Study on generation of surface wave from side into plain

Hiroaki Yokoyama
Research Institute of Technology, Konoike Construction Co., Ltd. Osaka, Japan

ABSTRACT: The wave enters from a side of the bedrock of peripheral mountains into the alluvial stratum adjacent thereto. As a result of array observation of the plain, it is clarified that this is the surface wave of alluvial stratum. This surface wave of alluvial stratum appears dozens seconds later than the main motion which repeats multiple reflections and this second wave which loses high frequency components appears and has a simple wave form, and its acceleration rate is considerably high. In this paper, by using the improved differential numerical method in potential domain, generation of Rayleigh type wave is researched mainly, assuming angle of incident wave which enters from a side to two different mediums, semi-infinite and sedimentary medium, as variables. Also, in addition to incident angle, depth, and zone of incident wave are changed variously.

As main results,
1) Generation of Rayleigh type surface wave is not influenced so much by incident angle.
2) Even if angle of incident wave is in horizontal level, Rayleigh type surface wave is formed.
3) Even if angle of incident wave is in horizontal level, generation process is as same as if in other direction.
4) In sedimentary medium, direct wave and first reflected wave at bottom form clearly Rayleigh type surface wave with time lag.
5) This calculation method is effective to know visually the generation of surface wave and process of the wave propagations.

1. INTRODUCTION

The wave motion of bedrock just under the plain enters the alluvial stratum, and reaches the ground surface, repeating the multiple reflection of S wave, thereby becoming the main movement of plain surface. And then, this wave contains the high-frequency component. The wave enters into the alluvial stratum adjacent to the peripheral mountain bedrock from the side of this bedrock. The seismological observation in the peripheral mountain bedrock and alluvial stratum revealed that this wave is surface wave of alluvial stratum. The wave transmits latently through the alluvial stratum, utilizing the free surface condition on the upper surface and the bedrock boundary conditions on the lower surface. This surface wave appears later than the main wave. It has the simple waveform without high-frequency component, but it causes a significant acceleration.

So as to analyze this phenomenon, the author has conducted the numerical differential calculation, using the displacement potential method. This method has been obtained by improving or modifying the method which Hones and Takahashi used to solve the heat conduction equation and wave equation. This analysis allows the author to express approximately the potential $\phi$ and $\psi$ of body waves having the longitudinal wave and transverse wave in media at specific time ($t = n$), using the time ($t$) and ($t - n$) potential $\phi$ and $\psi$.

This method enables to separate the displacement potential of body waves. It is gathered as displacement or stress at the boundary as necessary. And then, it transmits as P and S waves. Consequently, it affords the step-by-step explicit analysis.

Applying this method the author has reported already the mechanism of surface wave in the alluvial stratum and the behavior of the surface wave in the alluvial stratum having irregular form bedrock when the Sin 1 wave was inputted in the vertical direction on one point of ground surface.

In this research I have analyzed in what incidence conditions the surface wave is generated in the alluvial stratum at the side when the earthquake movement is transmitted from the surrounding mountain bedrock into the alluvial stratum as later phase "Atoyare" which is seen in earthquake observation. At first I have calculated the wave transmission in the condition that the wave enters from a side into the alluvial stratum at specified depth, specific angle and in specific incident range, and examined the generation of the Rayleigh wave in the alluvial stratum. In this case I have also compared it with the generation of Rayleigh wave in semi-infinite medium.

Fig. 1 Model

* The incident angle ($\theta$) is determined by the vertical component to horizontal component ratio ($W$, $U$).
2. NUMERICAL CALCULATION MODEL

The author has executed the numerical calculation of step-by-step wave propagation with the aid of 2-dimensional elastic model where the incident angle (θ), incident point depth (H), number of incident points (N), and thickness of alluvial stratum (H) are taken as parameters. (Fig. 1)

3. NUMERICAL CALCULATION AND RESULT

3.1 Wave Propagation

In order to analyze the Rayleigh wave generation process the wave propagation diagram was plotted. Figure 2 is taken as a reference diagram. It represents the distribution of displacement in the state where Sin 1 wave (20°) is inputted in the vertical direction on one point of ground surface which is semi-infinite medium. Figure 2 represents 60 × step (H = m, H = 0, H = 0°, N = 1). From the figure it is evident that P, S and Rayleigh waves are generated and transmitted.

