Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Seismic risk: Non-linear MDOF structures

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ABSTRACT: The conventional seismic hazard analysis does not offer a direct estimate of the annual risk of damage of multi-degree-of-freedom structures, whose behavior under a severe earthquake is highly non-elastic. This conventional approach is integrated here by introducing alternative magnitude and distance-dependent non-linear-response-based measures of the damage potential of earthquake ground motions. The expected value of these measures is virtually independent of magnitude and distance of the earthquake and its variability is small compared to the variability of the spectral acceleration. Any systematic effects of the input (e.g. spectral shape, duration, phasing, etc.), and all aspects of the non-linear behavior of the structure are implicitly taken into account. This modified seismic hazard analysis methodology can be employed with limited additional effort to predict seismic damage annual risks.

1. INTRODUCTION

The goal of the conventional seismic hazard analysis (SHA) is the prediction, based on the available geological and seismological information at the site, of the annual probability of exceedance of a specified ground motion parameter, which is often peak ground acceleration (PGA). Attenuation laws are used to correlate the ground motion parameter with site-to-source distance, site geology, source path characteristics, source directivity, and so on. Even if the PGA is replaced by an appropriate response spectral acceleration, the key assumptions in this approach are, first, that the single-degree-of-freedom (SDOF) response plays a major role in the global response of the structure, and, second, that the structure behaves elastically during the ground shaking. This approach does not offer a direct estimate of the annual risk of damage of complex multi-degree-of-freedom (MDOF) structures whose response behavior under a severe earthquake is highly non-elastic. Nor does it recognize directly or indirectly effects such as duration on non-linear response.

The purpose of this paper is the integration of the conventional SHA with the adoption of alternative magnitude and distance-dependent non-linear-response-based measures of the damage potential of earthquake ground motions. Conventional, well-studied linear response spectral ordinates will still be used, but they are coupled here with response-reduction factors that take into account both the MDOF characteristics and the non-linear behavior of the actual structure. The present study focuses mainly on the computation of the inelastic MDOF reduction factor associated with a given degree of post-elastic damage in a MDOF structure of given characteristics and on the potential dependence of this factor on magnitude and distance.

2. EVALUATION OF DAMAGE POTENTIAL OF GROUND MOTIONS ON MDOF STRUCTURES

The maximum response of a linear SDOF system to a specific record is described by the spectral acceleration S_a. However, severe damage prediction cannot be based on such linear spectrum alone because damage has been demonstrated to occur when structures experience sequences of non-linear response cycles. Therefore in order to improve the reliability of the damage prediction, two structure-dependent and record-dependent factors are introduced: C and F_{DM}.

The index C accounts for the differences in the response of the linear MDOF structure versus the response of a linear approximate SDOF model, while F_{DM} represents the reduction factor associated with a given degree of non-linear damage in the MDOF non-linear structure. These two indices are respectively defined as:

i)

$$C = \frac{DI_{SDOF}}{DI_{MDOF}} \qquad (1)$$

where, for instance in a building-like N-story structure,

$$DI = \max(|u_i|_{max}/u_{i,c})$$

in which $|u_i|_{max}$= maximum linear value of the i^{th} story drift; $u_{i,c} = i^{th}$ story linear drift capacity.

The linear damage index DI is computed twice, once correctly using all modes of vibration for the linear MDOF structure and once only approximately using the first mode in the linear SDOF structure. It is worth noting here that $DI = 1$ means that incipient yield has been reached in (at least) one story. Furthermore DI_{SDOF} turns out to be proportional to the linear SDOF elastic-response-based measure adopted in the analysis (e.g. S_a) C will be used to "correct" a SDOF-based prediction of the incipient yield.

In general, as stated above, C is dependent on both structural parameters and record characteristics; the latter in turn depend to some degree on event characteristics such as magnitude and source-to-site distance. Recently though the following important facts have been shown (Inoue and Cornell,1991) :

- the dependence of the average value of C, $E[C]$, on ground motions parameters is mild;
- the record-to-record variability in C is probabilistically dominated by the variability in S_a, not only when the first mode is dominant but also when contributions of higher nodes are important in the response of the structure.

Consequently the moments of C can be adequately estimated with a relatively small number of linear dynamic analyses of the MDOF structure.

ii) F_{DM}, defined originally by Kennedy et al. (1984) for SDOF non-linear systems, is the amount by which an earthquake record that causes incipient yield (here in at least one member), must be scaled up in order that this new, rescaled record will induce a specified (non-linear) damage level in the structure (e.g. ductility equal to 4).

