Fuzzy evaluation and statistical analysis of site intensity

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ABSTRACT: In fuzzy set analysis, site intensity is effectively used as a continuous variable. The degree of membership of site intensity is assumed to be represented by rth order polynomial functions. The corresponding (r+1) parameters are determined by the fuzzy properties of site intensity. The stochastic properties of site intensity are concerned with probability of earthquake occurrence frequency. Site intensity is related to peak ground acceleration. The peak ground acceleration distribution corresponding to a specified site intensity is obtained. An example problem is solved to determine the seismic hazard of Hubei area in China.

1 INTRODUCTION

The traditional methods for seismic hazard analysis are based on the probability theory to provide the necessary and sufficient tools in dealing with the uncertainty and imprecision of risk in decision analysis. The theory of fuzzy sets suggests that much of the uncertainty in risk analysis is based on the fuzziness of the information in the database and, more particularly, in the fuzziness of the underlying probabilities (Schmucker (1984)). In seismology and earthquake engineering, site intensity, as a discrete variable, is used for earthquake zoning. In fuzzy set analysis, it can be effectively handled in a continuous manner.

The estimation of seismic hazard in low and intermediate seismic activity regions, based on earthquake magnitude, site intensity and ground peak acceleration, has been a challenge for researchers for the past thirty years (Cornell (1975), McGgire (1977), Mortgat (1979), and Egozcue (1991)). However, no definite relationship has yet been established to correlate site intensity with earthquake occurrence frequency and peak ground acceleration. In this paper, a procedure for the statistical analysis of site intensity is proposed, which correlates the probabilities of site intensity with earthquake occurrence frequency and peak ground acceleration. An example problem is solved to determine the seismic hazard of Hubei area in China.

2 FUZZY EVALUATION OF SITE INTENSITY

2.1 Discrete and continuous sets of site intensity

In seismology and earthquake engineering, site intensity is used as a variable for earthquake zoning. Site intensity is defined as a 12 degree set as

\[ U_s = \{ S_1, S_2, \ldots, S_{12}\} = \{1, 2, \ldots, 12\} \]  

(1)

However, the set of site intensity is essentially continuous. In fuzzy analysis, site intensity can be effectively handled in a continuous manner. The definition of closed continuous set of site intensity is

\[ F_s = \{ S / S \in [0, 12]\} = [0, 12] \]  

(2)

Each index \( S_i \) of site intensity in discrete universe \( U_{si} \) should be a fuzzy subset in continuous universe \( F_s \). \( S_i \) is defined as the degree of fuzzy site intensity.

2.2 Degree of membership of fuzzy set site intensity

The degree of membership of site intensity is assumed to be represented by an rth order polynomial function (Shen (1988))

\[ \mu = a_0 + a_1 S + a_2 S^2 + \ldots + a_r S^r \]  

(3)

For example, based on the following conditions
\[ \mu_S = 0, \quad \text{for} \quad S = S_{i-1} \]
\[ \mu_S = 1, \quad \text{for} \quad S = S_i \]
\[ \mu_S = 0, \quad \text{for} \quad S = S_{i+1} \]

(4)

the 2nd order polynomial function of membership degree with respect to site intensity can be obtained as

\[ \mu_S = 1 - S_i^2 + 2S_i - S^2 \]

(5)

and

\[ S \in [S_{i-1}, S_i + 1] \]

(6)

The 2nd order polynomial function of membership degree is shown in Fig. 1.

![Fig. 1. The function of membership degree for site intensity](image)

3 STATISTICAL ANALYSIS OF SITE INTENSITY

3.1 Probability of earthquake occurrence frequency for a Specified Site Intensity

In general, if an event can be described by a discrete Poisson distribution, it can be concluded that the random variable representing the event is a positive integer, and the probability of the event is small. The discrete Poisson distribution of earthquake occurrence frequency can be given as

\[ P_{K S_i} = \frac{(\lambda S_i)^{K S_i}}{K S_i!} \exp(-\lambda S_i) \]

(7)

in which \( P_{K S_i} \) is the probability of earthquake occurrence with a frequency \( K S_i \), representing a random variable, for a specified site intensity \( S_i \) during a period of \( T \) years. \( \lambda S_i \) is earthquake occurrence frequency for the same site intensity \( S_i \) during a unit period.