![Wave Propagation Diagram](image)

**Fig. 2 Displacement Distribution in Semi-Infinite Medium**

(H = m, H = 0, H = 0°, N = 1)

Figure 3a and Figure 3b show the wave propagation in semi-infinite medium and sedimentary layer (X = 40, 50 and 60 × step). (The wave propagation in the sedimentary layer is condition of 100 × step is also shown). In this state Sin 1 wave (20°) enters in the horizontal direction (θ = 90°) on one point (1°, N = 1) at incident depth Hf = 1.0 λs (where λs is wavelength of transverse wave).

These figures reveal the following.

1. Even when the incident angle is horizontal (θ = 90°), the Rayleigh wave is generated at 30 to 60 × step (P).
2. The Rayleigh wave is generated even when the wave enters under the ground in the sedimentary layer. It is generated as follows. Namely, mainly the S wave is transmitted from the incident point, and the Rayleigh wave is generated depending on boundary conditions (free surface).
3. At the distance of 1.0 to 1.5 λs from the surface near the incident point, the Rayleigh wave is generated.
4. As shown in the lowest figure of Fig. 3b, mainly the S wave is reflected by the bedrock near the incident point in the sedimentary layer and is transmitted to the surface, thereby generating again the Rayleigh wave (R).

![Wave Propagation Diagram in Sedimentary Layer](image)

**Fig. 3 Displacement Propagation Diagram in Semi-Infinite and Sedimentary Medium**

(H = 1.0 λs, θ = 90°, N = 1): R: Rayleigh Wave

Then, the author has analyzed the generation of Rayleigh wave due to change of incident angle. Figure 4a shows the displacement distributions when the incident angle is set to = 0, 45, and 135° in the condition of the incident point depth Hf = 1.0 λs. It indicates the state after a layer of 60 × step. In Fig. 4b, the thickness of sedimentary layer is H = 2.0 λs. The comparison with the case where the horizontal incidence (θ = 90°) is taken into consideration (Fig. 3) revealed the following.

1. The change of incident angle affects less the generation of Rayleigh wave.
2. As shown in Fig. 3, the Rayleigh wave is generated even when the incident angle is horizontal (θ = 90°).

![Wave Propagation Diagram with Different Incident Angles](image)

**Fig. 4 Displacement Propagation Diagrams with Different Incident Angles**

(H = 1.0 λs, θ = 90°, N = 1): R: Rayleigh Wave

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(3) The surface wave generation process does not differ from other processes even when the incident angle is horizontal ($\theta = 90^\circ$).

(4) The displacement distribution after the lapse of 60 step in the sedimentary layer does not differ remarkably form that of semi-infinite medium.

(5) When the incident angle is $\theta = 135^\circ$, the displacement distribution of surface wave is contrary to that at incident angle ($\theta = 45^\circ$) in phase.

\textbf{Fig. 3} Displacement Propagation Diagrams in Semi-Infinite and Sedimentary Mediums ($H = 1.0 \lambda_s$, $\theta = 90^\circ$, $N_i = 1$). R: Rayleigh Wave.

\textbf{Fig. 4} Displacement Distributions in Semi-Infinite and Sedimentary Mediums after 60 $t$ Step ($H = 1.0 \lambda_s$, $\theta = 0^\circ$, $45^\circ$, $135^\circ$, $N_i = 1$). R: Rayleigh Wave.
3.2 Displacement Time History of Ground Surface

Figure 5 and Figure 6 show the vertical and horizontal displacement time history (W, U) on 6 or 7 typical points (point interval is 1.0 \(\lambda_d\)) of ground surface of sedimentary layer and semi-infinite mediums which are shown in Fig. 2 to 4. In this case, however, the layer thickness of sedimentary layer was assumed to be \(H = 3.0 \lambda\). The result of analysis is as follows.

(1) These time histories evidence more clearly that the Rayleigh wave is generated. Especially, the vertical displacement time history (W) is remarkable.