From this definition it follows immediately that several F_{DM} factors can be defined, depending on the specified level, the type of damage (ductility vs hysteretic energy absorbed), and damage measures such as local ductility vs global ductility. Again F_{DM} varies from record-to-record for any specific structure.

Moreover, in an MDOF system the specified damage level is not necessarily experienced by the same member that reached first the yield. For symple non-linear SDOF systems, Sewell (1988) extensively studied the dependence of F_{DM} on structural parameters, force-deformation relationships, structural damage measures (e.g. ductility, normalized hysteretic energy, etc), magnitude and source-to-site distance. His conclusions were that for a fixed damage level

- the dependence on the force-deformation and damage models is of secondary importance;
- based on more than 100 records, $E[F_{DM}]$ shows no strong systematic dependence on either magnitude or distance to the investigators' surprise;
- the record-to-record variability of F_{DM} is small compared to the variability in S_a.

The statistical dependence on magnitude and distance of F_{DM} for MDOF structures has been investigated so far only in a limited way and only for a small number of lumped-mass stick-modelled MDOF structures (Inoue and Cornell, 1991). These studies, and, as will shown, the present one, confirm that the same remarks on the lack of dependency of the mean of F_{DM} on magnitude and distance hold also in the MDOF case. Consequently $E[F_{DM}]$ can be predicted with sufficient accuracy by a very modest number of non-linear dynamic analyses of the MDOF structure, performed using ground motion records of magnitude and distance that portray the limits of the ranges contributing significantly to the hazard of the site. Moreover it must be emphasized that different F_{DM} factors can be evaluated using the same set of non-linear dynamic analyses.

The most important consequence of the weak dependence of $E[C]$ and $E[F_{DM}]$ on magnitude and source-to-site distance and the relatively small variability on C and F_{DM} compared with the variability in S_a, is that conventional SHA methodology can be employed, with limited additional effort, to produce damage risks.

The annual probability, λ, of exceeding a level x of a certain kind of post-elastic seismic damage DM in a non-linear MDOF structure is given by:

$$\lambda[DM > x] \cong \sum_{i=1}^{N} \nu_i \left\{ \int_{R_1}^{R_2} \int_{M_1}^{M_2} P_{m,r} f_{M,R}(m,r) dm dr \right\}_i \quad (2)$$

in which:

- $$P_{m,r} = P[DM > x|m,r] = P\left[\frac{S_a}{CF_{DM}} > S_{a_{yld}}|m,r\right] \quad (3)$$
- N is the number of seismic sources that present a threat to the site;
- ν_i is the mean annual rate of occurrence of eathquakes from source i with magnitude between M_1 and M_2;
- R_1 and R_2 are the minimum and maximum distances from source to site;

- $f_{M,R}(m,r)$ is the joint probability density function of magnitude M and distance R;
- S_a is the spectral acceleration at the first mode frequency, f_1, of the MDOF structure;
- C is the (linear) MDOF response factor previously defined by Equation (1);
- F_{DM} is the (non-linear) spectral reduction factor, and
- $S_{a_{yld}}$ is the "yield level spectral acceleration", which is a capacity measure defined as the value of S_a at which $DI_{SDOF} = 1$. This is an easily established property of the structure.

The relationship between the two lines of Equation (3) follows from the definitions above of factors C and F_{DM}.

The results obtained from this approach incorporate in the F_{DM} and C factors implicitly and empirically (not theoretically) any systematic effects of the input (e.g. spectral shape, duration, phasing, etc.), and all aspects of the non-linear behavior of the structure.

The variables S_a, C, and F_{DM} in Equation (3) are all random variables whose joint distribution, conditional on m and r, establishes the probability in the integrand and its functional dependence on m and r.

As suggested above, however, empirical evidence to date suggests that these variables are dominated by S_a, implying that to a close first approximation C and F_{DM} can be replaced by their (conditional) means, which may in fact be virtually independent of m and r. Following M[c] Guire (1974), the statistical properties of S_a versus m and r, have been studied by many investigators in recent years. This proposed method is practical for large, realistic structures provided only a relatively few non-linear MDOF analyses of one's structure are necessary to estimate adequately the values (or weak trends) of $E[C|m,r]$ and $E[F_{DM}|m,r]$. The current research is aimed at confirming that this condition holds. Note also that improved software and computation efficiency has now brought such analyses within the same accessibility and the same cost range as former linear dynamic analyses.

The factor C is close to one for the first-mode dominated structures that are prevalent in civil structural engineering, and hence meets the criterion in at least these cases. In any case $E[C]$ (or its mean-value function) is comparatively inexpensively established.

