

## Relation between maximum amplitude ratio ( $a/v$ , $ad/v^2$ ) and spectral parameters of earthquake ground motion

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**ABSTRACT:** In this study, we investigate statistically the relation of the ratios ( $a/v$  and  $ad/v^2$ ) to the spectral characteristics and the duration of ground motions, in which  $a$ ,  $v$  and  $d$  are peak acceleration, peak velocity and peak displacement, respectively. The dependence of the  $a/v$  ratio on the magnitude, the distance and the predominant period at the sites is also discussed. It is found from the analysis that the  $a/v$  ratio is excellent parameter representing the spectral characteristics and the duration of earthquake ground motions and that the  $a/v$  ratio depends on the earthquake magnitude, the epicentral distance and the predominant period at the sites.

### 1. INTRODUCTION

One of the most important factors in the earthquake resistant design of structures is the proper representation of the characteristics of earthquake ground motions. The characteristics of earthquake ground motions are roughly classified intensities, frequency contents and nonstationarities. Among those, the intensities represented by peak ground motions are the fundamentals in earthquake resistant design and are used for normalizing the traditional response spectrum. On the other hand, the frequency contents and the nonstationarities of ground motions are also important, because they give the serious effect on the responses of long-period structures and inelastic structures. In order to perform the rational aseismic design of structures, it is required that the parameters representing properly these characteristics are established.

From such a point of view, the peak ground motions ratios  $a/v$  and  $ad/v^2$ , in which  $a$  is the peak acceleration,  $v$  the peak velocity and  $d$  the peak displacement, have been paid attention and they have been taken into account for the classification of earthquake ground motions and for the normalization of response spectra. Newmark et al. (1973) noted the significance of the  $a/v$  ratio and indicated that the ratio for the rock site is different from that for the alluvium site. Seed et al. (1976) investigated the  $a/v$  ratio for the rock, the stiff soil and the deep cohesionless soil site and mentioned the relation between the ratio and the predominant period of earthquake ground

motion. Mohraz (1976) calculated the  $a/v$  and  $ad/v^2$  ratios of the records on the sites classified into 4 categories, and showed that the  $a/v$  ratio on the rock site is higher than that on the alluvium site, and that the  $ad/v^2$  ratio on the rock is also larger than on the alluvium site. Zhu et al. (1988a, b) classified earthquake ground motions into 3 groups by the  $a/v$  ratio and indicated that the ratio effects significantly on the inelastic responses of single-degree-of freedom degrading systems. Niagi et al. (1990) analyzed the  $a/v$  and  $ad/v^2$  ratios of the array records of the SMART1 in Taiwan and investigated especially the dependence of the  $ad/v^2$  ratio to the earthquake magnitude.

It has been pointed out from these studies that the  $a/v$  and  $ad/v^2$  ratios closely relate to the frequency content and the duration of earthquake ground motions. However, there are few studies which investigated the ratios systematically and discussed the properties of these ratio in detail.

In this study, we investigate statistically the relation of the ratios ( $a/v$  and  $ad/v^2$ ) to the spectral characteristics and the duration of earthquake ground motions. The spectral characteristics are represented by spectral parameters which include the peak frequency, the mean frequency, the coefficient of variation of frequency and the irregularity index obtained from the power spectrum of earthquake ground motions. The relation of the  $a/v$  ratio to the magnitude, the distance and the predominant period at the sites, is also discussed, in which the predominant period of sites can be calculated from blow-count ( $N$ -value) profiles by standard penetration tests.

## 2. CHARACTERISTIC PARAMETERS OF GROUND MOTION

### 2.1 Peak amplitude ratios $a/v$ and $ad/v^2$

In this study, the  $a/v$  and  $ad/v^2$  ratios are investigated statistically, in which  $a$ ,  $v$  and  $d$  are the peak acceleration, velocity and displacement of earthquake ground motions, respectively. The peak velocity and displacement can be obtained from once and twice integrations of acceleration records, in which the integration is performed in frequency domain.

The significance of the  $a/v$  ratio can be explained in the following. The peak velocity of the records on alluvium site is relatively larger than that on rock site, because the records on alluvium site include longer period component. Consequently, the  $a/v$  ratio of the records on the alluvium site is smaller than that on the rock site.

