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Results of observation of torsional ground motions and response analysis

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ABSTRACT: The purpose of this paper is to construct torsional spectra due to the rotational component of seismic ground motions. An average and a mean plus one standard deviation torsional spectrum for Taiwan area is presented for design. The effect of a rotational motion on a symmetric single stroy building is also studied. It is shown that the rotational component of excitation may have a very significant effect on the response. Dynamic coupling which causes a translational ground motion to excite the torsional mode are also studied.

1 INTRODUCTION

Torsional motions of a structure caused by earthquakes may lead to severe damages. The causes of torsional motion are generally ascribed to asymmetrics of the building. Recently, it has been pointed out that symmetrical buildings may be subjected to torsional motion if the phases of the seismic wave at different points of the ground on which the structure is build are different. This spatial variation of ground motion cause the torsional excitation of structure. It is believed that the travelling wave approach is the practical method to study the ground-motion induced torsional effects (Tso 1975). Since the establishment of strong-motion earthquake instrument array, more precise information on the spatial characteristics of the ground motion has been acquired, in particular, a measurement of the apparent horizontal velocities of propagation. The rotational motion of the ground during an earthquake can be studied from the array data.

The primary purpose of this paper is to review the travelling wave assumption on the estimation of ground rotation during earthquake. It begins the treatment of the subject by presenting a detail review of previous work on the effects of torsional ground motion and emphasizing the methods which have been proposed to develop torsional response spectra. The effect of rotational motion on the response of a symmetric single story building model is also studied. The influence of the ratio of the rotational motion frequency to the translational motion frequency on the structural response is also presented and discussed.

2 TORSIONAL SPECTRUM FOR SEISMIC MOTIONS

The equation of motion for a SDOF oscillator excited by torsional ground motion $\theta_g(t)$ is given by

$$\ddot{\theta}(t) + 2\xi\omega_0\,\dot{\theta} + \omega_0^2\,\theta = -\ddot{\theta}_g(t) \tag{1}$$

in which θ = angle of twist relative to ground rotation. ω_0 = torsional frequency of the oscillator, ξ = damping ratio. The torsional displacement spectrum is obtained as

$$R_d = |\theta(t)|_{max} \tag{2}$$

The torsional pseudo-velocity spectrum R_v and acceleration spectrum R_a are then defined by

$$R_v = \omega_0\,R_d \quad , \quad R_a = \omega_0^2\,R_d \tag{3}$$

When a rotational time history of a massless footing on a Winkler foundation is derived from a unidirectional horizontal displacement record, assuming horizontally propagating shear waves, it can be shown that [Rutenberg and Heiderbrecht 1985]

$$\theta_g(t) = \frac{1}{\alpha_x\,C_h}\,\dot{u}_g(t) = \frac{1}{C^*}\,\dot{u}_g(t) \tag{4}$$

in which $u_g(t)$ = translational displacement time history, C_h = apparent, horizontal shear wave velocity or phase velocity, and α_x is a shape factor given by $\alpha_x = (I_x + I_y)/I_y$, the ratio of moment of inertia. As the code-modified shape is usually denoted, the torsional spectral acceleration can be derived from its translational counterpart by factoring through $2\pi/(\alpha C_h T_\theta)$; or

$$R_a = \frac{2\pi}{\alpha C_h T_\theta}\,S_a \tag{5}$$

in which T_θ = natural period of the torsional oscil-

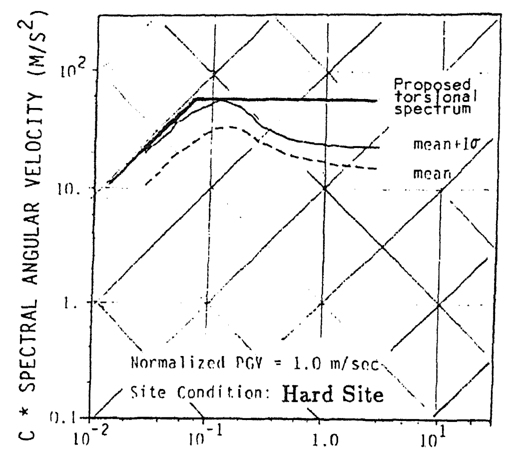

Fig. 1 Comparison between the calculated and proposed torsional spectrum for Taiwan area.