3. EVALUATION OF THE NON-LINEAR REDUCTION FACTOR F_{DM}

The key point in the described methodology for SHA of non-linear MDOF structures is therefore the evaluation of the statistics of the F_{DM} factors. In this section the computation procedure

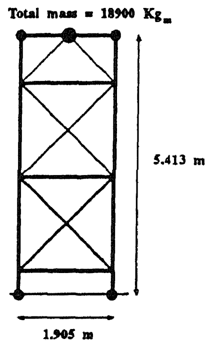

Figure 1: Jacket-type offshore platform 2D frame

of two types of non-linear damage reduction factors F_{DM} for a real jacket-type offshore platform is presented.

The four leg production facility prototype platform under consideration has been designed for 37 meters of water according to API criteria (American Petroleum Institute, 1991) for wave and earthquake conditions corresponding to Southern California. This structure is selected here because it has been extensively studied by others. A two-dimensional 5 : 48 scale test pinned frame of this structure, which is depicted in Figure 1, has been built, thoroughly dynamically tested, and analyzed at University of California, Berkeley, (Popov, et al. ,1985).

In the present study the analyses have been performed on a mathematical model of the 2D scaled frame. This geometrically scaled frame with mass distribution similar to the prototype, represents a model which is dynamically similar to the prototype offshore platform only if an appropriate time scaling factor is applied to the ground motion records. Non-linear models of structures, especially those involving slender members, remain open to debate.

For checking the sensitivity of the final results to the accuracy of the mathematical model describing the post-buckling behavior of the braces, two different finite element software packages have been used: one is an older, widely distributed, popular program, DRAIN2D (Kanaan and Powell, 1973) and the other is a state-of-the-art commercial program, KARMA.

In the finite element model the jacket legs have been described by using non-linear beam-column elements, the deck by linear beam elements, and the horizontal and diagonal braces by post-buckling truss elements. The post-buckling element available in DRAIN2D, developed by Jain and Goel (1978), is suitable only for very slender members (KL/r ratio greater than 60) and does not account for stiffness degradation, while the hystheresis behavior of the post-buckling element in KARMA can describe both slender and stocky braces and includes strength and stiffness degradation. These latter models have been shown to reproduce behavior such as that observed in the model tests.

The importance of modelling the braces as beams instead of as truss members has been also checked and, because the differences in the results have been found to be negligible, truss members have been retained in the final version of the model.

As previously mentioned, several reduction factors can be defined according to the different failure modes of interest. In this study two different types of post-elastic damage have been considered:

- a global-ductility-based damage measure; F_{DM1} is defined, for a given μ, as the ground motion record scaling factor necessary to achieve a deck displacement μ times larger than the displacement experienced by the deck when incipient yield occurs anywhere in the structure;

- a local-ductility-based damage measure; F_{DM2}, for a given μ, is defined as the ground motion scaling factor to achieve a local ductility μ in at least one element of the structure, over and above a ground motion which just initiates yield anywhere in the original structure.

From the previous definitions, it follows immediately that low values of F_{DM} correspond to ground motions that are particularly damaging for the structure.

To compute F_{DM} factors, a suite of 15 ground motion records has been selected in the range of magnitude and source-to-site distance of interest to a hypothetical site. The description of the records is reported in Table 1 and in Table 2, while the scatter plot of magnitude versus distance is presented in Figure 2.

The records are chosen to cover as uniformly as possible the (M, R) plane. This is a greater number of records than would be necessary in a practical application.

Table 1: List of ground motion records used in the analyses

Earthquake No.	Name	Station Name
1	Kern County, CA	Taft
2	Imperial Valley, CA	El Centro, Sta.9
3	Kern County, CA	Caltech Athen.
4	Lytle Creek, CA	Cal Edison
5	San Fernando, CA	Palos Verdes
6	Hollister, CA	Gilroy, Gavilan
7	Kalapana, Hawaii	Hilo, Univ Hawaii
8	Santa Barbara, CA	Freitas Bldg.
9	Tabas, Iran	Tabas
10	Imperial Valley, CA	El Centro, Arr.5
11	Imperial Valley, CA	El Centro, Arr.4
12	Livermore, CA	Livermore, Hosp.
13	Parkfield, CA	Lincoln school
14	Borrego Mountain,CA	San Onofre Plant
15	Mt. Hamilton, CA	Holl. City Hall

