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Is Mexico's long lasting ground motion made of gravity waves?

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ABSTRACT: The influence of gravity in wave propagation at Mexico City is investigated. Our objective is to elucidate whether gravity perturbations may help to explain the exceedingly long durations of strong ground motion observed at Mexico City during the September 19, 1985, earthquake, as suggested by Lomnitz (1990). Two possibilities are investigated: gravity perturbations on elastic waves in an extremely soft clay and, assuming that the clay behaves more like a fluid than a solid, gravity waves in an irregular fluid layer overlaying an elastic half-space. Our results show that gravity is a very unlikely explanation to the observed long lasting ground motion in Mexico City.

1 INTRODUCTION

Local site effects on seismic ground motion during the September 1985 earthquakes were at the origin of significant damage in structures and the loss of thousands of lives in Mexico City. The presence at the surface of a thin clay layer with very low S-wave velocity amplified ground motion by a factor as high as 50 in the frequency domain (Singh et al, 1988). High level amplification however, was only one side of site effects. Another one was a very important increase in the duration of strong motion records in the lake bed zone, due to conspicuous, long period and high amplitude late arrivals, that may have contributed significantly to the high levels of damage. In spite of significant research efforts, this duration increase has not yet received a satisfactory explanation.

One possibility that has not received much attention is an explanation in terms of gravity waves. This idea, advanced by Lomnitz in a series of papers (e.g. 1990), proposes that non-linear effects are at the origin of a "fluidification" of the surficial clay layer, leading to its behaving as a fluid with the subsequent propagation of gravity waves. Up to now there are no proofs of this behaviour, and Lomnitz himself does not produce observations or modeling of any kind. In this paper we investigate whether the influence of gravity may help to explain strong motion durations in Mexico City. Two possibilities are envisaged: gravity perturbed elastic waves in an very soft clay layer; and, assuming that non linear processes bring the rheology of the clay nearer to a viscous fluid than to a solid, gravity waves in a viscous fluid layer overlaying an elastic half-space.

2 GRAVITY PERTURBED WAVES IN AN ELASTIC SOLID

It has been shown (Bath and Berkhout, 1984) that gravity cannot affect body wave propagation in an infinite elastic space. The case of surface waves was examined by Ewing et al. (1957) who concluded that, if the vertical displacement at the surface is considered to be small, surface waves even in a fluid layer are not affected by gravity. However, Gilbert (1967) studied the problem of a half-space and a layer over a half-space and concluded that in incompressible, very soft sediments there is a gradual transition between Rayleigh waves and classical gravity waves. In this section we examine this possibility.

2.1 *Significance of $\overline{S}$ and $\overline{P}$ phases*

In an elastic half-space ground motion at the surface due to a buried source results from the direct arrivals, plus the Rayleigh wave contribution when horizontal distance to the source is about 5 times larger than source depth (Aki and Richards, 1980). The Rayleigh wave is the contribution of the only real root (baptized $\overline{S}$ par Gilbert and Laster, 1962) of Rayleigh's function. An additional contribution to the seismogram may come from the imaginary roots of Rayleigh's function, located on the non-physical Riemann sheet, the $\bar{P}$ pole in Gilbert's notation. This imaginary pole corresponds to the leaky mode of the problem. Although it exists in the non physical Riemann sheet, it may extend through analytical continuation into the physical sheet and affect the seismogram for certain values of the distance/depth ratio of the source. For incompressible solids, Gilbert (1967) defines $\varepsilon=g/(2\pi f\beta)$, where g=gravity acceleration, f=frequency and β=S-wave velocity, and proposes that as ε increases (i.e. at low frequencies and for soft soil sites) the significance of Rayleigh's pole diminishes, while the $\bar{P}$ pole becomes preponderant. He associates the $\bar{P}$ pole with prograde elliptic particle orbits and the corresponding phase velocity with that of water waves in a half space.

