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Energy input rate spectra of earthquake ground motions

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ABSTRACT: A technique of non-stationary spectral analysis termed "energy input rate spectrum" is proposed to visualize non-stationary changes of seismic damage potential contained in earthquake ground motions, and then applied to the two moderate earthquake motions, which actually induced different types of inelastic excursions in an identical steel braced frame model with intentionally reduced seismic strength.

1 INTRODUCTION

It is widely recognized that the amount of energy absorption is one of important factors in the seismic damage assessment of structural system and elements. Most of past research efforts have been made to evaluate the final amount of energy absorption or energy input exerted by earthquakes, and they have achieved energy-based design methods.(Akiyama, 1980) For more detailed understanding of the structural damage process, however, it is important to obtain not only the final values but also the dynamic change of the energy input, in other words, when, how long, how much, and at which frequencies, the potential to exert energy is contained in the ground motion. For such a purpose, non-stationary characteristics of seismic waves should be studied carefully. The goal of this paper is to propose a rigorous tool of spectral analysis for the evaluation of such a non-stationary damage potential.

2 ENERGY INPUT RATE SPECTRUM

To handle non-stationary signals, many researchers in various engineering fields have developed non-stationary spectral analyses suitable for their purposes: for example, Mark's physical spectrum, Wigner distribution, Page's instantaneous power spectrum, multifilter techniques, and so on. Among them, the multifilter spectrum (Kameda, 1975) is specifically related to the energy responses of linear oscillators subjected to a ground motion. Kameda's spectral value denoted by $S_K(t,\omega,h)$ indicates the sum of the kinetic energy and the potential energy which are held by the viscously damped linear oscillator with respect to its undamped natural circular frequency, ω.

On the other hand, a new spectrum termed energy input rate spectrum proposed here (abbreviated as "EIR spectrum") indicates the power of the effective excitation force acting on the viscously damped linear oscillator per unit mass. In other words, the EIR spectrum denoted by $S_E(t,\omega,h)$ indicates the energy input rate per unit mass exerted into the oscillator with respect to its undamped natural frequency. The difference between these two formulations is schematically illustrated in Fig. 1.

The EIR spectrum has close relationship also with Page's instantaneous power spectrum (Page, 1952). The recent study on the final value of the energy input was focused on interesting formulae to indicate the energy input in frequency-domain (Ohi, 1984). According to this study, the energy input to a certain vibrational system is obtained by integrating the Fourier square amplitudes of the ground acceleration together with the weighting function termed "energy admittance" as follows:

$$e_I = -\int_{-\infty}^{\infty} \dot{x}(\tau)\, \ddot{y}(\tau)\, d\tau \qquad (1)$$
(Time-domain integral)

$$e_I = \int_{-\infty}^{\infty} W(u)\, (|\ddot{Y}(u)|^2/2\pi)\, du \qquad (2)$$
(Frequency-domain integral)

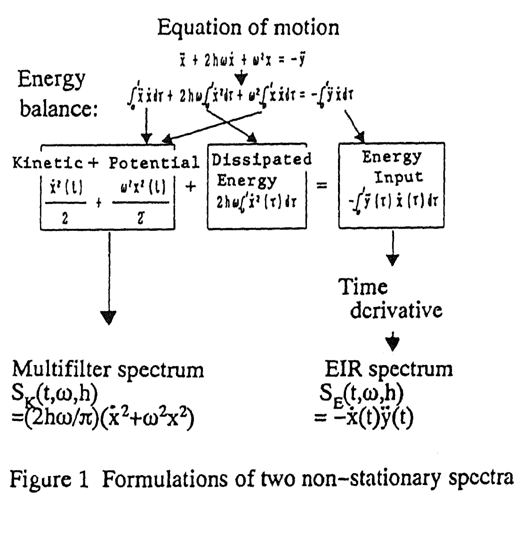


Figure 1 Formulations of two non-stationary spectra

where $W(u) = -\ \mathrm{Real}[H(u)]$ (3)
(Energy Admittance)

e_I : energy input per unit mass
$\dot{x}(\tau)$: response velocity
$\ddot{y}(\tau)$: ground acceleration
τ: time, u : circular frequency
$\ddot{Y}(u)$: Fourier transformation of $\ddot{y}(\tau)$
$H(u)$: transfer function to obtain velocity response from ground acceleration