It is also indicated from seismological studies that the attenuation of ground motion velocity with distance is generally slower than that of acceleration. As a result, the  $a/v$  ratio is high near an earthquake source and low at a large distance from the source.

As to the  $ad/v^2$  ratio, Mohraz (1976) indicated that the ratio is a measure of the width of spectrum and the ratio for alluvium site is smaller than that for rock site. From the fact mentioned above, it has been shown that the  $a/v$  ratio is lower and the  $ad/v^2$  ratio is smaller for alluvium sites than those for rock sites.

### 2.2 Spectral parameters

The spectral characteristics of earthquake ground motions are represented by spectral parameters in the following.

(1) Peak frequency :  $f_p$  (Hz)

The peak frequency,  $f_p$ , is the frequency corresponding to maximum peak in the power spectrum of acceleration records.

(2) Mean frequency :  $f_m$  (Hz)

The mean frequency,  $f_m$ , is the frequency corresponding to the centroid of the power spectrum of acceleration records.

(3) Mean frequency in log axis :  $f_{1.0\%}$  (Hz)

The mean frequency,  $f_{1.0\%}$ , is the frequency corresponding to the centroid of the power spectrum which is shown against log frequency axis. The mean frequency,  $f_{1.0\%}$ , is used in this study, because the power spectra of many records are roughly symmetric to this frequency in log axis.

(4) Coefficient of variation :  $\sigma_r/f_m$

The  $\sigma_r/f_m$  is the coefficient of variation of the frequency in the power spectrum of acceleration records, in which  $f_m$ =mean frequency mentioned above and  $\sigma_r$ =the standard deviation of the frequency respecting to the mean frequency,  $f_m$ , in the power spectrum of acceleration records. The  $\sigma_r/f_m$  is the parameter relating to the normalized width

of the spectrum.

(5) Irregularity index :  $q$

The irregularity index,  $q$ , is also a measurement of the normalized width of the spectrum and is defined in the following (Vanmarcke et al. (1972))

$$q = \{1 - \alpha_1^2 / (\alpha_0 \alpha_2)\}^{1/2} \quad (1)$$

where  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are spectral moments which are defined as follows.

$$\begin{cases} \alpha_0 = \int_0^{\infty} G(f) df \\ \alpha_1 = \int_0^{\infty} f G(f) df \\ \alpha_2 = \int_0^{\infty} f^2 G(f) df \end{cases} \quad (2)$$

in which  $G(f)$ =one-sided power spectrum of acceleration records and  $f$ =frequency.

### 2.3 Duration of ground motion : $T_d$ (sec)

In this study, we use the duration defined by Trifunac and Brady (1975), which can be calculated by

$$T_d = T_{95\%} - T_{5\%} \quad (3)$$

in which  $T_{95\%}$  and  $T_{5\%}$  are the times when the cumulative squared integral of acceleration record reaches to 5% and 95% of total power.

## 3. RELATION BETWEEN PEAK AMPLITUDE RATIOS AND CHARACTERISTIC PARAMETERS OF GROUND MOTIONS

### 3.1 Records used in analysis

The 148 acceleration records obtained in Japan are used in the numerical analysis, in which those are selected such that the earthquake magnitude,  $M$ , is larger than 5, the epicentral distance,  $\Delta$ , longer than 10 km and the blow-count ( $N$ -value) profiles are known. Fig.1 shows the relation of  $M$  and  $\Delta$  in the records used in the analysis.

An example of the acceleration, velocity and displacement time histories is shown in Fig.2 and the acceleration Fourier spectrum in Fig.3.

