Results of observation of torsional ground motions and response analysis

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ABSTRACT: The purpose of this paper is to construct torsional spectra due to the rotational component of seismic ground motions. An average and a mean plus one standard deviation torsional spectrum for Taiwan area is presented for design. The effect of a rotational motion on a symmetrical single story building is also studied. It is shown that the rotational component of excitation may have a very significant effect on the response. Dynamic coupling which causes a translational ground motion to excite the torsional mode are also studied.

1 INTRODUCTION

Torsional motions of a structure caused by earthquakes may lead to severe damages. The causes of torsional motion are generally ascribed to asymmetries of the building. Recently, it has been pointed out that symmetrical buildings may be subjected to torsional motion if the phases of the seismic wave at different points of the ground on which the structure is build are different. This spatial variation of ground motion cause the torsional excitation of structure. It is believed that the travelling wave approach is the practical method to study the ground-motion induced torsional effects (Tao 1975). Since the establishment of strong-motion earthquake instrument array, more precise information on the spatial characteristics of the ground motion has been acquired, in particular, a measurement of the apparent horizontal velocities of propagation. The rotational motion of the ground during an earthquake can be studied from the array data.

The primary purpose of this paper is to review the travelling wave assumption on the estimation of ground rotation during earthquake. It begins the treatment of the subject by presenting a detail review of previous work on the effects of torsional ground motion and emphasizing the methods which have been proposed to develop torsional response spectra. The effect of rotational motion on the response of a symmetric single story building model is also studied. The influence of the ratio of the rotational motion frequency to the translational motion frequency on the structural response is also presented and discussed.

2 TORSIONAL SPECTRUM FOR SEISMIC MOTIONS

The equation of motion for a SDOF oscillator excited by torsional ground motion \( \theta(t) \) is given by

\[
\ddot{\theta}(t) + 2\xi \omega_n \dot{\theta} + \omega_n^2 \theta = -\alpha \omega_n \dot{\theta}(t)
\]

in which \( \beta \) = angle of twist relative to ground rotation, \( \omega_n \) = torsional frequency of the oscillator, \( \xi \) = damping ratio. The torsional displacement spectrum is obtained as

\[
R_\theta = |\theta(t)|_{max}
\]

The torsional pseudo-velocity spectrum \( R_\nu \) and acceleration spectrum \( R_a \) are then defined by

\[
R_\nu = \omega_n^2 R_\theta, \quad R_a = \omega_n^4 R_\theta
\]

When a rotational time history of a massless footing on a Winkler foundation is derived from a uni-directional horizontal displacement record, assuming horizontally propagating shear waves, it can be shown that (Rutenberg and Heiderbrecht 1985)

\[
\theta(t) = \frac{1}{\alpha_n C_h} \xi_n(t) = \frac{1}{C_h^2} \xi_n(t)
\]

in which \( \xi_n(t) \) = translational displacement time history, \( C_h \) = apparent, horizontal shear wave velocity or phase velocity, and \( \alpha_n \) is a shape factor given by \( \alpha_n = (I_s + I_r)/I_r \), the ratio of moment of inertia. As the code-modified shape is usually denoted, the torsional spectral acceleration can be derived from its translational counterpart by factoring through \( 2\pi/(\alpha_n C_h T_s) \) or

\[
R_a = \frac{2\pi}{\alpha_n C_h T_s} S_a
\]

in which \( T_s \) = natural period of the torsional oscill-
The figure compares the calculated and proposed torsional spectrum for Taiwan area, showing a significant difference. The proposed system is designed to improve upon the existing spectrum, particularly at higher frequencies. The figure also illustrates the ratio of torsional spectrum amplitude to translational spectrum amplitude, highlighting the amplification of torsional motions relative to translation. The comparison in Figure 3 indicates that the proposed system offers a more balanced response, especially at shorter periods.

3 Dynamic Response of Building to Ground Rotational Motion

Consider a symmetrical structure consisting of a stiff rectangular platform with massless axially rigid columns and walls, and with a total mass m and its radius of gyration r, as shown in Figure 4. This system has three degrees-of-freedom in movement, namely, the horizontal lateral displacement Ux and Uy in the two principal directions and rotation Uz about a vertical axis. If the translational stiffness of the system along the x and y axes are, respectively, Kx and Ky, the torsional stiffness about the z-axis is Kt, and if the center of resistance of the system is defined by the static eccentricities ex and ey measured from the center of the mass, then the equations of motion are given by (Awad and Humar 1984):
\[
\begin{bmatrix}
\ddot{U}_y \\
r \ddot{U}_y \\
\ddot{U}_x \\
\end{bmatrix} =
\begin{bmatrix}
\omega_z^2 & -\gamma \omega_y^2 & 0 \\
-\gamma \omega_y^2 & \omega_y^2 & \gamma \omega_y^2 \\
0 & \gamma \omega_y^2 & \omega_y^2 \\
\end{bmatrix}
\begin{bmatrix}
\dot{U}_y \\
r \dot{U}_y \\
\dot{U}_x \\
\end{bmatrix} \\
\begin{bmatrix}
P_\gamma(t) \\
\end{bmatrix}
\]