However, when we consider an elastic solid instead of an incompressible one, we realize that Poisson's coefficient, σ, also affects the position of $\bar{P}$ pole in the complex horizontal slowness, p, plane. This is important because the farther the $\bar{P}$ pole is in the non-physical Riemann sheet, the less it will affect the synthetic seismograms.

2.2 *Free surface reflection coefficients including gravity*

Let us investigate gravity effects on ground motion for a very high Poisson's ratio solid (such as Mexico City clay, where P-wave velocity is about 1500 m/s while S-wave velocity can be as low as 40 m/s). If gravity has an effect it must be at the free surface where, for non-negligeable vertical displacements, it contributes a surface force distribution of wavelength similar to that of the waves in the solid, proportional to vertical displacement. This non-linear problem may be linearized following a classic scheme (Lliboutry, 1987). Instead of imposing free traction on an unknown free surface ($z=\zeta(x)$, Figure 1) we impose a distribution of forces proportional to vertical displacement given by $\rho g w$ (where ρ=density and w is vertical displacement) on z=0.

Using this modified boundary condition we have computed free surface reflection coefficients in the case of non-negligeable free surface displacements. Following Aki and Richards (1980) we compute reflection coefficients as the ratio of amplitudes of reflected to incident potentials ϕ and ψ. Our boundary conditions introduce an additional term in the denominator (and the numerator for coefficients $\acute{P}\grave{P}$ and $\acute{S}\grave{S}$) of the form $ig\nu\omega^2/\beta^4$ (where $\nu^2=(\omega/\alpha)^2-k_x^2$, k_x=horizontal wave number), proportional to ω^3 (whereas all other terms are proportional to ω^4). A non cancelling dependence on ω then appears quite clearly. The form of the additional term suggests that it will be significant especially at low frequencies and for small values of β. This is in fact the case as can be seen from Figure 2 where we compare the classical $\acute{P}\grave{P}$ coefficient, independent of frequency, with the one obtained modifying the boundary condition. There are significant differences at low frequencies. We observe a shift of Rayleigh's pole towards high p. The leaky mode pole, $\bar{P}$, shows a weaker dependence on frequency. Differences between the two coefficients disappear for frequencies higher than 0.2 Hz in the example shown.

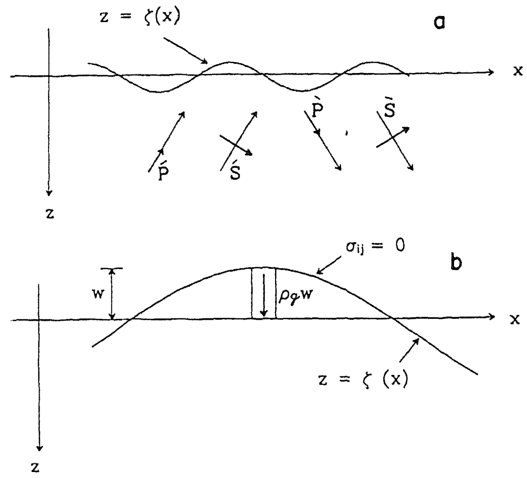

Fig. 1 (a) Incident and reflected waves of the in-plane problem. The free surface takes the form $z=\zeta(x)$. (b) Linearization of the boundary condition. We impose conditions on z=0 instead of $z=\zeta(x)$.

2.3 *Complete ground motion computations*

Gravity does affect free surface reflection coefficients, but its impact on ground motion depends strongly on the energy available at low frequencies and in certain slowness ranges. To determine gravity effects on complete synthetic seismograms we have simulated ground motion on the surface of a very thin, irregular, extremely soft layer. This layer is intended to represent that part of the clay that has undergone non linear softening. The method we use is that of Aki and Larner (1970) in the formulation given by Bard and Gariel (1986). Lacking constraints on the geometry and properties of a "fluidified" clay layer we have performed a series of analysis from which we show results for only one model. The reader is asked to believe that different geometries and parameters give results consistent with those presented here. More information and details may be found in