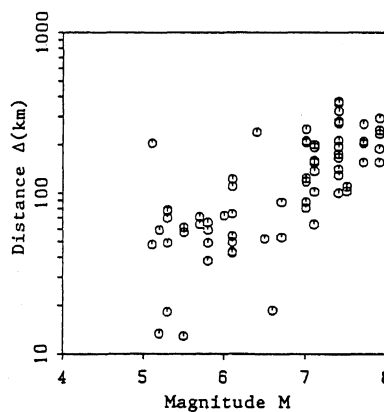


Fig.1 Relation between Magnitude and Distance

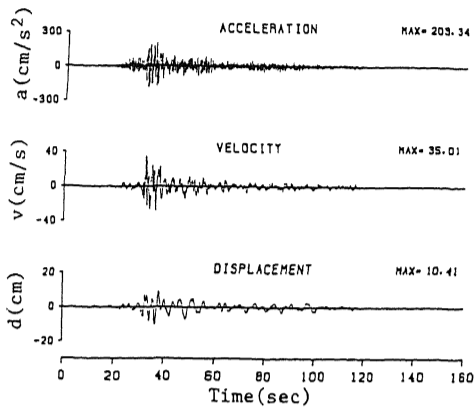


Fig.2 Example of Time Histories

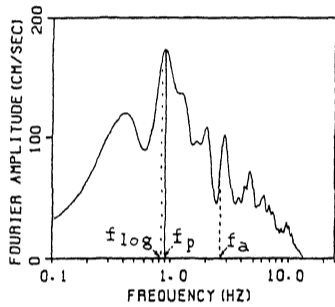


Fig.3 Example of Fourier Spectrum

### 3.2 Relation of a/v ratio to characteristic parameters

#### (1) a/v to $f_p$ , $f_a$ and $f_{log}$

Figs.4~6 are the scatter diagrams of the a/v ratio to the peak frequency ( $f_p$ ), the mean frequency ( $f_a$ ) and the mean frequency in log axis ( $f_{log}$ ), in which the horizontal axis is frequency and the vertical one the a/v ratio in logarithm. It is found from these figures that the a/v ratio increases in proportional to the peak and mean frequencies in log axis. Regression analysis of a/v to  $f_p$ ,  $f_a$  and  $f_{log}$  is performed, whose results are also shown in Figs.4~6. Correlation coefficients of each case are very high such as  $R=0.85$  (Fig.4),  $R=0.88$  (Fig.5), and  $R=0.92$  (Fig.6), as shown in the figures.

#### (2) a/v to $\sigma_r/f_a$ and $q$

Figs.7 and 8 show the relation of the a/v ratio to the coefficient of variation ( $\sigma_r/f_a$ ) and the irregularity index ( $q$ ), respectively, in which horizontal axis is  $\sigma_r/f_a$  (Fig.7) and  $q$  (Fig.8) in arithmetic axis, and vertical one the a/v ratio in log axis. The regression lines are also shown in the figures, whose correlation coefficients are  $R=0.88$  (Fig.7) and  $R=0.87$  (Fig.8),

respectively. It is found from the figures that the a/v ratio decreases as the  $\sigma_r/f_a$  or  $q$  value is larger. This fact indicates that the a/v ratio is small when the spectrum is wide-band, because large  $\sigma_r/f_a$  and  $q$  mean that the spread of frequencies in the spectrum are wide. Since, in general, the peak acceleration is considered to be large and the peak velocity be relatively small for wide-band processes, this result seems to be curious. However, considering that the wide-band spectrum includes lower frequency components and so the peak velocity becomes relatively large, it may be concluded that the above result is not inconsistent.

#### (3) a/v to $T_d$

Fig.9 shows the relation between the a/v ratio and the duration,  $T_d$ , of the ground motions. Although the scatter is considerably large, there is some intercorrelation between the a/v ratio and  $T_d$ , that is, the a/v ratio is lower as  $T_d$  is longer.