Table 2: Characteristics of the ground motion records

Quake No.	Date	Comp.	PGA (cm/sec^2)	Duration (sec)
1	07-21-52	EW	-46.46	54.4
2	05-19-40	S90W	210.14	53.5
3	07-21-52	S00E	-46.46	77.3
4	09-12-70	South	-40.20	42.0
5	02-09-71	S25E	-40.08	70.2
6	11-28-74	S23E	-94.11	12.0
7	11-29-75	N16W	-169.88	73.0
8	08-13-78	158	284.71	11.8
9	09-16-78	Long	795.77	29.0
10	10-15-79	230	367.21	39.4
11	10-15-79	140	229.63	19.5
12	01-27-80	038	60.62	20.0
13	06-27-66	S69E	11.23	72.4
14	04-08-68	N33E	40.03	45.2
15	04-24-84	271	71.06	29.6

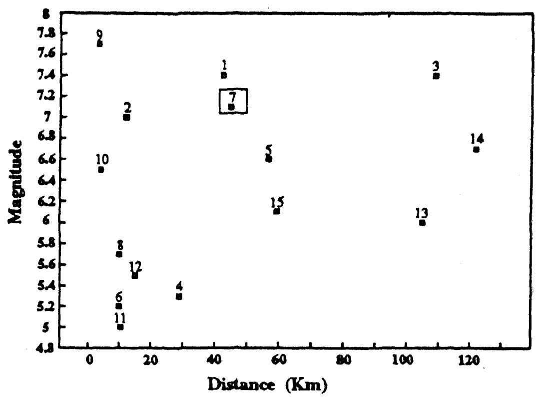

Figure 2: Magnitude-distance scattergram. Notes: (1) See Table 1 for earthquakes names; (2) Kalapana ground motion not included in the summary statistics.

The global-ductility-based reduction factors, F_{DM1}, for a given ductility equal to 4, found using KARMA are plotted versus source-to-site distance and magnitude of each record in Figure 3 and in Figure 4, respectively.

As previously found by Sewell (1988) and Inoue and Cornell (1991) for SDOF and for simple stick-type MDOF structures, the values of F_{DM} for a specified damage level show a little, if any, systematic dependence on distance and magnitude. The same conclusion holds also for the local- ductility-based reduction factor F_{DM2}, whose results are not included here (for brevity).

From Figure 3 and Figure 4 it is evident that the Kalapana ground motion is particularly ineffective for this structure, in terms of non-linear effects, i.e. its F_{DM1} value is very large. This fact could have been detected in advance of the calculations by simply looking at the response spectrum of this ground motion: in the region of 0.9 Hz and less, corresponding to the fundamental

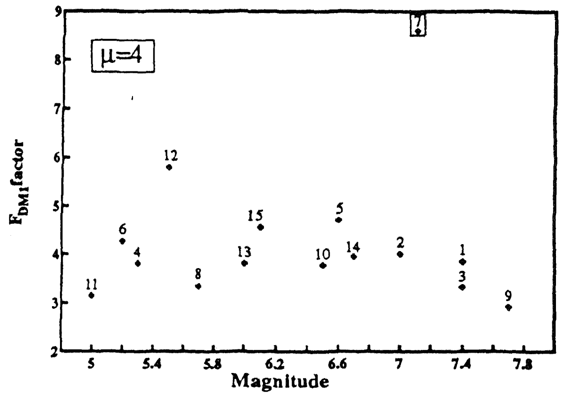

Figure 3: Correlation of F_{DM1} factor vs magnitude. Notes: (1) See Table 1 for earthquakes names; (2) Kalapana ground motion not included in the summary statistics.

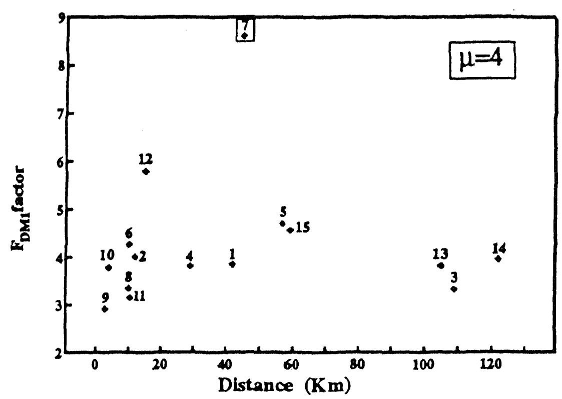

Figure 4: Correlation of F_{DM1} factor vs distance. Notes: (1) See Table 1 for earthquakes names; (2) Kalapana ground motion not included in the summary statistics.

