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A general algorithm for geometry probability P (Y > y | E_i, m) in the seismic hazard analysis

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ABSTRACT: Two equivalent algorithms for calculating geometry probability P (Y> y| E_i, m) in the seismic hazard analysis considering arbitrary shape of potential source area and attenuation relationships in different directions from an epicenter are presented: difinition algorithm and conjugate algorithm. The definition algorithm consists of the following two steps: Firstly, Set up an algorithm to make arbitrary N polygon divided into a lot of small enough elements; secondly, Set up a cret-rior to judge if a certain point is belonged to an interior one of some plane closed curve. The key to the conjugate algorithm lies in the calculation of the intersectimg area of two plane closed curves--arbitrary shaped contour of potential source area and arbitrary shaped attenuation curve. Therefore, the matter finding P(Y> y| E_i, m) is attributed to the calculation of the intersecting area of two plane closed curves. Howere, It should be pointed out that the attenuation curve or isoseismal curve here such an attenuation one that is rotated 180°about an epicenter and moved to a site. As mentioned above, using this algorithm, the intersecting area which two plane closed curves, including com-plex connected or even non-connected curves (e. g. abnormal intensity area), intersecting each other can be calculated. This paper gives an example at an algorithm's angle, the results calculated by two algotithms in this paper are compared with the ones by the analytic method, for intensity, the results by three methods are consistent with each other, for acceleration, there is a slight difference among them. Therefore, this paper will provide an udeful tool for the calculation of geometry probability P(Y> y| E_i, m) determined by the contour of potenial source area and regional or local attenuation curves on which to base the well-considered information on seismogology, seismicity, geophysical field and lateral inhomogeneity where seismic wave propagates in different directions from an epicenter etc. in a working region and its surroundings.

INTRODUCTION

Since the probabilistic method of the seismic hazard analysis was first advanced by Cornell taking seismic source as point one in 1968 [1], the research on the implementation of this analysis has technically made a great advance[2], [3]. this can be seen from extensively adopting the fault rutpture model and elliptic attenuation model. this paper is just based on these results.

At present, in the seismic hazard analysis, the potential source area is always delineated into a single convex polygon, such as convex quadrilateral or convex pentagon, however, the actual existing geological structures are rather complicated, obviously, it is not reasonable that one boundary of a potential source area which is tens or even one hundren kilometers or more long is expressed by a single straight line, on the other hand, as seen by the analysis of a large number of the firsthand data on macroseismic effect fields, there is a great difference between the actual attenuation of ground motion and the elliptic attenuation, because of the effect of lateral inhomogeneity of geological structures on the propagation of seismic wave, the attenuation of seismic wave is made to bring about complicated change, for example, even if the attenuation curves which are artifically smoothed look like an ellipse, they are often neither concentric nor coaxial. Furthermore, the inside and outside isoseismal curves of a great earthquake, or rather the shape of lower and higher intensity isoseismal curves isn't consistent with each other. Generally speaking, the shape of lower intensity isoseismal curves may deviate from further an ellipse than the shape of higher ones. In this case, it isn't in keeping with the actual situation that the characteristic of attenuation of seismic waves in different directions from an epicenter is discribed by a single elliptic attenuation relationship.

By adopting two algorithms in the paper, while considering arbirary shape of potential source area, the characteristic of attenuation of seismic waves in different directions from an epicenter can be considered. It should point out that whether earthquake intensity data are taken from the firsthand intensity data or isoseismal curve map by the statistics of attenuation law in different directions, because the epicentral distance is dealt with as a random variable, the statistic directions or the rays drawn from an epicenter should not too densely be taken in the statistics of attenuation relationship, In general, it can be done in 8 or 16 directions. Finally, only at algorithm's angle, this paper gives an example.

A BRIEF ACCOUNT OF SEISMIC HAZARD ANALYSIS

The aim of this paper is not thoroughly to deal with the entire process of seismic hazard analysis, and discuss how to delineat the potential source area and how to estimate the seismicity parameters, and also not to study how to set up the attenuation relationship, but only to account for the algorithm for geometry pobability P (Y> y| E_i, m) considering arbitrary shpe of potential source area and attenuation relationships in different directions from an epicerter, hence only a brief account of relevant parts of seismic hazard analysis is given.