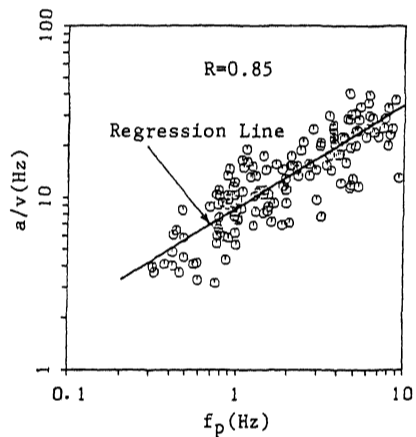


Fig.4 Relation between a/v and  $f_p$

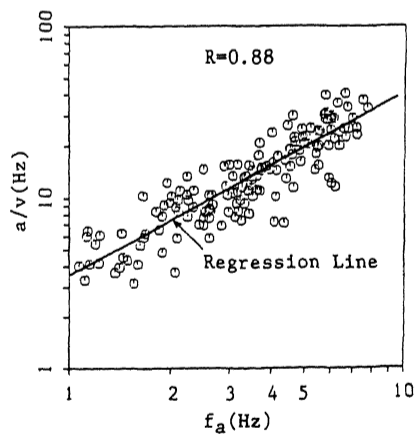


Fig.5 Relation between a/v and  $f_a$

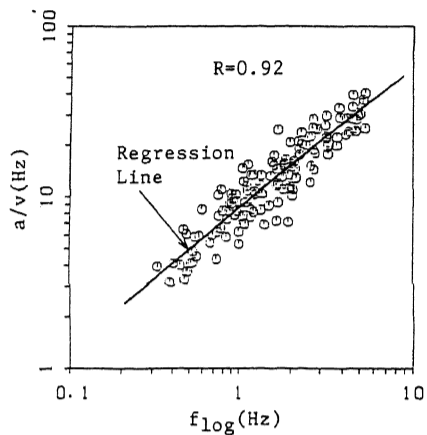


Fig.6 Relation between  $a/v$  and  $f_{log}$

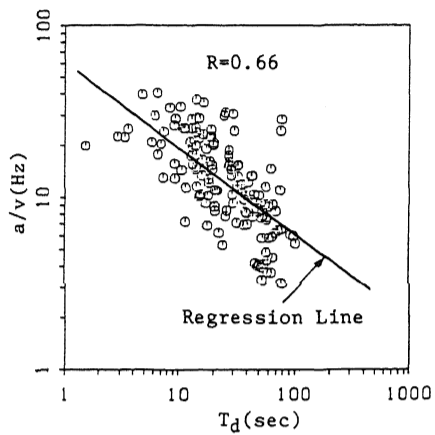


Fig.9 Relation between  $a/v$  and  $T_d$

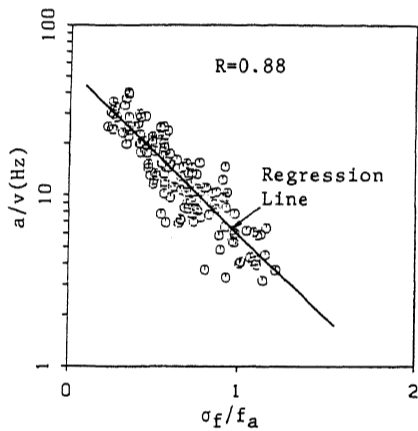


Fig.7 Relation between  $a/v$  and  $\sigma_f/f_a$

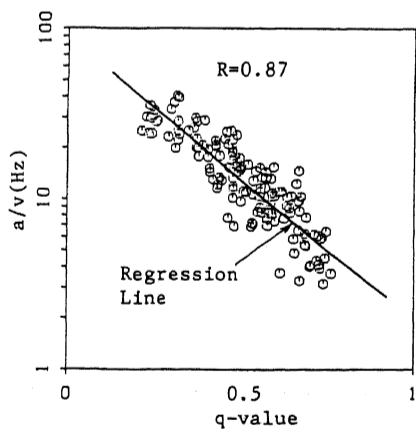


Fig.8 Relation between  $a/v$  and  $q$

### 3.3 Relation of $ad/v^2$ ratio to spectral parameters

The  $ad/v^2$  ratio is a direct measure of the width of the spectra (Mohraz, 1976). In this study, we represent the width of the spectra by using the standard deviation,  $\sigma_f$ , the coefficient variation,  $\sigma_f/f_a$ , and the irregularity index  $q$  of the frequency in the spectra. Among those,  $\sigma_f$  shows the width of the spectra in itself, while the latter two parameters are corresponding to the width normalized by the mean frequency or the similar one, which are unitless.

Here, we investigate the relation of the  $ad/v^2$  ratio to  $\sigma_f$ ,  $\sigma_f/f_a$  and  $q$ . Figs.10-12 show the relation between  $ad/v^2 - \sigma_f$ ,  $ad/v^2 - \sigma_f/f_a$  and  $ad/v^2 - q$ , respectively. From these figures, it is found that the relations are not so good as those of the  $a/v$  ratio (see Figs.7 and 8).