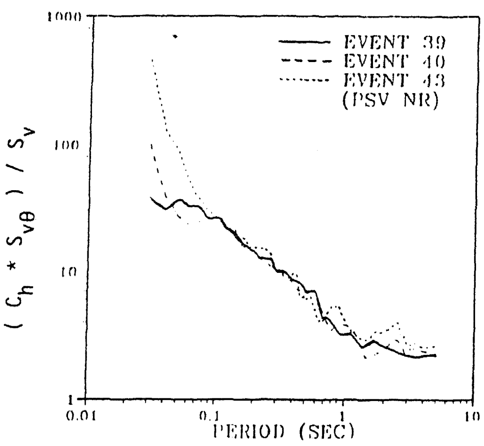

Fig. 2 Ratio of torsional spectrum amplitude to translational spectrum amplitude.

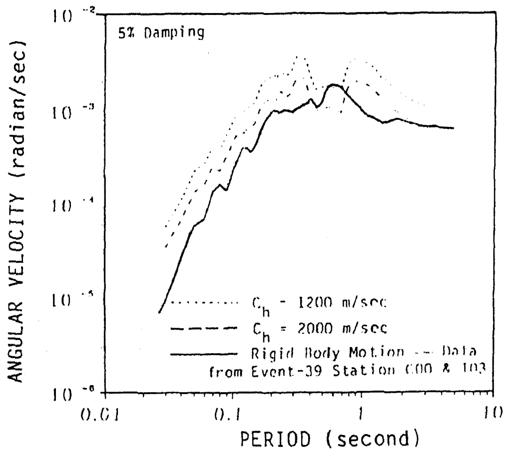

Fig. 3 Comparison on the torsional spectrum by two different method; travelling wave approach and rigid body motion of two points.

lator and S_a = spectral acceleration. Equation (5) also indicates that the ratio of torsional to translational spectral acceleration is high for the high frequency and almost decreases with decreasing frequency linearly.

For the purpose of computing the torsional spectrum, it is assumed that the ground motions at the site are truely the result of a planar shear wave propagating along the X-direction, then one can construct the torsional spectrum curve which is normalized for peak horizontal ground velocity $v = 1\ m/sec$. In Fig. 1, we depict the proposed design torsional spectrum and the mean and mean-plus-one-standard-deviation spectrum curves which are calculated based on the data from the hard site of SMA-1 Network in Taiwan. Figure 2 shows the ratio of torsional to translational spectral velocity for different periods (5% damping ratio). As indicated, the ratio is high at the high frequency end and decreases almost linearly to the low frequency end. This result is consistent with the analytical result of the ratio of the amplitude ψ_{13} to U_3, torsional component associated with the SH-wave and the amplitude of incident SH-wave, which can be written as [Lee and Trifunac 1985]

$$\left|\frac{\psi_{13}}{U_3}\right| = \frac{1}{2}\frac{\omega}{C_h} \tag{6}$$

Figure 3 shows the comparison of torsional response spectra by two different approaches. One is based on the travelling wave approach (specify apparent wave velocity) and the other is obtained from the dense array observation by estimating the rotation from horizontal motion of two stations. roughly estimate the shape of the torsional spectrum without loss of generality. However, the estimation of apparent shear wave velocity is the key parameter to the amplitude of torsional spectrum.

3 DYNAMIC RESPONSE OF BUILDING TO GROUND ROTATIONAL MOTION

Consider a symmetrical structure consisting of a stiff rectangular platform with massless axially rigid columns and walls, and with a total mass m and its radius of gyration r, as shown in Figure 4. This system has three degrees-of-freedom in movement, namely, the horizontal lateral displacement U_x and U_y in the two principal directions and rotation U_θ about a vertical axis. If the translational stiffness of the system along the x and y axes are, respectively, K_x and K_y, the torsional stiffness about the z-axis is K_θ, and if the center of resistance of the system is defined by the static eccentricities e_x and e_y measured from the center of the mass, then the equations of motion are given by (Awad and Humar 1984):