It is straightforward to obtain the energy input up to the time, t:

$$e_I = \int_{-\infty}^{\infty} W(u)\, (|\ddot{Y}_R(t,u)|^2/2\pi)\, du \tag{4}$$

where $(|\ddot{Y}_R(t,u)|^2/2\pi)$ is the running Fourier square amplitude spectrum defined as the Fourier square amplitude spectrum of the ground acceleration truncated at the time, t.

Here we denote the undamped natural frequency of the oscillator by ω and the damping constant by h. By differentiating eq. (4) with respect to the time, t, we obtain:

$$S_E(t,\omega,h) = \int_{-\infty}^{\infty} W(u,\omega,h)\, S_P(t,u)\, du \tag{5}$$

where $S_P(t,u)$ is Page's instantaneous power spectrum (Page, 1952) and equal to the time derivative of the running Fourier square amplitude spectrum, that is, $S_P(t,u) = d(|\ddot{Y}_R(t,u)|^2/2\pi)/dt$.

Thus the EIR spectrum is interpreted as an instantaneous power spectrum smoothed along the frequency axis by the set of spectral windows which are identical to the set of energy admittances for viscously damped linear oscillators with various natural frequencies and the specified damping constant. Especially in the case of no damping, the EIR spectrum is identical to the Page's instantaneous power spectrum, since the following energy admittance function asymptotically becomes Dirac's delta function as the damping constant approaches to zero.

$$W(u,\omega,h) = \frac{2\, h\, \omega\, u^2}{(u^2-\omega^2)^2 + 4\, h^2\, \omega^2\, u^2} \tag{6}$$

$$\rightarrow (\pi/2)\ \{\delta(u-\omega) + \delta(u+\omega)\} \qquad (h \rightarrow 0) \tag{7}$$

this leads to:

$$S_E(t,\omega,h{=}0) = \pi\, S_P(t,\omega) \tag{8}$$

In the actual calculation of the EIR spectrum, time histories of velocity responses are calculated in time-domain for the set of viscously damped linear oscillators with various natural frequencies and the specified damping constant, and they are then multiplied by the ground acceleration history with negative sign:

$$S_E(t,\omega,h) = -\ \dot{x}(t,\omega,h)\ \ddot{y}(t) \tag{9}$$

The EIR spectrum sometimes includes negative spectral values as the Page's instantaneous spectrum does. The physical meaning of such negative values is that the response velocity of the oscillator opposes in the direction of the effective excitation force, and this phenomenon is not strange at all in the deterministic earthquake responses. If a number of sample waves were available, and if ensemble average could be applied to these sample waves, such negative spectral values would disappear. When only one sample wave is given, moving time average techniques can be used to reduce such negative spectral values if desired. In the following examples, the smaller value of three times natural period and 1.5 seconds is adopted for the data window length for the moving average.

