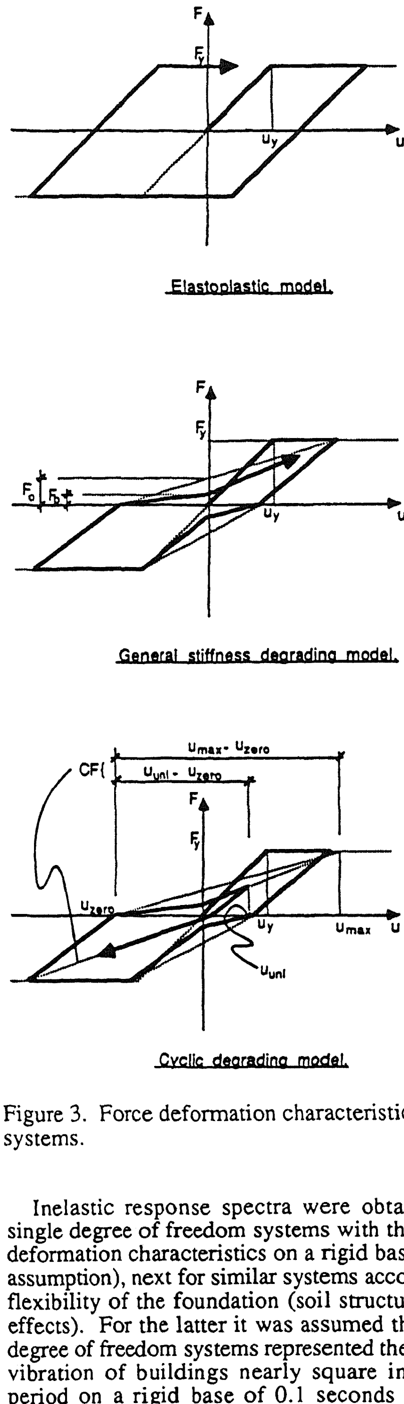


Figure 3. Force deformation characteristics of inelastic systems.

Inelastic response spectra were obtained first for single degree of freedom systems with the above force deformation characteristics on a rigid base (the normal assumption), next for similar systems accounting for the flexibility of the foundation (soil structure interaction effects). For the latter it was assumed that the simple degree of freedom systems represented the first mode of vibration of buildings nearly square in plan with a period on a rigid base of 0.1 seconds per story, an average story height of 3 m, and a mass of 800 kg/m$^2$ per story. To define the equivalent single degree of freedom systems a mass equal to eighty percent of the total mass and a height above the ground equal to 65% of the total building height were assumed. The soil was assumed to have a shear wave velocity of 80 m/sec, a shear modulus of 7.8 x 10$^6$ N/m$^2$ and a Poisson's ratio of 0.45. A depth to bedrock of 40 m was considered. Buildings with up to 6 stories were assumed to have shallow mat foundations; from seven to fifteen stories they were assumed to have 1 basement and a friction pile foundation. Buildings from sixteen to thirty stories were supported on bearing piles also with 1 basement. The values of the spring stiffnesses and dashpot coefficients for the mat foundations were obtained using the formulas suggested by Elsabee and Morray (1977) accounting for the finite depth of the soil stratum. The dynamic stiffnesses of the pile foundations were computed by Kim (1987) assuming an elastic soil with 2% damping according to recommendations made in a number of studies conducted after the 1985 earthquake. It was finally necessary to assume a value of the slenderness ratio (ratio of the total height to the base dimension) for the buildings. A slenderness ratio of 0.2 was considered appropriate for buildings with 1 to 4 stories; 0.5 for the buildings with 1 to 8 stories; 1.0 for buildings with 3 to 17 stories; 2.0 for buildings with 5 to 30 stories; and 3.0 for buildings with 8 to 30 stories. Over the common ranges the results obtained with the various slenderness ratios were smoothed and enveloped.