Assume that there are N potential source areas all round a site, all earthquake occured over them have a destructive effect on the site, and that v_i is the annual average occurrence rate of earthquake with magnitude $m \geq m_0$ over the source E_i, then the probability that the ground motion parameter Y exceeds a given value y in a site in a year is calculated as follows:

$$P_1(Y>y)=1-\ EXP[-\sum_{i=1}^{N} P(Y>y|E_i)v_i] \qquad (1)$$

in which v_i is very small, so approximately obtained:

$$P_1(Y>y)=\sum_{i=1}^{N} P(Y>y|E_i)v_i \qquad (2)$$

in which P (Y> y| E_i) denotes the probability that Y at the site caused by the events with magnitude $m \geq m_0$ occurred over the ith source exceeds y, here we assume that the occurrence of earthquake would obey the homogeneous' Poisson process. so only one seismicity parameter is used to describe it (namely, the annual average occurrence rate of earthquake with magnitude no lower than a specified one).

The exceedance probability in t years is calculated by the following formula:

$$P_t(Y>y)=1-\ [1-P_1(Y>y)]^t \qquad (3)$$

As seen above, the calculation of the formula(2) and(3) rests with the calculation of conditional probability P (Y> y| E_i). Because the distribution of the events with magnitude from m_0 to m_u occurred over the ith source is the probabilistic one, based on the total probabilistic formula, we have:

$$P(Y>y|E_i)=\int_{m_0}^{m_u} P(Y>y|E_i,m)f(m)dm \qquad (4)$$

in which, f (m) is the probability density function given by the magnitude frequency relationship, P (Y> y | E_i, m) denotes the probability that Y at a site caused by the event with magnitude m occurred over the ith source exceeds y, and is associated with the space distribution of earthquake, the geometric shape of potential source area and attenuation curve or isoseismal one . Therefore the key point finding P (Y> y| E_i, m) would result in the calculation of the intersecting area of two plane closed curves. If the potential source area is a simple convex polygon, the probabilitcs of occurrence of earthquake everywhere on it is same, and the attenuation relationship is an elliptic one, then P (Y> y| E_i, m) can be calculated by the analytic method [4] . If the geometric shape of potential source area and attenuation curve or isoseismal curve are optional arbitrary, e. g. the complex connected or nonconnected one , and the occurrence of earthquake over a potential source area is uniform, then can we find an effective algorithm to calculate P(Y> y| E_i, m) yet?, This is exactly what is to be studied in this paper.

(1) THE DEFINITION ALGORITHM

The basic idea of this paper is to divide the arbitrary shape of potential source area into a lot of small enough elements, and an event with magnitude m may occur over any one of them, and the probability of occurrence of earthquake over different elements can not be same, which is shown by different weight coefficients, an event over each element is checked if Y at a site is made to exceed y, all element areas over which the events can cause Y > y at a site are accumulated, the ratio of the accumulated area to the total area of the potential source area is just the geometric probability P (Y> y| E_i, m) . It should be point out that all the areas above are the weight area. In the actual process of calculation, all the potential source areas and earthquakes with magnitudes from m_0 to m_u must simultaneously be accounted for.

As seen above, the definition algorithm consists of the following two steps: Firstly, Set up an algorithm to make arbitrary N polygon divided into a lot of small enough elements; Secondly, Set up a criterion to judge if a certain point is belonged to an interior one of some plane curve.

The first step is realized through arbitrary N polygon to be divided into N-2 big triangles with original vertexes, and then every big triangle to be subdivided into a lot of small elements. see Fig. 1

The second step is to set up the criterior based on the following fact: If any curve drawing from a judged point to infinity passes through a closed curve, when the number of the intersecting points of the two curves is odd one, the judged point must be inside of the closed curve. On the contrary, even one, outside of it. See Fig. 2. the point A is an interion one, and point B is an exterior. It should be point out that an exterior point may be misjudged as an interion one when two curves circumscribes each other, because the number of the intersecting points is odd one. In order to avoid this misjudgement, we would draw more than two curves from an judged point to infinity.

Assume that the attenuation curve or isoseismal curve caused by an earthquake with magnitude m over a certain element of the ith potential source area is arbitrary shape of plane closed curve, and can be expressed by arbitrary shape of K polygon, if a site is included in it, the area of the element is multiplied by its weight and the product is counted in the effective weight SC, and the products which all the areas of elements are multiplied by the corresponding weights are counted in the total weight area SI. After all elements are judged,

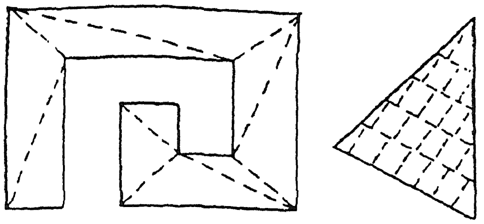

Fig. 1 arbitrary n-polygon can always be divided into n-2big triangles and every big one can be survided in various ways.

