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Frequency-wave number Fourier amplitudes of seismic ground motion in a multiple-layered half-space due to a Haskell-type source

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ABSTRACT: In this paper, analytic expressions for the frequency-wave number domain Fourier amplitudes of seismic ground motion at the free surface of a layered half-space are established. The displacement field is produced by a Haskell-type source having arbitrary orientation. The computational method used provides a complete description of ground motion (i.e. it accounts for all types of waves and there are no approximations involved) at any point of the ground surface (i.e. ground motion is computed both in the near-field as well as in the far-field). The derived analytic expressions for the Fourier amplitudes can be used to synthesize the displacement field due to an earthquake with known fault and rupture characteristics. In addition, they can be used to obtain the Fourier spectra, power spectral densities, frequency-dependent autocorrelation function and frequency-wave number spectra at any point on the ground surface.

1. INTRODUCTION

The spatial variation of seismic ground motion is important for the seismic design of elongated structures, such as long-span bridges, lifelines etc. The temporal and spatial variability of seismic ground motion are effectively described by the Fourier amplitudes of the displacement field in the frequency-wave number domain, which provide information concerning the correlational characteristics of earthquake motion. One way to infer the Fourier amplitudes is by using data obtained from seismograph arrays. This kind of data are very useful in characterizing a particular recorded event and in understanding the mechanism of rupture. They are, though, very dependent on the local ground properties and on the characteristics of the particular event, so that it is unrealistic to use the results of such an analysis for another region and a different seismic event. It is therefore obvious that there is need for establishing analytic expressions for the complex Fourier amplitudes as functions of several parameters describing the seismic source and the ground.

In this paper, closed form analytic expressions are established for the Fourier amplitudes of the displacement field at the free surface of a layered half-space caused by a seismic source described by Haskell's model. The computational method used provides a complete description of ground motion, i.e. it accounts for all types of waves and there are no approximations involved. In addition, ground motion is computed at the near-field, as well as in the far-field. It should be noted that this method provides the only currently available analytic description of the spectral content of ground motion in the vicinity (near-field) of an extended earthquake fault. The method is an extension of earlier work done by Lamb (1904), Bouchon (1979), Deodatis, Shinozuka and Papageorgiou (1990a and 1990b).

It is important to note that after establishing the analytic expressions for the Fourier amplitudes of the displacement field, it is possible to calculate the Fourier spectra, power spectral densities, frequency-dependent autocorrelation function and frequency-wave number spectra at any point on the ground surface. Finally, the displacement field itself can be computed by performing a triple Fourier transform on the Fourier amplitudes. The above-mentioned capabilities of the method are demonstrated in a companion paper (Deodatis and Theoharis, 1992).

2. BASIC ASSUMPTIONS

The ground is assumed to consist of a number of layers that are overlaying a half-space (see Fig. 1). Each layer is considered to be uniform and is characterized by the P-wave and S-wave velocities (denoted by α and β, respectively), the mass density ρ and the attenuation factor Q that summarizes the gross effect of internal friction (Aki and Richards, 1980).

According to the Helmholtz decomposition, the vector $\mathbf{u}$ of the displacement field can be written as:

$$\mathbf{u} = \nabla\phi + \nabla \times \mathbf{\Psi} \quad (1)$$

In the above equation, ϕ and $\mathbf{\Psi}(\Psi_1, \Psi_2, \Psi_3)$ are the displacement potentials which are the solutions of the wave equations:

$$\nabla^2\phi = \frac{1}{\alpha^2}\frac{\partial^2\phi}{\partial t^2} \qquad \text{P} - \text{wave equation} \quad (2)$$

$$\nabla^2\mathbf{\Psi} = \frac{1}{\beta^2}\frac{\partial^2\mathbf{\Psi}}{\partial t^2} \qquad \text{S} - \text{wave equation} \quad (3)$$

where α and β are the complex P-wave and S-wave velocities respectively, given by:

$$\alpha = c_p \cdot \left(1 + i\frac{1}{2Q}\right) \quad (4)$$

$$\beta = c_s \cdot \left(1 + i\frac{1}{2Q}\right) \quad (5)$$

with c_p and c_s representing the elastic compressional and shear wave velocities, respectively. The vector potential $\mathbf{\Psi}$ satisfies the additional equation:

$$\nabla \cdot \mathbf{\Psi} = 0 \quad (6)$$

Within a layer, the solution for simple harmonic motion of frequency ω can be written in the form of plane waves, as:

$$\phi(\omega) = A \cdot \exp[-i\kappa_x x - i\kappa_y y \pm i\nu z + i\omega t] \quad (7)$$

$$\mathbf{\Psi}(\omega) = \mathbf{B} \cdot \exp[-i\kappa_x x - i\kappa_y y \pm i\gamma z + i\omega t] \quad (8)$$

where κ_x and κ_y are the wave numbers in the x and y direction, respectively. The + and − signs in Eqs. 7 and 8 indicate upward and downward propagation, respectively. The quantities ν and γ are defined as:

$$\nu = \sqrt{\kappa_\alpha^2 - \kappa_x^2 - \kappa_y^2} \qquad \text{Im } \nu \leq 0 \quad (9)$$

$$\gamma = \sqrt{\kappa_\beta^2 - \kappa_x^2 - \kappa_y^2} \qquad \text{Im } \gamma \leq 0 \quad (10)$$

representing the vertical wave numbers of the propagating waves. The quantities κ_α , κ_β are the wave numbers along the direction of propagation of P and S waves respectively, given by:

$$\kappa_\alpha = \frac{\omega}{\alpha} \quad ; \quad \kappa_\beta = \frac{\omega}{\beta} \quad (11)$$

The motion is decoupled into P-SV and SH motions and the two problems are then treated separately (Bouchon, 1979). The decoupling equations are:

$$\psi_{SV} = \frac{\kappa_x}{\kappa}\Psi_2 - \frac{\kappa_y}{\kappa}\Psi_1 \quad (12)$$

$$u_{y'}^{SH} = i[\kappa\Psi_3 + \frac{\gamma}{\kappa}(\kappa_x\Psi_1 + \kappa_y\Psi_2)] \quad (13)$$

with Ψ_1, Ψ_2, Ψ_3 being the components of $\mathbf{\Psi}$ and

$$\kappa = \sqrt{\kappa_x^2 + \kappa_y^2} = \kappa_{x'} \quad (14)$$

3. CALCULATION OF FOURIER AMPLITUDES OF DISPLACEMENT FIELD

3.1 *Green's functions*

First, analytic expressions are established for Green's functions using the propagator-based formalism proposed by Chouet (1987). Specifically, the displacement components in the x, y and z directions (G_{xp}, G_{yp} and G_{zp}, respectively) at the free surface of the layered half-space, due to a unit impulse applied in the p-direction as shown in Fig. 2, are written in the following form:

$$G_{np}(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}_{np}(\kappa_x, \kappa_y, \omega) \cdot \mathrm{I} \qquad n = x, y, z \quad (15)$$

where:

$$\mathrm{I} = \exp[-i\kappa_x x - i\kappa_y y + i\omega t] d\kappa_x d\kappa_y d\omega \quad (16)$$

Analytic expressions for $\tilde{G}_{xp}$, $\tilde{G}_{yp}$ and $\tilde{G}_{zp}$ can be found in Theoharis, Deodatis and Shinozuka (1992).

3.2 *Double couple solutions*

The three displacement components at the ground surface due to a double couple associated with a pure strike slip are then computed as:

$$u_n^s(x,y,t) = M_{xy} \star G_n^1(x,y,t) + M_{xz} \star G_n^2(x,y,t) \quad ; \quad n = x, y, z \quad (17)$$

with:

$$G_n^1 = [(G_{nx,y_s^0} + G_{ny,x_s^0})]_{z=0} \quad (18)$$

$$G_n^2 = [(G_{nx,z_s^0} + G_{nz,x_s^0})]_{z=0} \qquad (19)$$

where M_{ij} are the components of the moment tensor, $\star$ is the convolution symbol, $_{,x_s^0}$, $_{,y_s^0}$ and $_{,z_s^0}$ denote partial differentiation with respect to x_s^0, y_s^0 and z_s^0 and (x_s^0, y_s^0, z_s^0) are the coordinates of the double couple.