The equation of motion for the single degree of freedom systems on a rigid base can be written as

$$m\ddot{u} + c\dot{u} + F = -m\ddot{u}_G \qquad (1)$$

Defining

$$z = \frac{u}{u_y} \qquad f = \frac{F}{F_y} = \frac{F}{ku_y} \qquad (2)$$

where k is the initial (elastic) stiffness and dividing both sides of the equation by $mu_y$

$$\ddot{z} + 2D\omega_o\dot{z} + \omega_o^2 f = \omega_o^2\,\ddot{u}_G/\ddot{u}_{max} \qquad (3)$$

where D is the initial (elastic) damping, $\omega_o$ is the natural frequency of the sytems and $\ddot{u}_{max} = F_y/m$ is the maximum response acceleration for an undamped system (maximum pseudo-acceleration).

Equation 3 can then be solved integrating in time step by step using the constant average acceleration method. This leads to

$$\begin{aligned}(1 + D\omega_o\,\Delta t + \tfrac{1}{4}\,\omega_o^2\,\Delta t^2\,\tfrac{k_{ti}}{k})\,\ddot{z}_{i+1} &= -R_{i+1} - P_i - 2D\omega_o\,(\dot{z}_i + \tfrac{1}{2}\Delta t\,\ddot{z}_i) \\ &\quad - \omega_o^2\,\tfrac{k_{ti}}{k}\,(z_i + \Delta t\,\dot{z}_i + \tfrac{1}{4}\Delta t^2\,\ddot{z}_i)\end{aligned} \qquad (4)$$

$k_{ti}$ is the tangent stiffness at time $i\Delta t$

$$R_{i+1} = \omega_o^2\,\ddot{u}_G\,(t_{i+1})/\ddot{u}_{max} \qquad (5)$$

and

$$P_i = \omega_o^2\,(f_i - \frac{k_{ti}}{k}\,z_i) \qquad (6)$$

It should be noticed that the maximum value of the variable z over time is the required ductility. For a given natural frequency $\omega_o$ and damping D one can perform the above solution for different values of $F_y$ or scaling the accelerogram $\ddot{u}_G(t)$ by different scaling factors. One can thus obtain the variation of the required ductility $z_{max}$ with $F_y$ or $\ddot{u}_{max}$. To obtain the value of $\ddot{u}_{max}$ corresponding to a specified ductility $\mu$ one can then interpolate between the values in this curve.

When including the effects of soil structure interaction the equations of motion are

$$M\ddot{U} + C\dot{U} + KU = -R \qquad (7)$$

where

$$M = \begin{bmatrix} m & m & mh \\ m & m & mh \\ mh & mh & mh^2 \end{bmatrix} \qquad C = \begin{bmatrix} c & 0 & 0 \\ 0 & c_x & 0 \\ 0 & 0 & c_\phi \end{bmatrix}$$

$$K = \begin{bmatrix} k & 0 & 0 \\ 0 & k_x & 0 \\ 0 & 0 & k_\phi \end{bmatrix} \qquad R = \begin{Bmatrix} m \\ m \\ mh \end{Bmatrix}\ddot{u}_G$$

and $$U = \begin{Bmatrix} y \\ u_o \\ \phi \end{Bmatrix} \qquad (8)$$

Writing these expressions in dimensionless form, using again the constant average acceleration as the integration procedure, and defining

$D = \dfrac{c}{2m\omega_o}$ structural damping

$D_x = \dfrac{c_x}{2m\omega_x}$ damping in swaying

$D_\phi = \dfrac{c_\phi}{2mh^2\omega_\phi}$ damping in rocking

$$\omega^2{}_o = \frac{k}{m} \qquad \omega^2{}_x = \frac{k_x}{m} \qquad \omega^2{}_\phi = \frac{k_\phi}{mh^2} \qquad (9)$$

where $k_x$ $k_\phi$ are horizontal and a rocking spring representing the foundation stiffnesses, $c_x$ $c_\phi$ are corresponding dashpots and h is the height of the mass above the foundation.