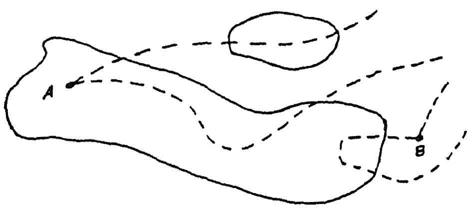

Fig. 2 the judgement of the interior and exterior points by the odevity.

we can get:

$$P(Y>y \mid E_i, m) = \frac{SC}{SI} \qquad (5)$$

This is the definition formular of geometric probability

(2) THE CONJUGATE ALGORITHM

Assume that the contour of potential source area is arbitrarily shaped plane curve, and can be expressed by N polygon. The attenuation curve or isoseismal curve is also arbitrarily shaped plane curve, and by K polygon, then the effective area above can be obtained from the intersecting area of these two polygons, but this K polygon should be such an attenuation curve that is rotated 180 °about an epicenter and moved to an site. For the symmetrical attenuation curve about an epicenter, as ellipse, it can directly be moved from the epicenter to a site. Generally speaking, if the graphical symmetry isn' t accounted for, or the attenuation curve is unsymmetrical one, it should be rotated 180°about an epicenter and moved to the site, and then the intersecting area of two polygons is again calculated.

As mentioned above, in fact, the matter finding P ($Y > y \mid E_i$, m) corresponds to the one calculating the intersecting area of two plane closed curves, at first sight, the possible cases of intersection of two plane curves are very complicated, but as seen by configration, later we shall see that this is not the case. In measure' s sight, Fig. 3 and Fig. 4 are not same at all, but in the sight of configration, they are same, only a matter of the sides of K polygon (dotted curve) is to multiply enter in and pass through the N polygon(solid curve), and divide it into two parts: left hand and right hand. We are just based on this to set up a general algorithm for calculating the intersecting area of two plane closed curves.

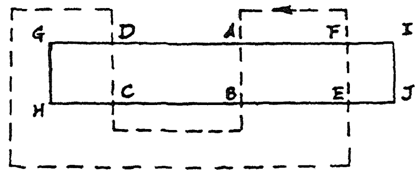

Fig. 3 this is the configuration relation of fig. 4

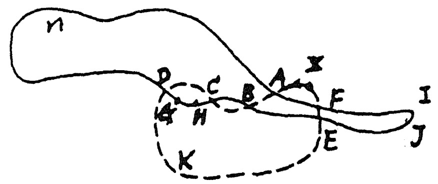

Fig. 4 this is the measure relation of fig. 3

The concrecte process of the cojugate algorithm is as follows: The curve K(arbitrary shape of isoseismal curve or cutting curve) along its forward direction (counter-clockwise direction) cuts the curve N(arbitrary contour of potential source area or cutted curve), in the first layer, the curve N is first divided into left hand and right hand, at this moment, the area of left hand is temporarily counted in SC and the right hand is temporarily given up. Then entering into the second layer, analysing the left hand in the first layer again, if it may be subdivided into the left hand and right hand, the area of the right hand in this layer should be subtracted from SC, because all the areas of the left hand and right hand in this layer have been counted in SC in the first layer,. Keeping on doing so, if the left hand in a certain layer can no longer be subdivided into the left hand and right hand, we would return to the analysis of the right hand in the preceding layer again, if it may be subdivided into the left hand and right hand. then the area of the left hand in this layer should be counted in SC, because all the areas of the left hand and right hand in this layer have been given up in the preceding layer. In this recursive analysis, from the beginning of the first layer, the preceding layer is continuously followed by the following layer and the following layer continuously returns to the preceding layer, until the concrete performing process is made to end up at the right hand in the first layer.