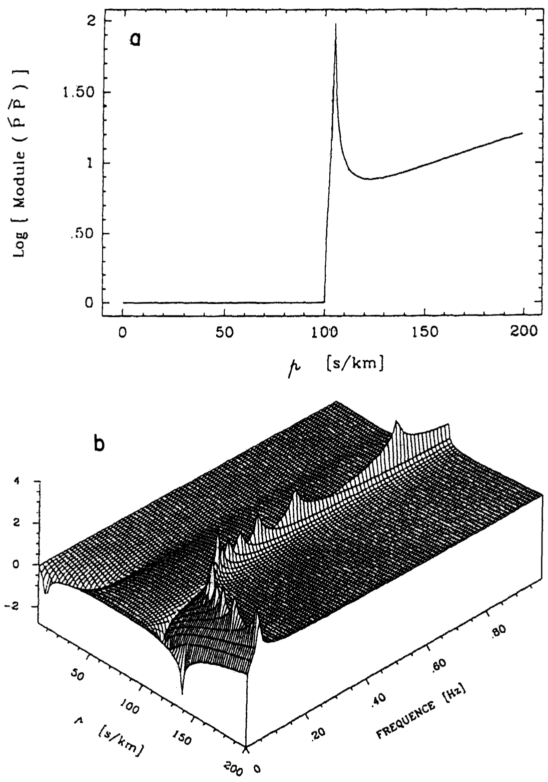

Fig. 2 Module of $\acute{P}\grave{P}$ as a function of p for the classical case (a) and as a function of p and frequency for the modified boundary condition case (b). P-wave velocity is 1500 m/s and S-wave velocity is 10 m/s (thus σ=0.49998).

Chávez-García (1991). The model used is displayed on Figure 3. A very thin, irregular, extremely soft layer represents that part of the clay that has undergone extreme non linear softening. (We do not adress, in this paper, the physical plausibility of such an extreme softening; we just investigate if it would lead to strong effects on signal duration.) Material properties are given on Table 1. Figure 4 shows the resulting

Table 1. Mechanical properties of the model

Layer	α	β	ρ	σ	Q_P	Q_S
	m/s	m/s	gr/cm^3			
1	1500	2	1.2	0.49999	50	10
2	1500	40	1.5	0.4996	50	25
3	1500	500	2.0	0.4375	100	50

α=P-wave velocity, β=S-wave velocity, ρ=density, σ=Poisson's ratio, Q=attenuation factor for P or S waves.

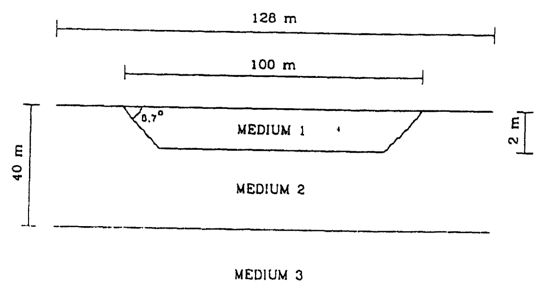

Fig. 3 Geometry chosen for a complete simulation of ground motion including gravity. The irregular layer represents the softened part of the clay.

transfer functions at 4 points at the surface. We compare results using the classical formulation with those including our modified free surface reflection coefficients. There are no significant differences between the two cases neither for vertical nor for horizontal displacement. Finally we have computed synthetic seismograms. The incident signal is a Ricker pulse with 0.25 Hz central frequency. Results are shown on Figure 5 for vertical incidence of SV waves. The most striking difference between the two sets of synthetics is the difference in the phase velocity of surface waves generated by the lateral irregularities. In the classical coefficients case it is of 5 m/s whereas in the modified free boundary case it is 13 m/s. We observe no fundamental change between the two sets of results, particle orbits (not shown) are essentially the same and there are no significant variations of ground motion duration.