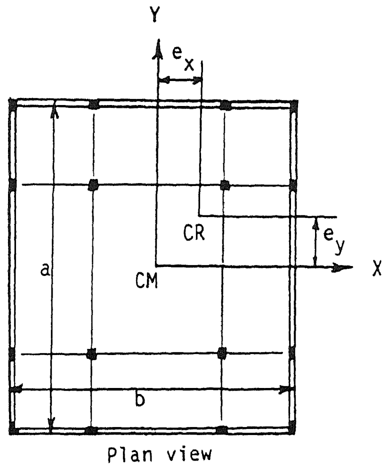

Fig. 4 Plan view of one story building shows the mass center and the center of stiffness.

$$\begin{bmatrix} \ddot{U}_x \\ r\ddot{U}_\theta \\ \ddot{U}_y \end{bmatrix} + \begin{bmatrix} \omega_x^2 & -\frac{e_y}{r}\omega_x^2 & 0 \\ -\frac{e_y}{r}\omega_x^2 & \omega_\theta^2 & \frac{e_x}{r}\omega_y^2 \\ 0 & \frac{e_x}{r}\omega_y^2 & \omega_y^2 \end{bmatrix} \begin{bmatrix} U_x \\ rU_\theta \\ U_y \end{bmatrix}$$

$$= - \begin{bmatrix} \ddot{U}_{gx} \\ r\ddot{U}_{g\theta} \\ \ddot{U}_{gy} \end{bmatrix} \qquad (7)$$

where $\omega_x = \sqrt{K_x/m}$ and $\omega_y = \sqrt{K_y/m}$ are the uncoupled translational frequencies, $\omega_\theta = \sqrt{K_\theta/mr^2}$ is the uncoupled torsional frequency, $\ddot{U}_{gx}$ and $\ddot{U}_{gy}$ are the ground accelerations in the x and y directions respectively, and $\ddot{U}_{g\theta}$ is the torsional acceleration of the ground. The natural frequencies, ω_n and mode shapes, $\{\alpha_n\}$, of the system can be solved with the rigid-hand side of Eq. (7) equal to zero. The modal equations are obtained by introducing the following transformation,

$$\begin{bmatrix} U_x \\ rU_\theta \\ U_y \end{bmatrix} = \sum_{n=1}^{3} Y_n(t) \begin{bmatrix} \alpha_{xn} \\ \alpha_{\theta n} \\ \alpha_{yn} \end{bmatrix} = \sum_{n=1}^{3} Y_n(t)\{\alpha_n\} \quad (8)$$

where $Y_n(t)$ are the modal coordinates. The resulting equations are

$$\ddot{Y}_n(t) + 2\xi_n \omega_n \dot{Y}_n(t) + \omega_n^2 Y_n(t) = P_n(t)/M_n$$

$$n = 1, 2, 3 \qquad (9)$$

in which $M_n = \{\alpha_n\}^T[M]\{\alpha_n\}$; $[M] = m[I]$; and $P_n(t) = -\{\alpha_n\}^T P(t)$. Note that the damping is introduced in this modal equation through the damping ratio ξ_n for n-th mode. For ground excitation in the x-direction only,

$$P_n(t) = -m\, \alpha_{xn}\, \ddot{U}_{gx} \qquad (10)$$

Using response spectrum, the absolute maximum value of the modal coordinate for the n-th mode is given by

$$Y_n(t) = \frac{\alpha_{xn}}{\omega_n^2} S_{ax}(\omega_n, \xi_n) \qquad (11)$$

in which $S_{ax}(\omega_n, \xi_n)$ is the translation spectral acceleration.

For a ground motion in the x-direction only, the modal maximum of the base shears and the base torque are given by

$$\begin{bmatrix} V_{xn}^x \\ T_n^x \\ V_{yn}^x \end{bmatrix} = m\omega_n^2 \begin{bmatrix} \alpha_{xn} \\ r\alpha_{\theta n} \\ \alpha_{xn} \end{bmatrix} Y_n(t)_{max}$$

$$= m\, S_{ax}(\omega_n, \xi) \begin{bmatrix} \alpha_{xn}^2 \\ r\alpha_{xn}\alpha_{\theta n} \\ \alpha_{xn}\alpha_{yn} \end{bmatrix} \qquad (12)$$

In a similar manner, for a rotational ground motion about the z-axis the modal maxima are

$$\begin{bmatrix} V_{xn}^\theta \\ T_n^\theta \\ V_{yn}^\theta \end{bmatrix} = m\, S_{a\theta}(\omega_n, \xi_n) \begin{bmatrix} r\alpha_{xn}\alpha_{\theta n} \\ r^2\alpha_{\theta n}^2 \\ r\alpha_{yn}\alpha_{\theta n} \end{bmatrix} \qquad (13)$$

in which $S_{a\theta}(\omega_n, \xi_n)$ = the spectral rotational acceleration. For purposes of numerical analysis, a symmetric building model ($30m \times 30m$) will be considered in next section.