where \(\omega_z, \omega_y\) are the uncoupled translational frequencies, \(\omega_z = \sqrt{K_z/m}\) is the uncoupled translational frequency, \(\omega_y = \sqrt{K_y/m}\) is the uncoupled torsional frequency, \(\ddot{U}_x\) and \(\ddot{U}_y\) are the ground accelerations in the \(x\) and \(y\) directions respectively, and \(\ddot{U}_\gamma\) is the torsional acceleration of the ground. The natural frequencies, \(\omega_z, \omega_y\) and mode shapes, \(\{\alpha_n\}\), of the system can be solved with the rigid-body side of Eq. (7) equal to zero. The modal equations are obtained by introducing the transformation,

\[
\begin{bmatrix}
\ddot{U}_y \\
r \ddot{U}_y \\
\ddot{U}_x \\
\end{bmatrix} =
\sum_{n=1}^{3}
\begin{bmatrix}
\alpha_{n} \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
Y_n(t) \alpha_{n} \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\begin{bmatrix}
\alpha_{n} \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\]

where \(Y_n(t)\) are the modal coordinates. The resulting equations are

\[
\ddot{Y}_n(t) + 2\zeta_n \omega_n Y_n(t) + \omega_n^2 Y_n(t) = P_n(t) / M_n \\
\]

where \(\zeta_n = \alpha_n^T [M]^{-1} [\alpha_n]; (M) = m[I]^{-1}\) and \(P_n(t) = -\alpha_n^T P(t)\). Note that the damping is introduced in this modal equation through the damping ratio \(\zeta_n\) for \(n\)-th mode. For ground excitation in the \(z\)-direction only,

\[
P_z(t) = -m \alpha_n \ddot{U}_\zeta(t) \\
\]

Using response spectrum, the absolute maximum value of the modal coordinate for the \(n\)-th mode is given by

\[
Y_n(t) = \frac{2m \alpha_n S_{\gamma}(\omega_n, \zeta_n)}{\omega_n^2} \\
\]

in which \(S_{\gamma}(\omega_n, \zeta_n)\) is the translation spectral acceleration.

For a ground motion in the \(z\)-direction only, the modal maximum of the base shears and the base torque are given by

\[
\begin{bmatrix}
V_{x_n}^z \\
V_{y_n}^z \\
T_z^z \\
\end{bmatrix} =
\begin{bmatrix}
\alpha_{n} \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\begin{bmatrix}
2m \omega_n^2 \\
r \alpha_{n} \\
\alpha_{n} \\
\end{bmatrix} Y_n(t) \omega_n^2 \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\]

In a similar manner, for a rotational ground motion about the \(z\)-axis the modal maxima are

\[
\begin{bmatrix}
V_{x_n}^z \\
V_{y_n}^z \\
T_z^\gamma \\
\end{bmatrix} =
\begin{bmatrix}
\alpha_{n} \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\begin{bmatrix}
r \omega_n \omega_n \\
r \alpha_{n} \\
\alpha_{n} \\
\end{bmatrix} S_{\gamma}(\omega_n, \zeta_n) \\
\alpha_{n} \\
\alpha_{n} \\
\end{bmatrix}
\]

in which \(S_{\gamma}(\omega_n, \zeta_n)\) = the spectral rotational acceleration. For purposes of numerical analysis, a symmetrical building model (30m x 30m) will be considered in next section.
 Fig. 6 Response of one-way torsional coupled system to true acceleration spectrum (shown in Fig. 5) as a function of uncoupled frequency ratio \( \omega_y / \omega_x \) for different values of plane eccentricity \( e_x / b \); (a) consider the base shear per unit mass as a function of frequency ratio, (b) consider the base torque as a function of frequency ratio.

Fig. 7 Response of one-way torsional coupled system to true torsional spectrum (calculate by wave travelling approach) as a function of uncoupled frequency ratio for different plane eccentricity \( e_x / b \); (a) consider base shear per unit mass as a function of frequency ratio, (b) consider the base torque as a function of frequency ratio.

4 NUMERICAL EVALUATION OF TORSIONAL RESPONSES

Consider a symmetrical building with dimension 30 m x 30 m in plan view. The data of SMART-1 of event-39 at station C00 is chosen as horizontal excitation \( a_x(t) \) of which the spectral acceleration is shown in Fig. 5 and its corresponding response of one way torsionally coupled system is examined. For purpose of comparison, plots of base shear and base torque vs. frequency ratio, \( \omega_y / \omega_x \), for different value of plan eccentricity are depicted in Figs. 6a, 6b and 7a, 7b. Figures 6a and 6b consider the horizontal acceleration spectrum \( S_{ax}(\omega) \) only, which Figures 7a and 7b only consider the torsional spectrum \( S_{ax}(\omega) \). It is found that \( V_x^* \) is much greater than \( V_x^* \) for such a symmetric building. As expected, value of plan eccentricity will induce greater value of base torque even one-way excitation is considered. The simple relationship between the translational and rotational spectra obtained in this section can be shown by defining the nondimensional eccentricity \( \varepsilon' \):

\[
\varepsilon' = \frac{\varepsilon}{\beta} = \frac{M_y}{V_x^*}
\]

in which \( \beta = \frac{l}{V_x} \) is the spectral torque and spectral base shear respectively. Figure 8 shows the relationship of \( \varepsilon' \) vs. \( \omega_y / \omega_x \). It can be visualized that the accidental eccentricity for symmetric building is quite obvious when \( \omega_y / \omega_x \approx 1.0 \).

5. CONCLUSIONS

In the absence of direct field measurements on torsional motion, the travelling wave hypothesis is
Fig. 8 Plot of eccentricity, $e = M_0/Vh$, as a function of uncouple frequency ratio for different values of plan eccentricity subject to true translational spectral acceleration only.

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REFERENCES