Note that the moment tensor M_{ij} is assumed to have a time-dependence described by a ramp function with rise time t_r. The Fourier amplitudes of $u_n^s(x,y,t)$ in the frequency-wave number domain are given by:

$$\tilde{u}_n^s(\kappa_x, \kappa_y, \omega) = \bar{M}_{xy} \cdot \tilde{G}_n^1(\kappa_x, \kappa_y, \omega) + \\ + \bar{M}_{xz} \cdot \tilde{G}_n^2(\kappa_x, \kappa_y, \omega) \ ; \ n = x, y, z \qquad (20)$$

where super bar indicates Fourier transform with respect to frequency and super tilde indicates Fourier transform with respect to both frequency and wave numbers. Analytic expressions for $\tilde{G}_n^1$ and $\tilde{G}_n^2$ can be found in Theoharis, Deodatis and Shinozuka (1992).

The corresponding equations for a double couple associated with a pure dip slip are:

$$u_n^d(x,y,t) = M_{yy} \star G_n^3(x,y,t) + \\ + M_{zz} \star G_n^4(x,y,t) + \\ + M_{yz} \star G_n^5(x,y,t) \ ; \ n = x, y, z \qquad (21)$$

where:

$$G_n^3(x,y,t) = [G_{ny,y_s^0}]_{z=0} \qquad (22)$$

$$G_n^4(x,y,t) = [G_{nz,z_s^0}]_{z=0} \qquad (23)$$

$$G_n^5(x,y,t) = [G_{ny,z_s^0} + G_{nz,y_s^0}]_{z=0} \qquad (24)$$

and:

$$\tilde{u}_n^d(\kappa_x, \kappa_y, \omega) = \bar{M}_{yy} \cdot \tilde{G}_n^3(\kappa_x, \kappa_y, \omega) + \\ + \bar{M}_{zz} \cdot \tilde{G}_n^4(\kappa_x, \kappa_y, \omega) + \\ + \bar{M}_{yz} \cdot \tilde{G}_n^5(\kappa_x, \kappa_y, \omega) \ ; \ n = x, y, z \qquad (25)$$

Again, analytic expressions for $\tilde{G}_n^3$, $\tilde{G}_n^4$ and $\tilde{G}_n^5$ can be found in Theoharis, Deodatis and Shinozuka (1992).

3.3 *Haskell model solution*

The Haskell model is a shear dislocation propagating with constant velocity over a rectangular fault. The fault and its geometry are shown in Fig. 3. The dislocation can be a dip slip, a strike slip or a combination of the two.

First, the double couple solution obtained in section 3.2 is geometrically transformed to a coordinate system that is attached to the fault plane (see Fig. 4). Then, the solution for Haskell's model is obtained by integration of the transformed double couple solution over the rectangular fault surface.

After some algebra, the displacement field at the free surface is expressed as:

$$U_n^i(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{U}_n^i(\kappa_x, \kappa_y, \omega) \cdot \mathrm{I} \\ n = x, y, z \quad ; \quad i = s, d \qquad (26)$$

where I was defined in Eq. 16, subscript n denotes the displacement component and superscript i indicates whether the slip is pure strike or pure dip. The Fourier amplitudes are given by:

$$\tilde{U}_n^i(\kappa_x, \kappa_y, \omega) = \int_{z_1^{0'}}^{z_2^{0'}} \int_{L_1}^{L_2} \tilde{u}_n^i(\kappa_x, \kappa_y, \omega, x_0', z_0') \\ dz_0' dx_0' \quad ; \quad n = x, y, z \quad ; \quad i = s, d \qquad (27)$$

where $z_1^{0'}$, $z_2^{0'}$, L_1 and L_2 are defined in Fig. 4. Analytic expressions for the Fourier amplitudes $\tilde{U}_n^i(\kappa_x, \kappa_y, \omega)$; $n = x, y, z$; $i = s, d$ can be found in Theoharis, Deodatis and Shinozuka (1992).