$$\begin{aligned}(I + \tfrac{1}{2}\Delta t\,C + \tfrac{1}{4}\Delta t^2\,K^*_{ti})\,\ddot{U}^*_{i+1} &= -R^*_{i+1} - P_i - C^*(\dot{U}^*_i + \tfrac{1}{2}\Delta t\,\ddot{U}^*_i) \\ &\quad - K^*_{ti}\,(U^*_i + \Delta t\,\dot{U}^*_i + \tfrac{1}{4}\Delta t^2\,\ddot{U}^*_i)\end{aligned} \qquad (10)$$

with

$$U^* = \begin{Bmatrix} z \\ u_o/u_y \\ \phi/u_y \end{Bmatrix}$$

$$C^* = \begin{bmatrix} 2D\omega_o & 0 & 0 \\ 0 & 2D_x\omega_x & 0 \\ 0 & 0 & 2D_\phi\omega_\phi \end{bmatrix}$$

$$K^*ti = \begin{bmatrix} \omega_o^2 \frac{k_{ti}}{k} & 0 & 0 \\ 0 & \omega_x^2 & 0 \\ 0 & 0 & \omega_\phi^2 \end{bmatrix}$$

$$R^*_{i+1} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \omega_o^2 \frac{\ddot{u}_G\,(t_{i+1})}{\ddot{u}_{max}}$$

$$P_i = \omega_o^2 \begin{Bmatrix} f_i - \frac{k_{ti}}{k} z_i \\ 0 \\ 0 \end{Bmatrix} \qquad (11)$$

z represents again the relative deformation of thes tructural spring, $u_o$ is the horizontal displacement of the foundation and $\phi$ is the foundation's rotation.

## 3 RESULTS

The results for the general degrading and the cyclic degrading models were always very similar. These systems had the largest force requirements for a given ductility. The results for the Takeda model were normally intermediate between those of the elato-plastic system and the degrading models.

Figure 4 shows the inelastic response spectra for an elasto-plastic and a degrading single degree of freedom system with 5% initial (elastic) damping subjected to the accelerogram of the 1985 Mexico City earthquake recorded at the University on firm ground. The spectra are shown for ductilities of 2 and 4. In both cases the differences in the results for the two systems are very small in agreement with the conclusions of Riddell and Newmark (1979) and Al Sulaimani (1982) from their studies with broad banded motions. Figure 5 shows the corresponding results for the SE component of the motion recorded at the SCT in the lake zone. The difference in the response of the 2 systems is very pronounced in this case particularly around 2 seconds for a ductility of 2 and 1.5 seconds for a ductility of 4. For this latter value of ductility the response acceleration of the elasto-plastic system never exceeds the peak ground acceleration whereas for the degrading system it reaches a peak (around 1.5 seconds) with an amplification of about 1.5.

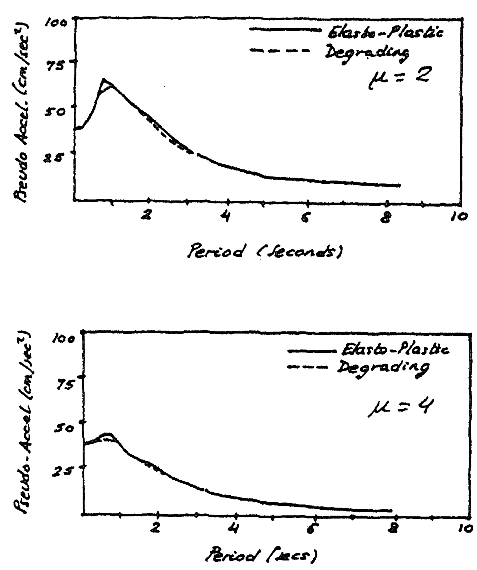


Figure 4. Inelastic response spectra on firm ground. Mexico City earthquake 1985.

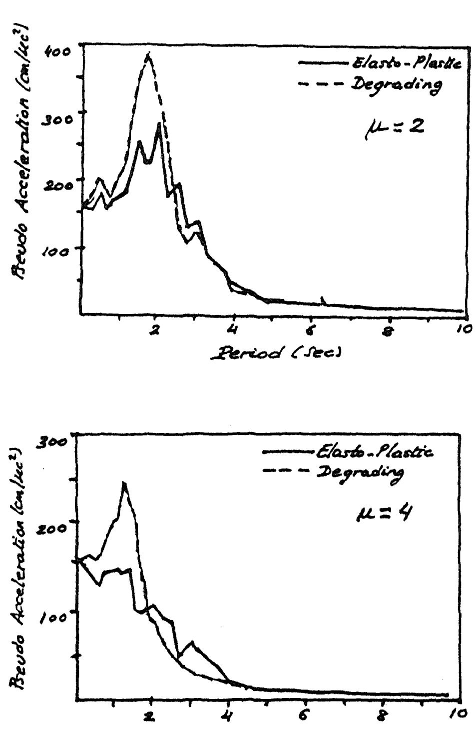


Figure 5. Inelastic response spectra on soft soil. Mexico City earthquake 1985.