In Hubei area of China (Yener and Shen (1991)), \( \lambda_5 = 0.1133, \lambda_6 = 0.0233, \lambda_7 = 0.0076, \lambda_8 = 0.0016 \). For \( T = 100 \) years, \( P_{1.5} = 1.3604 \times 10^{-4}, P_{1.6} = 0.2267, P_{1.7} = 0.3554, P_{1.8} = 0.1878 \). The distributions of frequencies for \( S_i = 5, 6, 7, \) and \( 8 \) are shown in Fig. 2. Through the \( \chi^2 \)-test, the earthquake occurrence data of Hubei area can be shown to be represented by the Poisson distribution with a percent point of 0.05.

3.2 Probability of site intensity

Site intensity is considered to be an extreme value random variable. On the basis of the favorable comparison of analytical results and the actual data, the writers propose the use of the following modified extreme value distribution (MEVD) to describe site intensity

\[ f_{S_i} = \exp[-\exp(-\nu_i)] \]

(8)

in which \( f_{S_i} \) is the probability of site intensity at a specified value \( S_i \), and \( \nu_i \) is given as

\[ \nu_i = a_1 + a_2 S_i + a_3 S_i^2 \]

(9)

in which \( a_1, a_2, \) and \( a_3 \) are factors that are determined by least square approximations. In general, \( \nu_i \) is defined as a linear function of \( S_i \). However, the analytical results obtained in the present investigation indicate that the inclusion of the third term on the right hand side of (9) yields a better correlation with the actual data. In (9), \( a_1, a_2, \) and \( a_3 \) are factors determined from the following expressions, which are developed by using least square approximations.

The values of \( a_1, a_2, \) and \( a_3 \), determined on the basis of the earthquake data of the Hubei area during the past 300 years (Yener and Shen (1991)), are \( a_1 = 36.7572, a_2 = 5.3973, \) and \( a_3 = -89.3933 \). The distributions of site intensity are shown in Fig. 3.
3.3 Distribution of site intensity associated with a specified earthquake occurrence frequency

The distribution of site intensity during a 100 year period with a specified earthquake occurrence frequency is

\[ F_{S|\xi} = \exp(\mathcal{P}_{S|\xi}(1 - \sum_{i=5}^{12} P_i)) \quad (10) \]

in which \( F_{S|\xi} \) is the probability distribution of site intensity with a specified earthquake occurrence frequency.

The probability density of site intensity corresponding to (10) is

\[ f_{S|\xi} = \frac{a_3}{\ln 10 S_i} \exp(-\nu) F_{S|\xi} P_{S|\xi} f_S \quad (11) \]

in which \( f_{S|\xi} \) is the probability density function of site intensity. The probability density of site intensity for the Hubei area is shown in Fig. 4.

4 DISTRIBUTION OF PEAK GROUND ACCELERATION WITH SPECIFIED SITE INTENSITY

For structural seismic design and reliability analysis, the following statistical relationship between site intensity and peak ground acceleration is used

\[ A_S = 10^{(5/6^2 - 0.01)} \quad (12) \]

in which \( A_S \) is the peak ground acceleration with a specified site intensity \( S_i \).

The deterministic relationship in (12) is useful in estimating the preliminary earthquake loading for a specified site intensity. However, actually peak ground acceleration needs to be considered as a
random variable for a specified site intensity $S$. Based on historical earthquake data gathered by Yener and Shen (1991), the writers propose the use of the following modified extreme value distribution to describe peak ground acceleration in order to analyze the distribution of peak ground acceleration with specified site intensity.

$$P(A_S) = \exp(-\exp(-b_S))$$ (13)

in which $P(A_S)$ the probability of peak ground acceleration at a specified value $S$, and $b_S$ is given as

$$b_S = c_1 + c_2 A_S + c_3 \log A_S$$ (14)

in which $c_1$, $c_2$, $c_3$ are factors determined by least square approximations. Table 1 lists the computed statistical values of $c_1$, $c_2$, and $c_3$. In Table 1, $\mu_A$ is the mean value of peak ground acceleration and $\sigma_A$ is the variance of the peak ground acceleration.

To examine this probability distribution, the $\chi^2$-test is used. Table 2 lists the $\chi^2$ values for MEVD for the specified percent point of 0.05.

5. CONCLUSIONS

In earthquake engineering, some variables can be seen as fuzzy set to analysis. Use polynomial function to evaluate the degree of membership is a effective and convenient method. Earthquake risk analysis is a difficult topic because of the random characteristics of earthquake. This article handles it successfully by the probability method.

REFERENCES