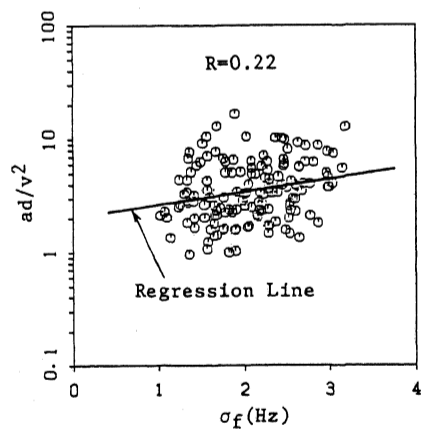


Fig.10 Relation between  $ad/v^2$  and  $\sigma_f$

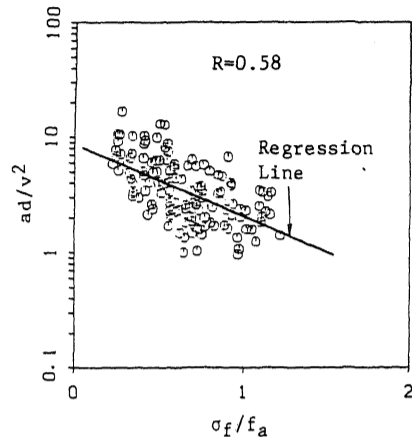


Fig.11 Relation between  $ad/v^2$  and  $\sigma_f/f_a$

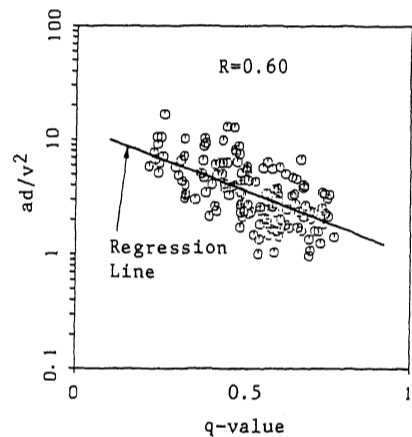


Fig.12 Relation between  $ad/v^2$  and  $q$

#### 4. REGRESSION ANALYSIS OF $a/v$ RATIO

##### 4.1 Quantification of site conditions

In the preceding section, we have indicated that the  $a/v$  ratio is the excellent parameter for the spectral characteristics, especially the predominant frequency ( $f_p, f_a$  and  $f_{0.5}$ ) of ground motions. In this section, the dependence of the  $a/v$  ratio on the magnitude, the distance and the geological condition at the site is also investigated. Then, it is important to quantify the geological condition. In the following, we quantify the geological condition by the predominant period of the site. In Japan road bridge specification(1990), the predominant period of a site,  $T_p$ (sec), can be calculated by

$$T_p = 4 \sum_{i=1}^n H_i / V_{s,i} \quad (4)$$

in which  $n$ =number of layers constructing the ground above the base at the site,  $H_i$ = thickness of  $i$ -th layer, in m, and  $V_{s,i}$ =S-wave velocity of  $i$ -th layer, in m/sec. If the blow-count ( $N$ -value) profiles are obtained from standard penetration tests at the site, the  $V_{s,i}$  can be calculated by following equations (Japan road bridge specification).

$$\begin{cases} V_{s,i} = 100N_i^{1/3} & (1 < N_i < 25); \text{ for clay} \\ V_{s,i} = 80N_i^{1/3} & (1 < N_i < 50); \text{ for sand} \end{cases} \quad (5)$$

in which  $N_i$ = $N$ -value of  $i$ -th layer. If S-wave velocities are obtained by well shooting tests, they are directly used for eq.4. The base is defined as the layer where the  $N$ -value is larger than 25 for clay and 50 for sand, and/or S-wave velocity larger 300m/sec.

##### 4.2 Dependence of $a/v$ ratio on magnitude, distance and predominant period of site

We investigate how the  $a/v$  ratio depends on the earthquake magnitude (JMA scale),  $M$ , the epicentral distance,  $\Delta$  (km), and the predominant period,  $T_p$  (sec), at the site. Figs.13~15 show the relations between  $a/v$ - $M$ ,  $a/v$ - $\Delta$  and  $a/v$ - $T_p$ , respectively. These figures indicate that the  $a/v$  ratio has the negative interrelation to  $M$ ,  $\Delta$  and  $T_p$ , respectively, which means that the  $a/v$  ratio is lower as  $M$ ,  $\Delta$  and  $T_p$  are larger.