2.4 *Conclusions*

We have incorporated gravity effects in ground motion computations for an elastic solid. We have assumed that due to non linear effects, the S-wave velocity comes down to 2 m/s (σ=0.499999, as α remains constant) in the "fluidified" clay. However, no significant modification of ground motion was observed. It appears clearly that there is no transition between Rayleigh waves and gravity waves for an elastic solid.

The results we have presented so far however, do not allow to rule out the possibility that gravity may affect ground motion. There remains the question whether the rheology of a "fluidified" clay layer be nearer to a fluid than to a solid. To answer this question we have substituted the irregular thin clay layer by a viscous fluid. This model is presented in the next section.

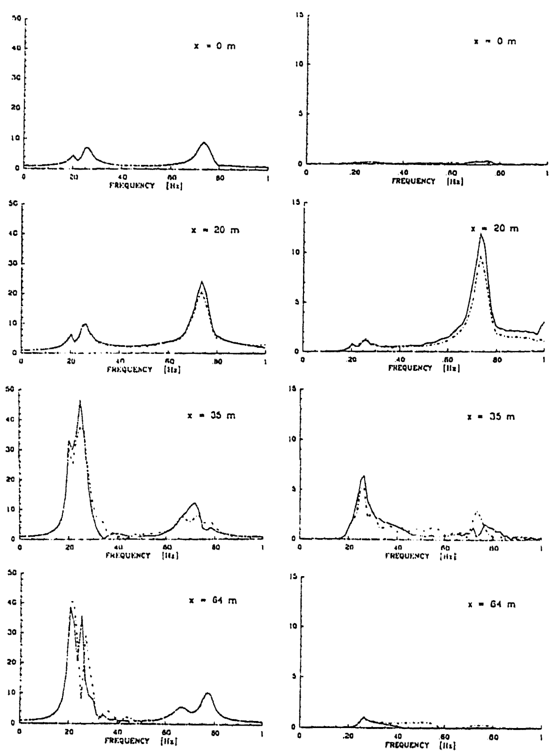

Fig. 4 Transfer functions at 4 points on the surface of the model shown on Figure 3 for vertical incidence of *SV* waves. Left column: horizontal component. Right column: vertical component. Continuous line: classical free surface coefficients. Dotted line: modified boundary condition coefficients.

3 COUPLING BETWEEN AN ELASTIC SOLID AND A FLUID LAYER

Gravity waves in fluids have been studied for a long time. However, when surface gravity waves are considered, it is generally accepted to neglect body waves in the fluid, supposed incompressible (e.g. Lamb, 1945). If, on the contrary, the interest is in body waves, gravity effects are neglected (e.g. Fehler, 1982). We have formulated the problem of an irregular viscous fluid layer overlaying an elastic half space. Our purpose is to evaluate the possibility that gravity waves propagate in Mexico City's clay because its behavior would be, for a reason still to be established (see Lomnitz, 1990), nearer to a fluid than to a solid, due to the postulated non linear effects. For our computations, based on Aki and Larner's method, it will be sufficient to write the diffracted field in the fluid as a superposition of plane waves. We follow the notation and developements of Sommerfeld (1971).

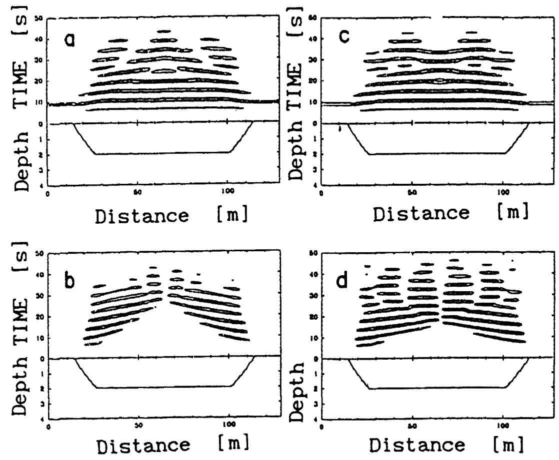

Fig. 5 Amplitude contours of synthetic seismograms on the surface of the model shown on Figure 3. Excitation is a Ricker pulse of 0.25 Hz central frequency. Only positive values are contoured at intervals of 10% of maximum amplitude. (a) and (b) Horizontal and vertical component, classical reflection coefficients. (c) and (d) Horizontal and vertical component, modified free surface reflection coefficients.