Finally, the total displacement field at the free surface $U_n^T(x,y,t)$ is obtained by summing up its pure strike slip and pure dip slip components:

$$U_n^T(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{U}_n^T(\kappa_x, \kappa_y, \omega) \cdot \mathrm{I} \\ n = x, y, z \qquad (28)$$

where I was defined in Eq. 16 and:

$$\tilde{U}_n^T(\kappa_x, \kappa_y, \omega) = \tilde{U}_n^s(\kappa_x, \kappa_y, \omega) + \tilde{U}_n^d(\kappa_x, \kappa_y, \omega) \qquad (29)$$

4. CONCLUSIONS

In this paper, analytic expressions for the Fourier amplitudes of the seismic ground motion at the free surface of a layered half-space were established. The displacement field was produced by a Haskell-type source having arbitrary orientation.

The derived expressions can be used to synthesize the displacement field due to an earthquake with known fault and rupture characteristics. In addition, they can be used to obtain the Fourier spectra, power spectral densities, frequency-dependent autocorrelation function and frequency-wave number spectra at any point on the ground surface.

The computational method used provides a complete description of ground motion, i.e. it accounts for all types of waves and there are no approximations involved. Finally, it should be pointed out that ground motion is computed at the near-field, as well as in the far-field.

5. ACKNOWLEDGMENTS

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6. REFERENCES

Aki, K. and Richards, P.G. (1980). *Quantitative Seismology: Theory and Methods (Vol. I)*, Freeman and Company, New York.

Bouchon, M. (1979). "Discrete Wave Number Representation of Elastic Wave Fields in Three-Space Dimensions," *Journal of Geophysical Research*, Vol. 84, No. B7, pp. 3609-3614.

Chouet, B. (1987). "Representation of an Extended Seismic Source in a Propagator-Based Formalism," *Bulletin of the Seismological Society of America*, Vol. 77, No. 1, pp. 14-27.

Deodatis, G. and Theoharis, A. (1992). "Simulation of Seismic Ground Motion in the New Madrid Area Using Analytically Derived Frequency Wave Number Fourier Amplitudes," *Proceedings of the 10th World Conference on Earthquake Engineering*, Madrid, Spain, July 19-24.

Deodatis, G., Shinozuka, M. and Papageorgiou, A. (1990a). "Stochastic Wave Representation of Seismic Ground Motion. I: F-K Spectra," *Journal of Engineering Mechanics*, ASCE, Vol. 116, No. 11, pp. 2363-2379.

Deodatis, G., Shinozuka, M. and Papageorgiou, A. (1990b). "Stochastic Wave Representation of Seismic Ground Motion. II: Simulation," *Journal of Engineering Mechanics*, ASCE, Vol. 116, No. 11, pp. 2381-2399.

Lamb, H. (1904). "On the Propagation of Tremors at the Surface of an Elastic Solid," *Philosophical Transactions of the Royal Society of London*, Vol. A203, pp. 1-42.

Theoharis, A., Deodatis, G. and Shinozuka, M. (1992). "Seismic Ground Motion From a Haskell-Type Source in a Multiple-Layered half-Space," NCEER Technical Report.