These differences are even more pronounced when accounting for the flexibility of the foundation (soil structure interaction effects). Results for ductility ratios of 2, 4 and 6 are presented in Figure 6. The natural period is the initial (elastic) period of the structure on a rigid base and the initial damping of 5% applies to the same condition. In this case the response acceleration of the degrading system can be twice that of the elasto-plastic system over a certain range of natural periods.

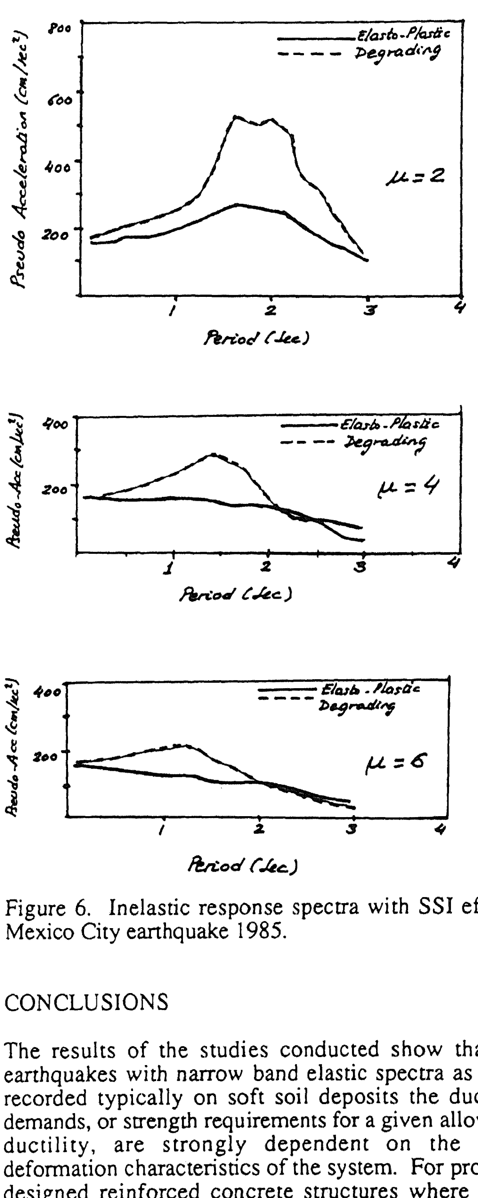


Figure 6. Inelastic response spectra with SSI effects. Mexico City earthquake 1985.

CONCLUSIONS

The results of the studies conducted show that for earthquakes with narrow band elastic spectra as those recorded typically on soft soil deposits the ductility demands, or strength requirements for a given allowable ductility, are strongly dependent on the force deformation characteristics of the system. For properly designed reinforced concrete structures where shear failure is avoided and the behavior is primarily flexural the Takeda model seems to be a more appropriate idealization than either the elasto-plastic or the degrading models. Figure 7 shows the inelastic spectra for a Takeda system. The results are intermediate between those of the other 2 models.

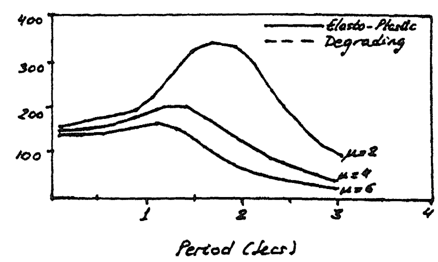


Figure 7. Inelastic response spectra Takeda model. Mexico City earthquake 1985.

Direct application of Newmark's rules starting from the elastic response spectrum (ductility of one) is not possible in this case. One can, however, apply Newmark's criteria reasonably well defining a fictitious equivalent elastic spectrum as done by Tarquis (1988). Constructing inelastic design spectra (or seismic design coefficients) for soft sites requires special considerations, including the type of nonlinear model most appropriate to reproduce the behavior of the structure, beyond the simple steps adopted in most model codes (extending the horizontal plateau over a longer range of periods).

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