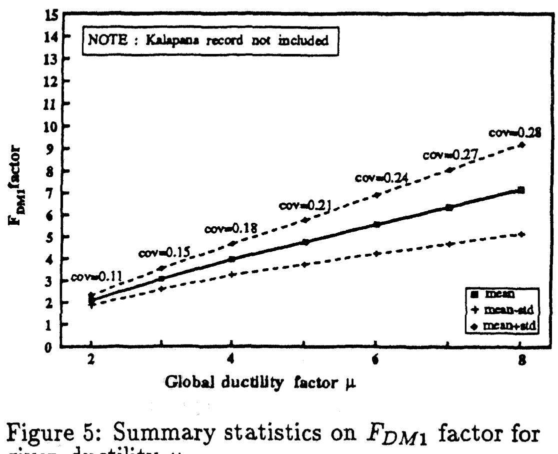

Figure 5: Summary statistics on F_{DM1} factor for given ductility μ

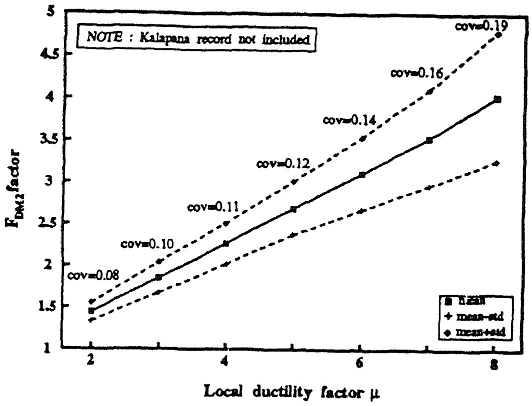

Figure 6: Summary statistics on F_{DM2} factor for given ductility μ

frequency of the scaled model (2.8 Hz) divided by the time scaling factor of $\sqrt{48/5}$, the ordinates reduce dramatically. Thus, as the structure softens because of the non-linear damage induced by the ground shaking, the power of the dynamic excitation drops very quickly, benefiting the structure. Therefore the Kalapana record has been (conservatively) omitted in the computation of the expected value of the reduction factors.

Statistics for F_{DM1} and F_{DM2} are reported, for the entire range of interest of the damage measures considered here, in Figure 5 and Figure 6 respectively.

As predicted based on previous experience, the results on both reduction factors show a relatively small dispersion around the mean. As a matter of fact, the coefficient of variation of the global-ductility-based reduction factor, F_{DM1}, at μ equal to 4 is in good agreement with the results found by Sewell (1988) for a 1 Hz SDOF system by using more than 100 ground motion records.

A similar set of results have been found using DRAIN2D and, even though there is some difference on a record by record basis, the final summary statistics on F_{DM1} and F_{DM2} show comparable means and very similar coefficients of variation for the entire range of damage levels considerd. That fact leads to the conclusion that reduction factors F_{DM}'s are also quite robust with respect to the use of different post-buckling models for the brace behavior.

Finally, as it follows from the inspection of the modal analysis results, the first mode is clearly predominant in the response of this structure. Therefore, in this case, the value of the factor C can be considered equal to one and independent of M and R and, moreover, its variability is small compared to that of S_a.

4. CONCLUSIONS

For a realistic model af an actual structure it has been confirmed by a relatively large sample of events chosen for their representativeness of a wide range of magnitudes and distances that the critical statistics, $E[F_{DM}|m,r]$ and the coefficient of variation of F_{DM}, meet the necessary criteria to permit a simplified probabilistic analysis of the annual probability of exceeding a specified non-linear damage. Those criteria are (1) that there is no significant functional dependence of $E[F_{DM}|m,r]$ on magnitude, m, and distance, r, (as confirmed by inspection of Figure 3 and Figure 4), and (2) that the coefficient of variation is relatively small, e.g. less than about 40%, compared to the coefficient of variation of S_a, given m and r. These results were confirmed for a broad range of ductility levels and for both a global and a local damage measure. The latter was particularly convincing because it was obtained for a "sensitive" brace member with complex hysteretic behavior, by using two different software packages, and for two mechanical models of the brace. With these two criteria met, F_{DM} and C can be replaced by their (constant) means and moved to the right hand side of the inequality within Equation (3). Then the annual probability of exceeding a non-linear damage level, ductility $\mu = 4$ say, is just the result of a conventional hazard analysis for the probability the S_a exceeds the value $S_{a_{yld}}E[C]E[F_{DM}]$, where $E[F_{DM}]$ is associated with level μ.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the kind contribution of computational support by Dr. J. Gidwani of ISEC, Inc., San Francisco, CA, (KARMA is a proprietary program developed by ISEC, Inc.) and the financial support of the industrial contributors of Stanford's Reliability of Marine Structures Program: Amoco, A.S Veritas Research, Bureau Veritas, Chevron, Conoco, Elf Aquitaine, Exxon, Mobil, Saga, Shell, Statoil and Norsk Hydro.

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