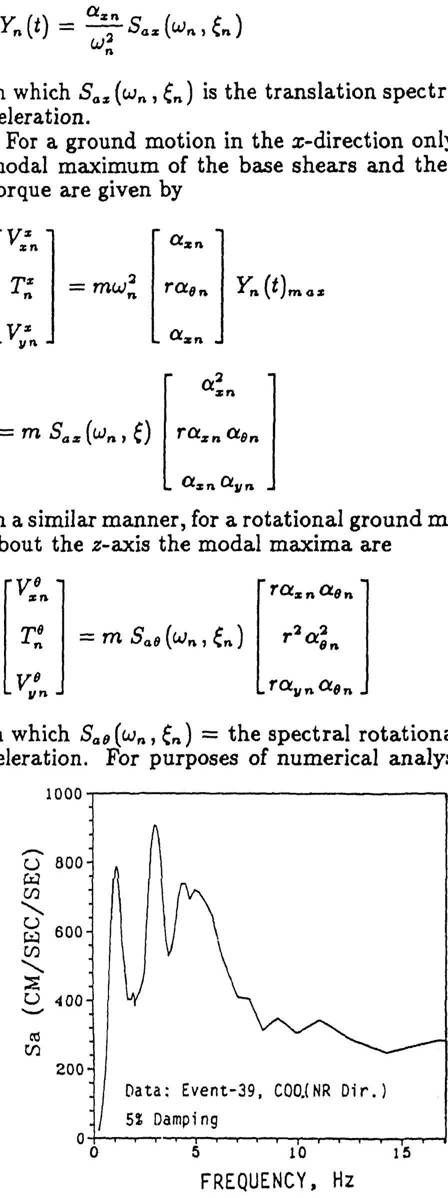

Fig. 5 Acceleration response spectrum of Jan. 16, 1986 earthquake at station C00 of SMART-1 array (5% damping ratio).

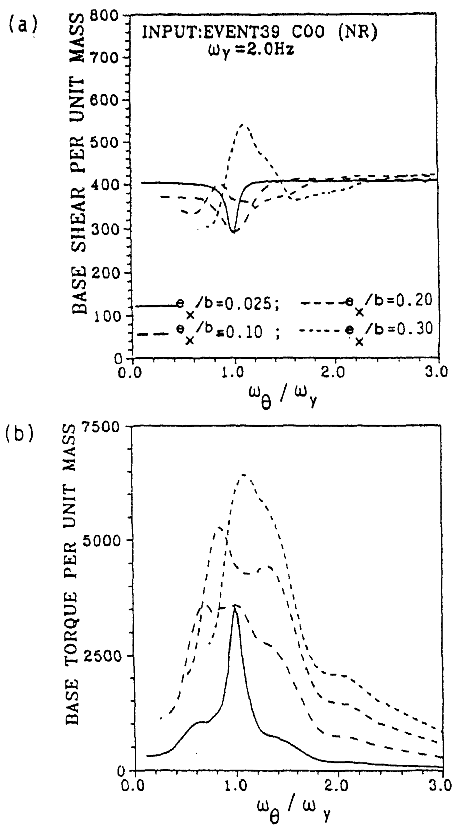

Fig. 6 Response of one-way torsional coupled system to true acceleration spectrum (shown in Fig. 5) as a function of uncoupled frequency ratio ω_θ/ω_y for different value of plane eccentricity e_x/b; (a) consider the base shear per unit mass as a function of frequency ratio, (b) consider the base torque as a function of frequency ratio.