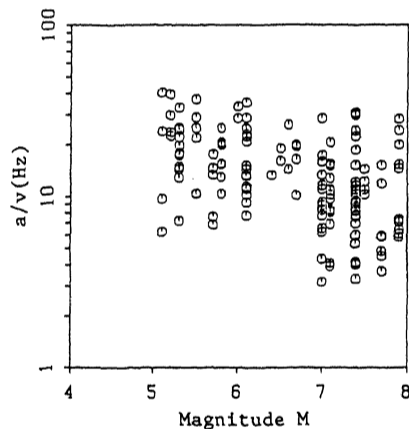


Fig.13 Relation between  $a/v$  and Magnitude

##### 4.3 Regression analysis

The following regression equation is assumed from the preceding fact

$$a/v = b_0 * 10^{b_1 M} * \Delta^{b_2} * T_p^{b_3} \quad (6)$$

The regression analysis is performed using 148 data obtained from the accelerograms in Japan. The results is as follows,

$$a/v = 41 * 10^{-0.07 M} * \Delta^{-0.12} * T_p^{-0.51} \quad (7)$$

The multiple correlation coefficient in eq.7 is 0.76. Fig.15 shows the relation between the  $a/v$  ratio and the predominant period,  $T_p$ , together with 4 regression lines, in which each line corresponds to the cases

of  $(M, \Delta) = (5, 20), (6, 50), (7, 100)$  and  $(8, 200)$ , respectively. This figure shows that the regression lines considerably fit the data.

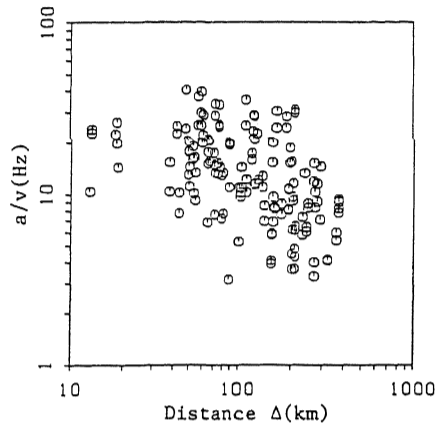


Fig.14 Relation between  $a/v$  and  $\Delta$

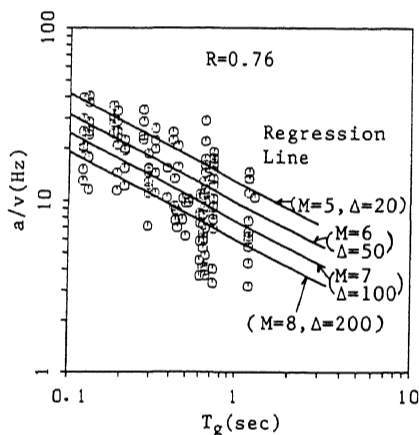


Fig.15 Result of Regression Analysis

## 5. CONCLUSIONS

In this study, the relation of the peak amplitude ratios  $a/v$  and  $ad/v^2$  to the spectral parameters and the duration of ground motions has been investigated statistically.

The dependence of the  $a/v$  ratio on the earthquake magnitude, the epicentral distance and the predominant period of the site has been also examined by the regression analysis. The results obtained in this study are summarized as follows.

1. The  $a/v$  ratio is the excellent parameter which represents the spectral characteristics and the duration of ground motions. The  $a/v$  ratio is lower as the predominant frequency ( $f_p, f_s$  and  $f_{1.0g}$ ) is small-

er, the width of spectra ( $\sigma_r/f_s$  and  $q$ ) broader and the duration longer.

2. The  $a/v$  ratio depends on the earthquake magnitude, the epicentral distance and the predominant period at the site, in which the  $a/v$  ratio has the negative intercorrelation to those parameters. The regression analysis of the  $a/v$  ratio has been performed and we have presented the regression equation whose multiple correlation coefficient is 0.76.

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