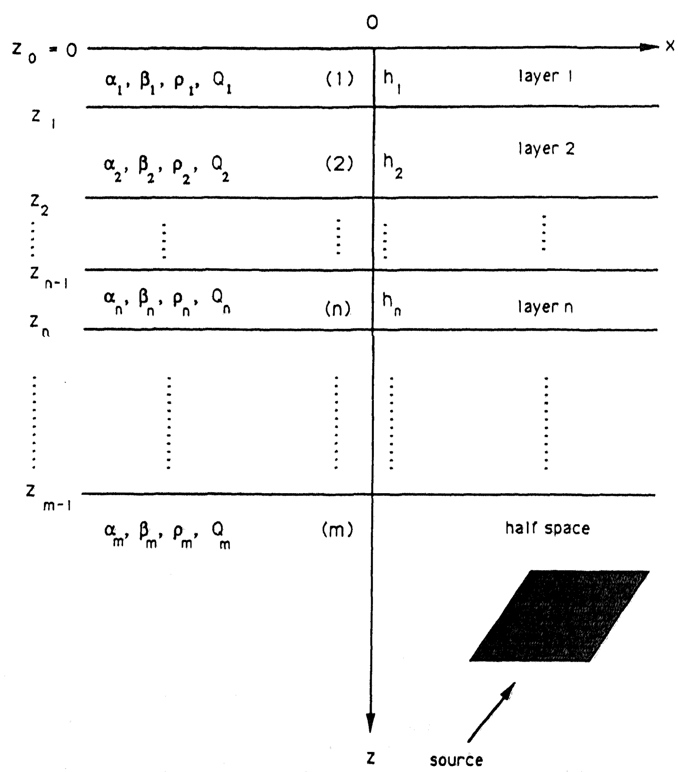

Fig. 1

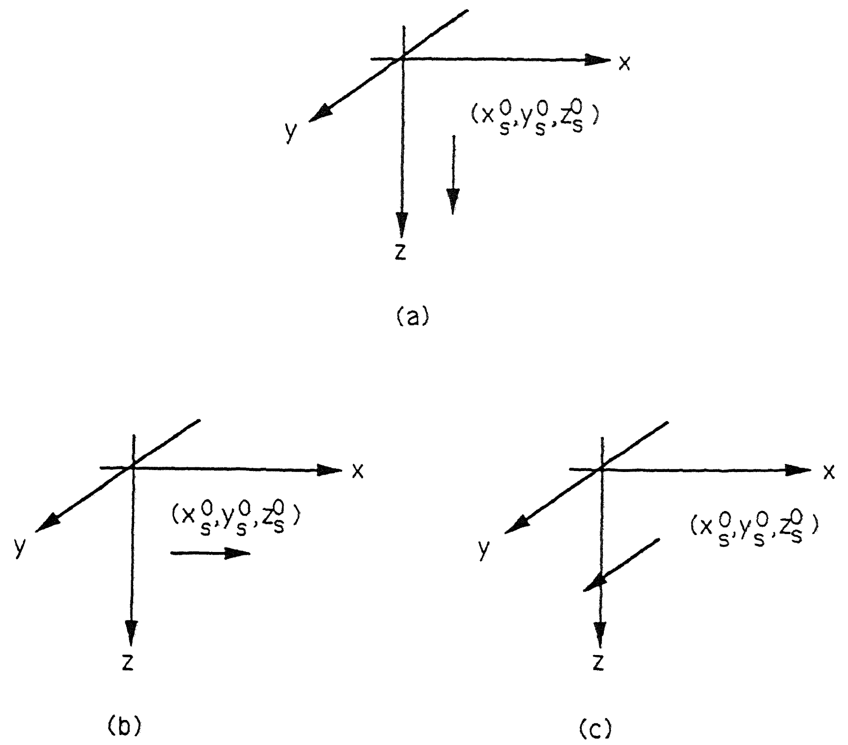

Fig. 2 Unidirectional unit impulses applied at point (x_s^0, y_s^0, z_s^0) in the (a) z-direction, (b) x-direction and (c) y-direction.