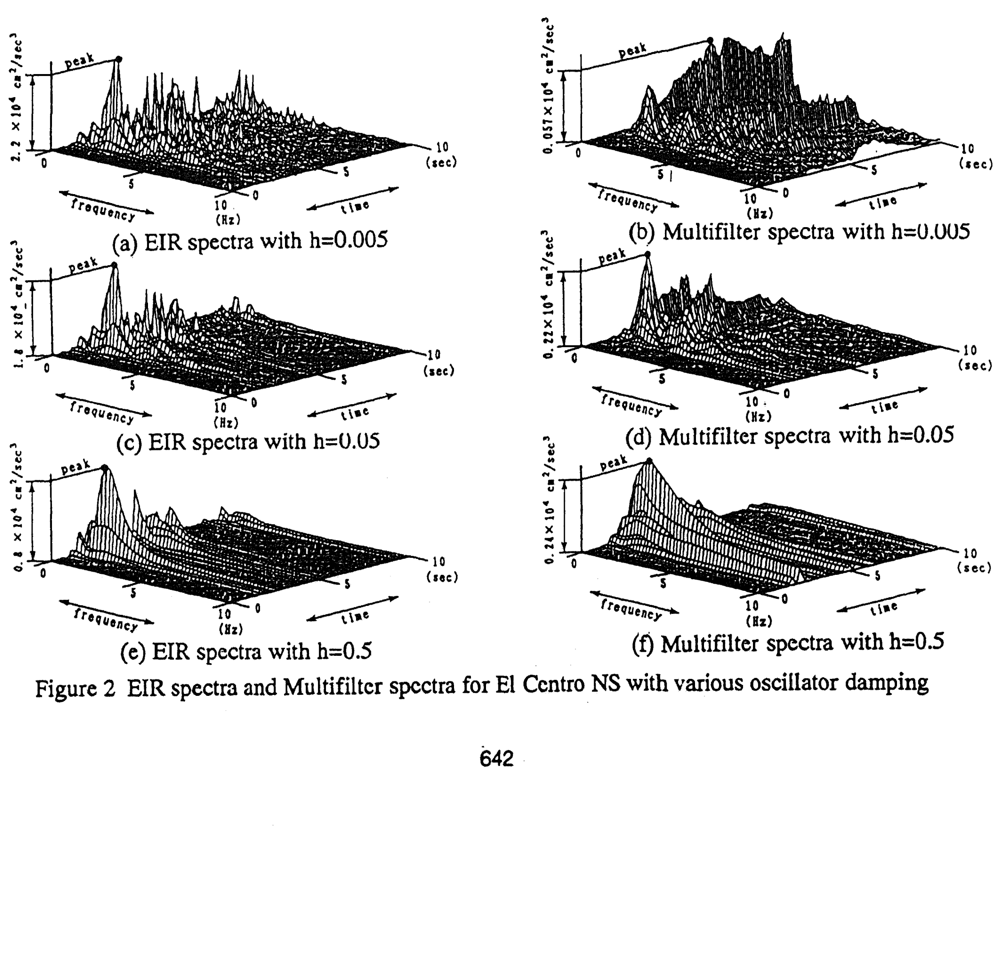


Figure 2 EIR spectra and Multifilter spectra for El Centro NS with various oscillator damping

3 EXAMPLES

3.1 Oscillator damping effects

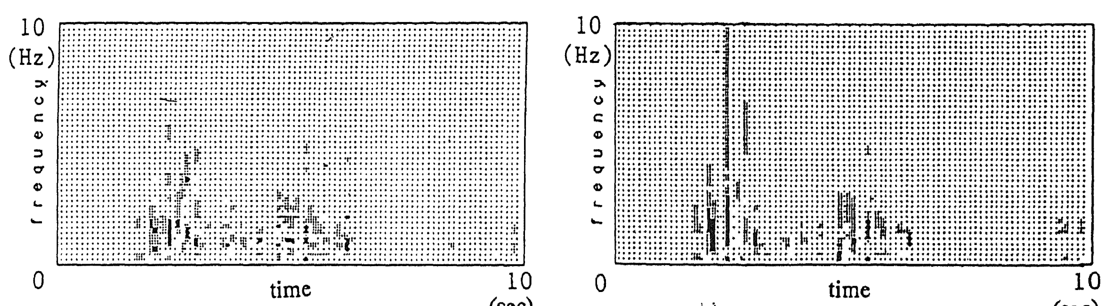
Figs. 2(a) to 2(f) show EIR spectra and Kameda's multifilter spectra calculated for the N-S component of El Centro 1940 with three damping constants. It is seen that the oscillator damping provides the Kameda's spectra with the smoothing effects along the frequency-coordinate and the time-lag effects as well, while such time-lag effects are not induced by the oscillator damping in the EIR spectra.