3.1 *Formulation of the problem*

Stresses in a viscous fluid can be written as

$$\sigma_{ij} = -p\delta_{ij} + p_{ij} \tag{1}$$

where p is the pressure, δ_{ij} is Kronecker's symbol and p_{ij} the stress related to strain rate. For a Newtonian fluid

$$p_{ij} = 2\mu\dot{\varepsilon}_{ij} \tag{2}$$

where μ is viscosity and $\dot{\varepsilon}_{ij}$ is the strain rate tensor. The equation of motion in the fluid is then given by

$$\rho\dot{\bar{v}} = F - \nabla p + \mu\nabla\nabla\bar{v} + \mu\nabla^{2}\bar{v} \tag{3}$$

where $\bar{v}$ = velocity vector, F= volume force vector and the point indicates derivative with respect to time. We have four unknowns (3 velocity components plus pressure). The fourth equation is given by mass conservation which can be written as (Morse and Feshbach, 1953)

$$p = -\kappa \operatorname{div} \bar{\xi} \tag{4}$$

where κ=fluid compressibility and $\bar{\xi}$= displacement vector. The system of equations is decomposed using Helmhotz potentials and accepts a solution in terms of plane waves

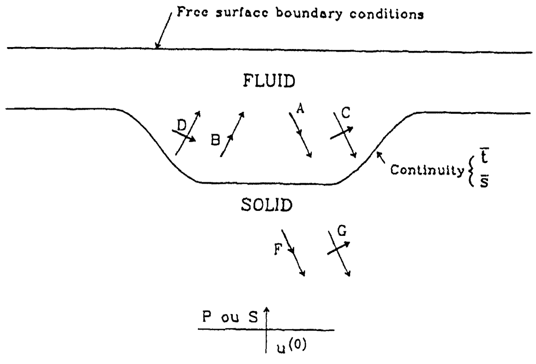

Fig. 6 Diagram showing the different waves in the case of an irregular viscous fluid layer on an elastic half-space. A and B are the pressure waves in the fluid. C and D are shear waves in the fluid. F and G are diffracted elastic waves in the half-space.

if

$$\omega^2/c^2=k^2+g_1^2 \tag{5}$$

and

$$-\rho i\omega/\mu=k^2+g_2^2 \tag{6}$$

where $c^2=(\kappa+2\mu i\omega)/\rho$ and g_1, g_2= vertical wave numbers.

We need now impose boundary conditions to the problem. The free surface is linearized in the same way as in the preceding section, which allows to obtain free surface reflection coefficients for the waves propagating in the fluid. Resulting expressions are too lengthy to be given here. Continuity conditions along the irregular boundary between the solid and the fluid are satisfied numerically using Aki and Larner's method.

3.2 *Results*

We lack again constraints on the geometry or the mechanical properties that a layer of "fluidified" clay would have. We can only be sure that our model must include some amount of viscosity (for example, glycerine viscosity is 2.33 Pa s, and clay must be more viscous than glycerine). We have again performed a series of tests, from which we present results for only one geometry and vertical incidence of SV waves (our model is almost transparent for P waves). Other results are consistent with the one presented. The model we have chosen, shown on Figure 6, is an irregular viscous fluid layer, whose depth varies from 5 m at the edges of the model to 15 m at the center of the model. The resulting transfer functions for zero viscosity and vertical incidence of *SV* waves are shown on Figure 7. It can be seen that there is a very significant amplification of surface motion, but only at low frequencies (under 0.2 Hz). As there is no viscosity, all motion at the surface come from diffraction of elastic waves along the irregular interface solid/fluid. Synthetics for this case (not shown) consist predominantly of surface waves with a phase velocity of 9 m/s and 50 m wavelength; very clearly gravity waves. Results in the time domain are summarized in Figure 7. Synthetic seismograms are shown for 3 points at the surface of the model and for the two motion components, considering 4 different viscosities. In the case of zero viscosity, high amplitude, surface wave arrivals give rise to very long seismograms, but they are rapidly attenuated in presence of viscosity and practically disappear for μ=300 Pa s.

