Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Fuzzy evaluation and statistical analysis of site intensity

H. Shen & M. Yener
Utah State University, Logan, USA

ABSTRACT: In fuzzy set analysis, site intensity is effectively used as a continuous variable. The degree of membership of site intensity is assumed to be represented by rth order polynomial functions. The corresponding (r+1) parameters are determined by the fuzzy properties of site intensity. The stochastic properties of site intensity are concerned with probability of earthquake occurrence frequency. Site intensity is related to peak ground acceleration. The peak ground acceleration distribution corresponding to a specified site intensity is obtained. An example problem is solved to determine the seismic hazard of Hubei area in China.

1 INTRODUCTION

The traditional methods for seismic hazard analysis are based on the probability theory to provide the necessary and sufficient tools in dealing with the uncertainty and imprecision of risk in decision analysis. The theory of fuzzy sets suggests that much of the uncertainty in risk analysis is based on the fuzziness of the information in the database and, more particularly, in the fuzziness of the underlying probabilities (Schmucker (1984)). In seismology and earthquake engineering, site intensity, as a discrete variable, is used for earthquake zoning. In fuzzy set analysis, it can be effectively handled in a continuous manner.

The estimation of seismic hazard in low and intermediate seismic activity regions, based on earthquake magnitude, site intensity and ground peak acceleration, has been a concern for researchers for the past thirty years (Cornell (1975), McGgire (1977), Mortgat (1979), and Egozcue (1991)). However, no definite relationship has yet been established to correlate site intensity with earthquake occurrence frequency and peak ground acceleration. In this paper, a procedure for the statistical analysis of site intensity is proposed, which correlates the probabilities of site intensity with earthquake occurrence frequency and peak ground acceleration. An example problem is solved to determine the seismic hazard of Hubei area in China.

2 FUZZY EVALUATION OF SITE INTENSITY

2.1 *Discrete and continuous sets of site intensity*

In seismology and earthquake engineering, site intensity is used as a variable for earthquake zoning. Site intensity is defined as a 12 degree set as

$$U_{S_i}=\{S_{i_1},S_{i_2},......S_{i_{12}}\}=\{1,2,......12\} \qquad (1)$$

However, the set of site intensity is essentially continuous. In fuzzy analysis, site intensity can be effectively handled in a continuous manner. The definition of closed continuous set of site intensity is

$$F_{S_i}=\{S_i/S_i\in[0,12]\}=[0,12] \qquad (2)$$

Each index S_i of site intensity in discrete universe U_{Si} should be a fuzzy subset in continuous universe F_{Si}. S_i is defined as the degree of fuzzy site intensity.

2.2 *Degree of membership of fuzzy set site intensity*

The degree of membership of site intensity is assumed to be represented by an rth order polynomial function (Shen (1988))

$$\mu_s=a_0+a_1S+a_2S^2+...+a_rS^r \qquad (3)$$

For example, based on the following conditions

$$\mu_S=0, \quad for \quad S=S_{i-1}$$
$$\mu_S=1, \quad for \quad S=S_i \tag{4}$$
$$\mu_s=0, \quad for \quad S=S_{i+1}$$

the 2nd order polynomial function of membership degree with respect to site intensity can be obtained as

$$\mu_S=1-S_i^2+2S_iS-S^2 \tag{5}$$

and

$$S\in[s_i-1,S_i+1] \tag{6}$$

The 2nd order polynomial function of membership degree is shown in Fig. 1.

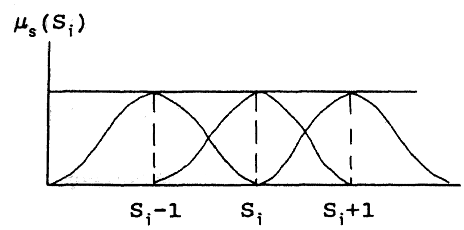

Fig. 1. The function of membership degree for site intensity

3 STATISTICAL ANALYSIS OF SITE INTENSITY

3.1 *Probability of earthquake occurrence frequency for a Specified Site Intensity*

In general, if an event can be described by a discrete Poisson distribution, it can be concluded that the random variable representing the event is a positive integer, and the probability of the event is small. The discrete Poisson distribution of earthquake occurrence frequency can be given as

$$P_{K_{S_i}S_i} = \frac{(\lambda_{S_i}T)^{K_{S_i}}}{K_{S_i}!}\exp(-\lambda_{S_i}T) \tag{7}$$

in which $P_{K_{S_i}S_i}$ is the probability of earthquake occurrence with a frequency K_{S_i}, representing a random variable, for a specified site intensity S_i during a period of T years. λ_{S_i} is earthquake occurrence frequency for the same site intensity S_i during a unit period.

In Hubei area of China (Yener and Shen (1991)), λ_5 = 0.1133, λ_6 = 0.0233, λ_7 = 0.0076, λ_8 = 0.0016. For T = 100 years, $P_{1,5}$ = 1.3604×10^{-4}, $P_{1,6}$ = 0.2267, $P_{1,7}$ = 0.3554, $P_{1,8}$ = 0.1878. The distributions of frequencies for S_i = 5, 6, 7, and 8 are shown in Fig. 2. Through the χ^2-test, the earthquake occurrence data of Hubei area can be shown to be represented by the Poisson distribution with a percent point of 0.05.