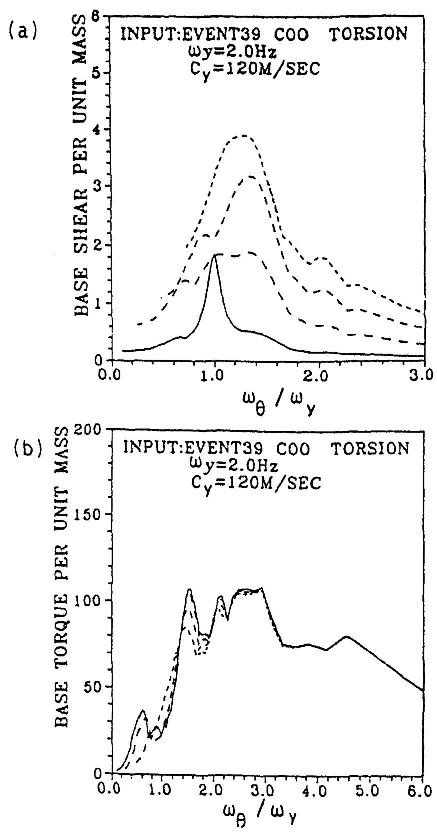

Fig. 7 Response of one-way torsional coupled system to true torsional spectrum (calculate by wave travelling approach) as a function of uncoupled frequency ratio for different plan eccentricity e_x/b; (a) consider base shear per unit mass as a function of frequency ratio, (b) consider the base torque as a function of frequency ratio.

4 NUMERICAL EVALUATION OF TORSIONAL RESPONSES

Consider a symmetrical building with dimension $30\ m \times 30\ m$ in plan view. The data of SMART-1 of event-39 at station C00 is chosen as horizontal excitation $\ddot{u}_{gx}(t)$ of which the spectral acceleration is shown in Fig. 5 and its corresponding response of one way torsionally coupled system is examined. For purpose of comparison, plots of base shear and base torque vs. frequency ratio, ω_θ/ω_x, for different value of plan eccentricity are depicted in Figs. 6a, 6b and 7a, 7b. Figures 6a and 6b consider the horizontal acceleration spectrum $S_{ax}(\omega)$ only, which Figures 7a and 7b only consider the torsional spectrum $S_{a\theta}(\omega)$. It is found that V_{xn}^x is much greater than $V_{\theta n}^x$ for such a symmetric building. As expected, value of plan eccentricity will induce greater value of base torque even one-way excitation is considered. The simple relationship between the translational and rotational spectra obtained in this section can be shown by defining the nondimensional eccentricity e^*:

$$e^* = \frac{e}{l} = \frac{M_\theta}{l\, V_h} \tag{14}$$

in which l = length of building perpendicular to the direction of the base shear, M_θ and V_h are the spectral torque and spectral base shear respectively. Figure 8 shows the relationship of "e" vs. (ω_θ/ω_y). It can be visualized that the accidental eccentricity for symmetric building is quite obvious when $\omega_\theta/\omega_y \doteq 1.0$.

5. CONCLUSIONS

In the absence of direct field measurements on torsional motion, the travelling wave hypothesis is

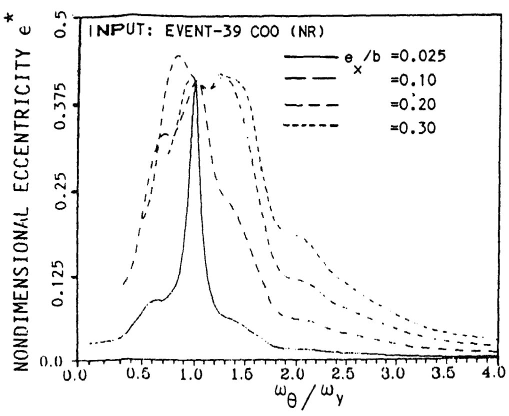

Fig. 8 Plot of eccentricity, $e = M_\theta / Vh$, as a function of uncouple frequency ratio for different value of plan eccentricity subject to true translational spectral acceleration only.

adopted to develop the torsional spectrum provided in Taiwan area. The estimation of the effective phase velocity of the seismic waves is directly related to the amplitude of torsional spectrum. Torsional motion in a building subjected to earthquake motion is also studied. It is shown that the most sensitive parameters to this torsional response are the eccentricity e_x or e_y and the ratio of the rotational frequency to the translational frequency, ω_θ / ω_y. Symmetric buildings may undergo significant torsional motion due to the presence of ground rotation, the geometry of the mass center and stiffness center, and the coupling effect of torsion and translation.

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