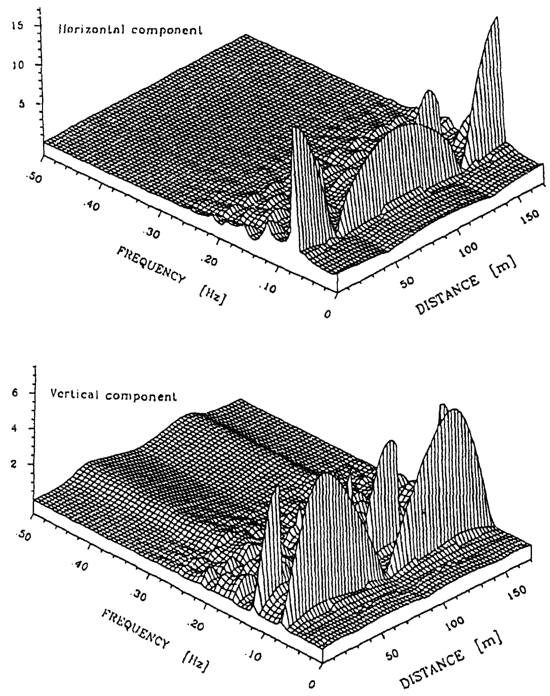

Fig. 7 Displacement transfer functions on the surface of the model shown on Figure 6 for vertical incidence of *SV* waves. The fluid is non viscous.

4 CONCLUSIONS

Concerning gravity effects in an elastic medium we may conclude that if gravity effects in an elastic solid are related to the P pole (as supposed by Gilbert, 1967), the influence of this pole is very small for the high Poisson ratios we have investigated

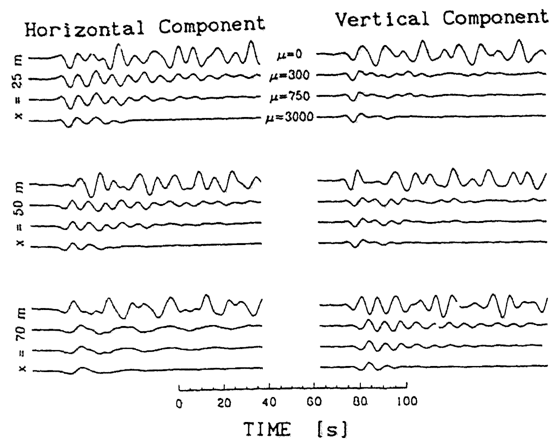

Fig. 8 Synthetic seismograms at *3* points on the surface of the model shown on Figure 6 for 4 different values of viscosity μ.

(and Mexico City clay has a very high σ). We have shown that gravity affects phase and group velocities of Rayleigh waves but that there is no change in the nature of ground motion, nor is the duration of motion at the surface affected in a significant way.

We have successfully modeled gravity waves in fluids as the diffraction of elastic waves on an irregular interface solid/fluid, and shown that elastic/fluid/gravity coupling effects in closed basins are extremely efficient. We believe this model to be of utility to study hydrodynamic pressures in dams or other closed fluid reservoirs. As regards the application of this model to Mexico City, we have shown that gravity surface waves in the fluid are strongly affected by moderate amounts of viscosity. Thus, if Mexico City's clay behaves as a fluid, the viscosity it would probably have inhibits gravity waves and duration of simulated ground motion falls too short of observations.

We have gone a long way to introduce the effect of gravity in wave propagation phenomena. We believe that we have covered all reasonable possibilities. According to our results an explanation of the long period high amplitude late arrivals in Mexico City strong motion data in terms of gravity effects is extremely unlikely.

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