3.2 *Probability of site intensity*

Site intensity is considered to be an extreme value random variable. On the basis of the favorable comparison of analytical results and the actual data, the writers propose the use of the following modified extreme value distribution (MEVD) to describe site intensity

$$f_{S_i} = \exp[-\exp(-v_i)] \tag{8}$$

in which f_{S_i} is the probability of site intensity at a specified value S_i, and v_i is given as

$$v_i = a_1 + a_2S_i + a_3lgS_i \tag{9}$$

in which a_1, a_2, and a_3 are factors that are determined by least square approximations. In general, v_i is defined as a linear function of S_i. However, the analytical results obtained in the present investigation indicate that the inclusion of the third term on the right hand side of (9) yields a better correlation with the actual data. In (9), a_1, a_2, and a_3 are factors determined from the following expressions, which are developed by using least square approximations.

The values of a_1, a_2, and a_3, determined on the basis of the earthquake data of the Hubei area during the past 300 years (Yener and Shen (1991)), are a_1 = 36.7572, a_2 = 5.3973, and a_3 = -89.3933. The distributions of site intensity are shown in Fig. 3.

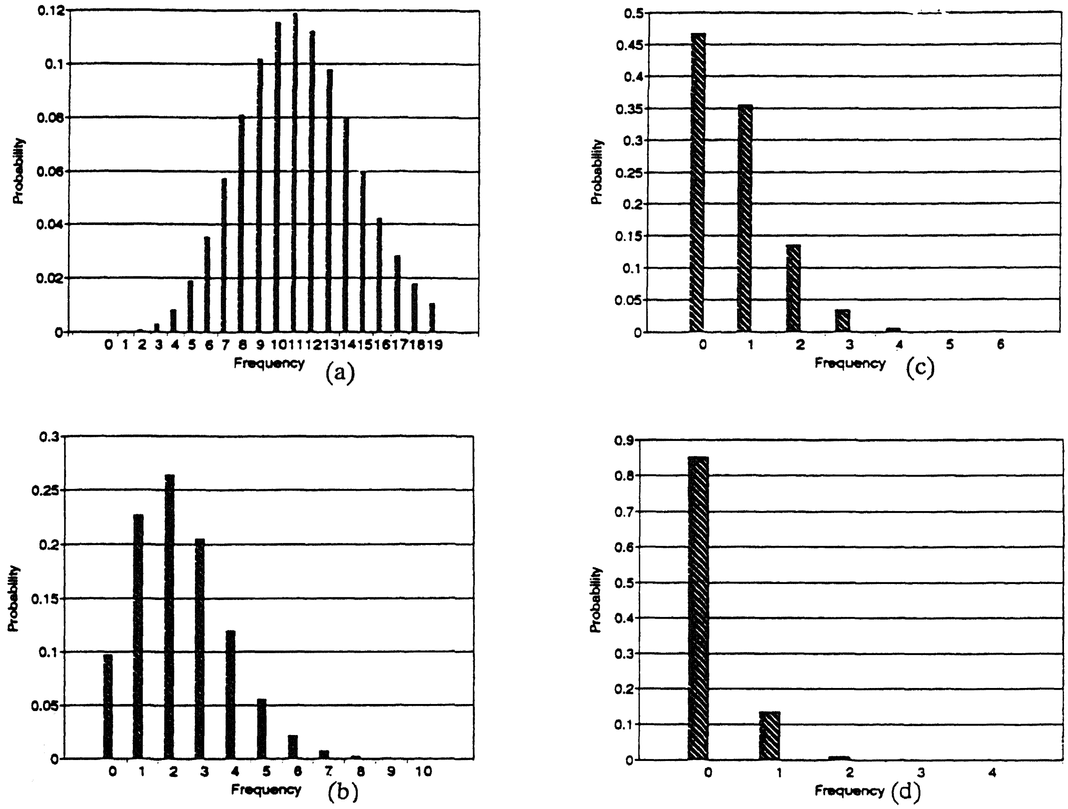

Fig.2 Distributions of earthquake occurrence: (a) S=5; (b) S=6; (c) S=7; (d) S=8

3.3 *Distribution of site intensity associated with a specified earthquake occurrence frequency*

The distribution of site intensity during a 100 year period with a specified earthquake occurrence frequency is

$$F_{S/K_i} = \exp(P_{K_S,S_i}(1-\sum_{i=5}^{12} P_{f_i})) \tag{10}$$

in which F_{S/K_i} is the probability distribution of site intensity with a specified earthquake occurrence frequency.

The probability density of site intensity corresponding to (10) is

$$f_{S/K_i}=(a_2+\frac{a_3}{\ln 10 S_i})\exp(-v_i)F_{S/K_i}P_{K_S,S_i}f_{S_i} \tag{11}$$

in which f_{S/K_i} is the probability density function of site intensity. The probability density of site intensity for the Hubei area is shown in Fig. 4.