3.2 Non-linear oscillators

A non-linear oscillator with hysteresis can be used instead of the viscously damped linear oscillators if desired. The EIR spectrum shown in Fig. 3(b) is calculated for the El Centro NS 1940 with the elastic-plastic oscillators, the yield shear forces of which are set to 0.3 times elastic peak responses. The pattern of the damage potential is similar to the pattern in the linear-elastic case shown in Fig. 3(a), except that the damage potential near the first shock is increased for the high frequency region.

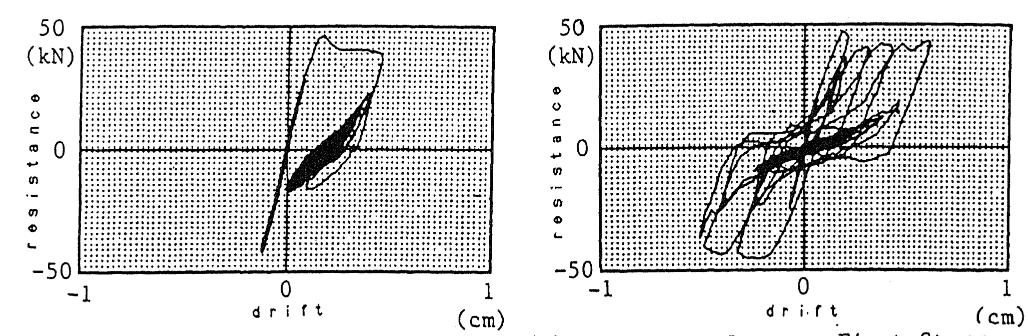
3.3 Two moderate earthquakes recorded at Chiba

A long-term observation project on a scaled steel braced frame model with intentionally reduced seismic strength is on-going since 1983 at Chiba, Japan (Ohi, 1989). Two moderate earthquakes observed on October 4, 1985 and June 24, 1986 caused almost the same peak acceleration of ground shaking, about 70cm /sec^2, but the hysteretic behaviors of the model were considerably different as shown in Figs. 4(a) and 4(b). To characterize these two events, the EIR spectra are calculated for the base acceleration records, as shown in Figs. 5 (a) and (b). It is seen that the 1986 event has long lasting damage potential enough to supply ample energy into the frame model, while the 1985 event has a pulse-like damage potential. The response velocity records are also chosen as the input signals for the EIR spectra shown in Figs. 5 (c) and (d), where the change of response frequency due to yielding can be well visualized by the same technique.



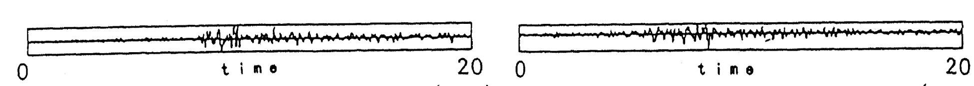
(a) Linear-elastic with h=0.05 (b) Elastic-plastic with h=0.05

Figure 3 Contours of EIR spectra with linear and non-linear oscillators



(a) Hysteresis Loop at First Story (Oct. 4, 1985)

(b) Hysteresis Loop at First Story (June 24, 1986)



(c) Base Acceleration (Oct. 4, 1985; Peak:71cm/s/s)

(d) Base Acceleration (June 24, 1986; Peak: 74cm/s/s)

Figure 4 Inelastic responses of steel braced frame observed during two moderate earthquakes