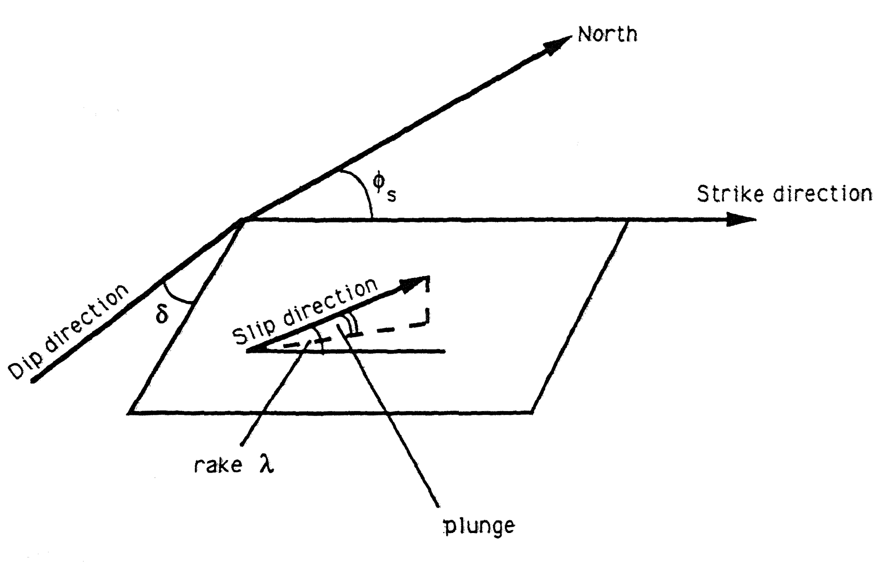

Fig. 3 The fault and its geometry.

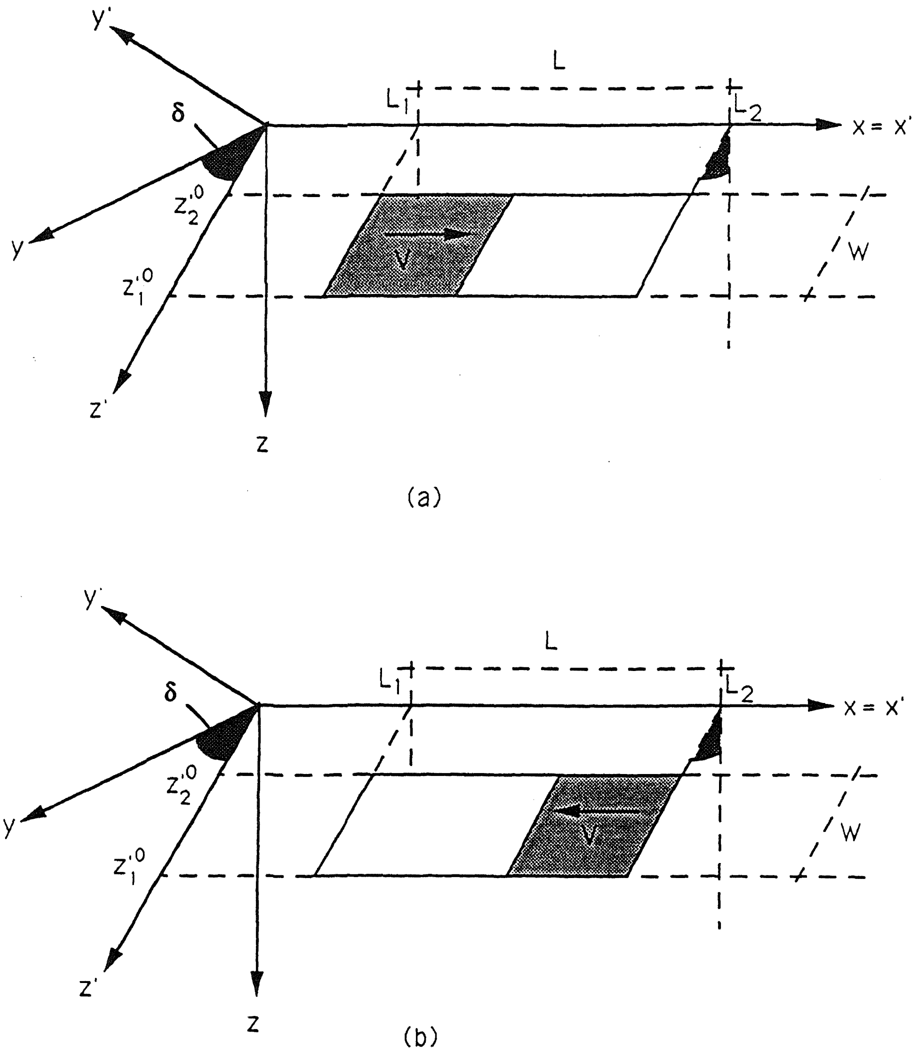

Fig. 4 Propagating shear dislocation in the (a) positive x-direction and (b) negative x-direction.