For example, in the sight of the configuration, Fig. 3 and Fig. 4 are equivalent, so the calculation of the intersecting area in Fig. 4 can be replaced by the calculation of the intersecting area in Fig. 3, but the measure ' s relationship reflects still in Fig. 4. The concrete process is as follows(See Fig. 3)

From beginning of any exterior point(e. g. point X) on the cutting curve k and outside of the cutted curve n, the curve k along forward direction cuts the curve n. In the first layer, the curve n is divided into the left hand

(ABJI) and right hand(AGHB), at the moment, the area ABJI is temporarily counted in SC, and the area AGHB is temporarily given up, then entering the second layer, analysing the left hand(ABJI) in the first layer, it can be subdivided into the left hand (EFAB) and right hand (EJIF), and then the area EJIF is subtracted from SC. the same procedure above is applied to the analysis of the right hand(AGHB) in the first layer, it can be divided into the left hand (CDGH) and right hand (CBAD), at the moment, the area of the left hand CDGH is counted in SC, the area of the right hand CBAD is given up. Analysing the left hand(CDGH) in the second layer, it can no longer be subdivided, and analysing the right hand (CBAD) again, it cannot also be subdivided. finally, analysing the right hand (EJIF) in the first layer again, it can no longer be subdivided, up to now, the entire analysing process is at an end, the area counted in SC is sum of areas EFAB and CDGH, this is the intersecting area that is to be found. If the weight function of space distribution of earthquake occurrence is given, such as the definition algorithm, then we will find the weight area.

As a special example, if the space distribution of earthquake occurrence is uniform, because the shape cutted out in each layer is all the the r-polygon, SC can fast be calculated by the following formula:

SC (the area temporarily counted in or subtracted from)

$$=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}+\frac{1}{2}\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}+\ldots+\frac{1}{2}\begin{vmatrix} x_r & y_r \\ x_1 & y_1 \end{vmatrix} \qquad (6)$$

in which x_1, y_1, x_2, y_2, ... x_r y_r are the coordinates of vertexes of the r-polygon.

(4) EXAMPLE

For lack of the actual data associated with arbitrary shape of potential source area and attenuation relationships in different directions from an epicenter, the example here is only an account of the algorithms themself that are presented in this paper. the basic data are taken from the part of seismic hazard analysis in" Report on the seismic microzonation in Deyang" [5]. The difference of them lies in adopting the potential source area of simple convex polygon, as quadrilateral or pentagon and the elliptic attenuation relationship in" Report on the seismic micrizonation in Deyang", but, in this example, the potential source area is divided into a lot of small element whose sides are no more than 10-20 kelometer long, and the elliptic attenuaton curves are replaced by inscribed 8-polygon and 16-polygon. We will compare the results by two algorithms in this paper with the ones by analytic method. See Table for the comparision of calculating results. It can be seen from above, for intensity, the calculating results by three methods are consistent with each other, but, for acceleration, there is slight difference among them. Generally speaking, the results by the definition algorithm is greater than the ones by the analytic method, the maximum percentage by difference among them is 8%, and the results by the conjugate algorithm is less than the ones by the analytic method, the maximum percentage by difference among them is 5%.

the comparison of calculating results

method \ number of side \ parameter		ntensity	acceleration (g)
analytic method		6.3	0.060
definition algorithm	8	6.2	0.064
	16	6.3	0.065
conjugate algorithm	8	6.2	0.057
	16	6.2	0.058

As seen above, (a) The intensity is not so sensitive to the algorithm as the acceleration, this is somewhat similar to the effect of variance on the corrected results of intensity and acceleration in the seismic hazard analysis; (b) As far as acceleration is cencerned, the results calculated as inscribed 16-polygon is greater than the ones as inscribed 8-polygon by the conjugate algorithm, and is less than ones as the elliptic attenuaby analytic method, this is because the area of an inscribed 16-polygon is greater than the one of an inscribed 8-polygon and less than the elliptic area. In general, the results by the definition algorithm is greater than ones by analytic method, this is because we have adopted a way" a point is relied upon instead of an area" in the paper. The definition algorithm is to judge if a site is included in an attenuation curve radiated from the centre of figure of a certain element(the first moment of area about it equals zero), if it is included in it, then the area is counted in the effective area, this will often make the effective area increased, this is because, in this case, the centre of figure of a certain element must be included in the attenuation curve. Provided the attenuation curve is rotated 180°about it and moved to the site, but entire area of the element is probably not included in the attenuation curve, hence" reply upon a point instead of an area" may make an effective area increased, however, in the sight of practical application, the calculating results given in this paper are acceptable.

(5) CONCLUSION

A general algorithm for the geometric probability P $(Y>y|\ E_i, m)$ in the seismic hazard analysis presented in this paper is to be able to account for the space inhomogeneity of earthquake occurrence, and provided an useful tool for the seismic hazard analysis for adopting the potential source area and attenuation relationships for well-considered the information on seismogeology, geophysical field, seismicity, lateral inhomogeneity where seismic wave propagates in different directions from an epicentre. At present, we haven't seen any related research results yet, and we hope that the algorithms in this paper will be put in practice with the advance of geonomy and earthquake engineering.

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