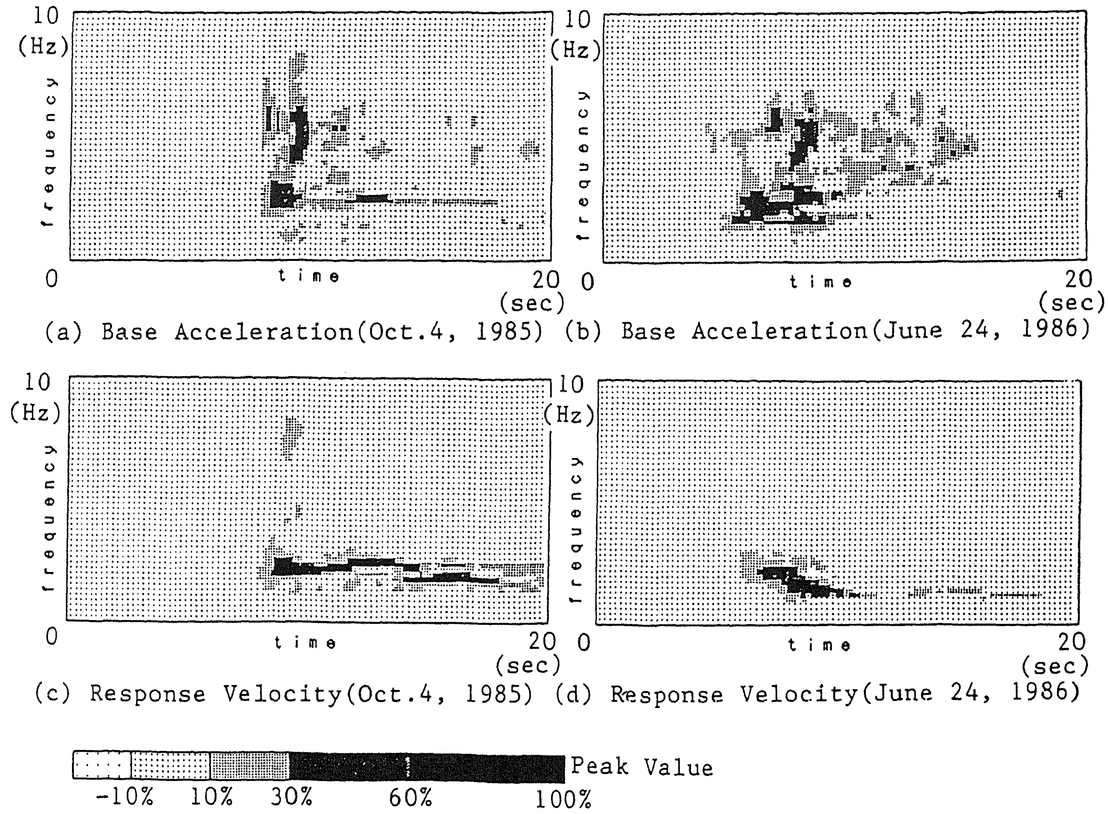


Figure 5 Contours of EIR spectra for the base acceleration and the response velocity records

4 CONCLUDING REMARKS

A new concept of non-stationary spectral analysis termed "energy input rate spectrum (EIR spectrum)" is introduced to express the rate of the energy input fed by the earthquake ground motion. Features of this spectrum are demonstrated theoretically and by several examples as well:

(1) EIR spectrum is mathematically equivalent to the Page's instantaneous power spectrum smoothed along the frequency axis.

(2) Fourier transformation techniques are not needed to calculate the EIR spectrum. Velocity responses of viscously damped linear oscillators are calculated in time-domain and multiplied by the original signal with negative sign. Non-linear oscillator can be used if desired.

(3) Damping constant in the EIR spectrum influences only on the smoothness along the frequency axis, and does not induce the time-lag effects.

This spectrum is suitable to visualize the frequency-time distribution of the non-stationary seismic damage potential, when it is applied to the ground acceleration record. Furthermore, this spectrum has the above-mentioned useful features as a non-stationary spectrum, and it can be generally applied also to other non-stationary signals, such as structural response records.

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