(2) When the incident angle is horizontal (\(\theta = 90^\circ\)), the surface wave generation process is fastest among all generation processes at another incident angles (lag time \(\Delta t = 6 \tau\), and moreover its vertical amplitude is 0.6 to 0.8 times higher than the vertical amplitude in case of vertical incidence (\(\theta = 0^\circ\)).

(3) As shown in Fig. 6, the time histories evidence that the surface wave converted from direct wave and the bedrock reflected wave is clearly generated in the sedimentary layer (R, P').

(4) In the sedimentary layer the ratio of amplitude of surface wave generated by bedrock reflection to that of direct wave (R/R) reduces down to 0.45 to 0.65.

(5) As the depth of incident point (\(H_f\)) is increased, the surface wave generation is delayed, and the amplitude is reduced. But it is really generated. Although this fact is not shown in the figure, it has been proved at the incident depth of \(H_f = 8.0 \lambda\).

Fig. 5 Displacement Time Histories (W, U) in Semi-Infinite Mediums (\(H = \infty\), Ni = 1) R: Rayleigh Wave  P: P wave

Fig. 6 Displacement Time Histories (W, U) in Sedimentary Mediums (\(H = 3.0 \lambda\), \(H_f = 1.0 \lambda\), Ni = 1)
The change of the number of incident points (nodes) at side surface and incident angle relates to the problem of efficiency of incidence into the sedimentary mediums (boundary profile of surrounding mountains and sedimentary layers). Hence, the multi-point incidence was also calculated. Figure 7 shows the displacement time histories in case of multi-point incidence which was analyzed with the aid of semi-infinite mediums model (H = ∞). In Fig. 7a and 7b, \( H = 1.0 \lambda_s \) and \( N_i = 3 \). Fig. 7a shows the case where incident angle \( \theta_i = \theta_s = 90^\circ \), whereas Fig. 7b shows the case where \( \theta_i = 45^\circ, \theta_s = 90^\circ, \theta_t = 135^\circ \). Fig. 7c shows the case where \( H = 1.0 \lambda_s, N_i = 6 \), and \( \theta_i = 90^\circ, \theta_s = 90^\circ, \theta_t = 270^\circ \). As is evident from the figures:

(1) In the case where incidence direction is not the same (Fig. 7b) the amplitude ratio of Rayleigh wave is smaller as compared to the incidence in the same direction, namely 0.4 to 0.8. That is, the same direction incidence is the favorable condition to generate the Rayleigh wave even when \( \theta_i = 90^\circ \).

(2) To generate more intensely the Rayleigh wave, it is better to enter the wave as a couple of forces (\( N_i = 6, 8 \), to \( \theta_i = 90^\circ, \theta_s = 90^\circ, \theta_t = 270^\circ \)) as shown Fig. 7c. The amplitude ratio is about 3 times higher than the ratio shown in Fig. 7a.

Figure 8 shows, as a reference data, the ground surface displacement waveforms in the visco-elastic semi-infinite and sedimentary mediums in the state where the ground surface and inside of ground are vibrated within the range of about 9 \( \lambda_s \) from input point. Fig. 9 shows an example of \( \phi \) and \( \psi \) potential wave components.

4. CONCLUSION

The generation of surface wave in the sedimentary layers when it is entered from the surrounding mountains has been revealed fairly.

(1) Even when the incident angle is horizontal, the Rayleigh wave is generated.

(2) At the distance of 1.0 to 1.5 \( \lambda_s \) from the surface near the incident point, the Rayleigh wave is generated.

(3) The change of incident angle affects the generation of Rayleigh wave.

(4) The surface wave converted from direct wave and the bedrock reflected wave is clearly generated in the sedimentary layer.

(5) The same direction incidence is the favorable condition to generate the Rayleigh wave even when \( \theta_i = 90^\circ \).

The calculation method applied by the author enables to visually comprehend the generation of surface wave and to understand the process of wave propagation.

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Fig. 8 Displacement Time Histories (W, U) in Semi-Infinite and Sedimentary Mediums for References

Fig. 9 Displacement Time Histories (W, U) of φ Potential Wave Components in Semi-Infinite Medium (H = ∞, Hf = 1.0 λd, Ni = 1, θ = 90°)