4 DISTRIBUTION OF PEAK GROUND ACCELERATION WITH SPECIFIED SITE INTENSITY

For structural seismic design and reliability analysis, the following statistical relationship between site intensity and peak ground acceleration is used

$$A_{S_i} = 10^{(S_i lg2 \ - \ 0.01)} \tag{12}$$

in which A_{S_i} is the peak ground acceleration with a specified site intensity S_i.

The deterministic relationship in (12) is useful in estimating the preliminary earthquake loading for a specified site intensity. However, actually peak ground acceleration needs to be considered as a

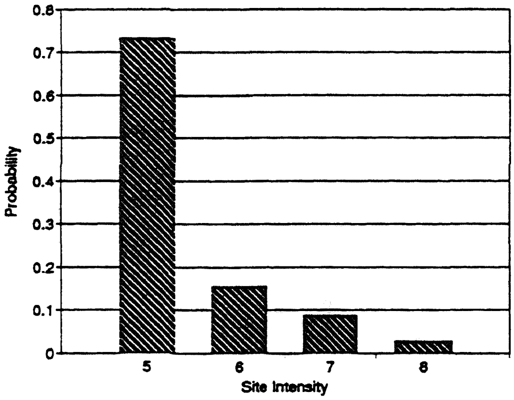

Fig.3 Probability of site intensity

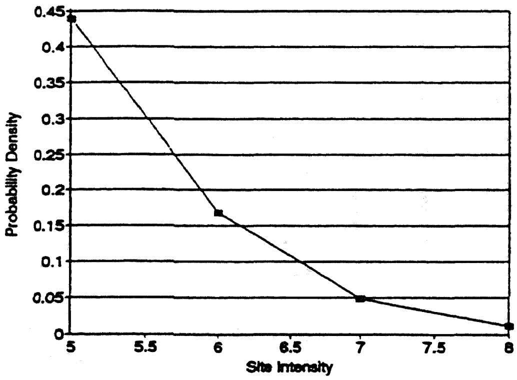

Fig.4 Hazard result of Hubei area

random variable for a specified site intensity S_i. Based on historical earthquake data gathered by Yener and Shen (1991), the writers propose the use of the following modified extreme value distribution to describe peak ground acceleration in order to analyze the distribution of peak ground acceleration with specified site intensity.

$$P(A_{S_i}) = \exp(-\exp(-b_i)) \qquad (13)$$

in which $P(A_{S_i})$ the probability of peak ground acceleration at a specified value S_i, and b_i is given as

$$b_i = c_1 + c_2 A_{S_i} + c_3 lg A_{S_i} \qquad (14)$$

in which c_1, c_2, c_3 are factors determined by least square approximations. Table 1 lists the computed statistical values of c_1, c_2, and c_3. In Table 1, μ_A is

Table 1. Statistical Parameters of Acceleration for MEVD

S_i	No	μ_A	σ_A	c_1	c_2	c_3
5	40	38.09	30.06	-5.25	0.00	3.88
6	32	80.42	72.99	-3.26	0.06	1.89
7	72	146.18	81.88	-4.45	0.01	1.71
8	7	265.41	173.96	-4.83	0.00	2.20

Table 2. Distribution Test for MEVD

S_i	MEVD-I χ^2
5	0.821
6	0.847
7	1.268

the mean value of peak ground acceleration and σ_A is the variance of the peak ground acceleration.

To examine this probability distribution, the χ^2-test is used. Table 2 lists the χ^2 values for MEVD for the specified percent point of 0.05.

5. CONCLUSIONS

In earthquake engineering, some variables can be seen as fuzzy set to analysis. Use polynomial function to evaluate the degree of membership is a effective and convenient method. Earthquake risk analysis is a difficult topic because of the random characteristics of earthquake. This article handles it successfully by the probability method.

REFERENCES

Cornell, C.A. 1975. Seismic risk analysis of Boston. *J. Struct.* div. ASCE 101, 2027-2043.

Egozcue, J.J. 1991. A method to estimate intensity occurrence probabilities in low seismic activity regions. *Earthquake Engrg. Struct. Dyn.*, Vel 2, 43-60.

McGgire, R.K. 1977. Effects of uncertainty in seismicity on estimates of seismic hazard for the east coast of United States. *Bull. Seism. Soc. Am.* 67, 827-848.

Mortgat, C.P. & H.C. Shah 1979. A Bayesian model for seismic hazard mapping. *Bull. Seism. S.soc. Am.* 69, 1237-1251.

Schmucker, K.J. 1984. *Fuzzy sets, natural language computations, and risk analysis*. Computer Sci. Press.

Shen, H. 1989. Fuzzy wind evaluation of strong sites. *Proc. of 3th conf. Wind Engrg. in China.*

Yener, M. & H. Shen 1991. Earthquake risk analysis for the Hubei area based on magnitude and PGA. *Struct. Engrg. and Mech. Report, CEE-SEMD-91-3*, Utah State Univ., Logan, Ut 84322-4110.