Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Influence of system initial conditions on elastic and inelastic response spectra

C.E.Ventura
Department of Civil Engineering, The University of British Columbia, Vancouver, B.C., Canada

R.Blázquez
Departamento de Ingeniería y Morfología del Terreno, Universidad Politécnica de Madrid y Instituto Eduardo Torroja, CSIC, Spain

ABSTRACT: The influence of the initial conditions on the computation of response spectra for elastic and inelastic, damped single-degree-of-freedom systems is considered here. Significant characteristics of the spectra resulting from various combinations of nonzero initial conditions are first evaluated for the responses for linear elastic systems. The discussion is then extended to the response of a type of inelastic systems. The results show that, in general, the long period region of the displacement and absolute acceleration response spectra for elastic and inelastic systems is more sensitive to the effects of the initial conditions than the middle and low period regions. The displacement and acceleration spectral values for long period systems with initials conditions are generally larger than those from systems initially at rest.

1 INTRODUCTION

The response spectrum method of analysis is widely used for evaluating the dynamic response of structural systems subjected to earthquake motions. In this approach, the peak response of a system due to a prescribed earthquake motion is generally computed assuming that the system is initially at rest; that is, assuming zero initial conditions. Although the significant aspects of response spectra computed assuming zero initial conditions have been extensively studied (Veletsos and Newmark, 1964), the key aspects for spectra including nonzero initial conditions are not well understood.

Nonzero initial conditions may arise for systems already undergoing vibrations when an earthquake occurs, or from evaluation of earthquake records where a segment at the beginning of the record is not available (Peknold and Ridell, 1978, Blázquez and Kelly, 1988 and Uang and Bertero, 1990).

In many practical applications, the initial conditions are not included in the computation of response spectra, and often this can be reasonably justified. There is, however, a paucity of information on the consequences of ignoring these effects when the response spectrum method is used for the dynamic analysis of structures.

The objectives of this paper are: 1) to help understand better the influence of initial conditions on the response spectra for elastic and inelastic, damped single-degree-of-freedom (SDOF) systems; and 2) to asses the design implications of considering or neglecting the effects of the initial conditions in a response spectrum analysis.

2 STATEMENT OF PROBLEM

The dynamic response of base-excited, viscously damped, SDOF systems is considered here. Each system is already vibrating at the reference time $t=0$ when a ground acceleration, $\ddot{y}(t)$ is applied. The relative displacement and velocity of each system at $t=0$ are known and identified as U_o and $\dot{U}_o$, respectively. A general form of the equation of motion of the SDOF system can be written as

$$m\ddot{u}(t) + c\dot{u}(t) + R(t) = -m\ddot{y}(t) \quad (1)$$

in which m is the mass, c is the coefficient of viscous damping, $R(t)$ is the restoring force which depends on the displacement amplitude at time t, and $\dot{u}(t)$ and $\ddot{u}(t)$ are the system's relative velocity and acceleration with respect to the ground motion.

For the case of elastic systems, the restoring force is expressed as $R(t)=ku(t)$, where k is the stiffness coefficient and $u(t)$ is the relative displacement of the system. In this case, Eq. 1 can be expressed as

$$\ddot{u}(t) + 2\zeta p\dot{u}(t) + p^2u(t) = -\ddot{y}(t) \quad (2)$$

where $p=\sqrt{(k/m)}=2\pi/T$ is the undamped circular natural frequency of the system (T is the undamped natural period), and $\zeta=c/2pm=c/2\sqrt{(km)}$ is the fraction of critical damping.

It is of interest to evaluate the influence of U_o and $\dot{U}_o$ on the absolute value of the numerically largest values of relative displacement, U, and absolute acceleration, SA, for ground excited systems.

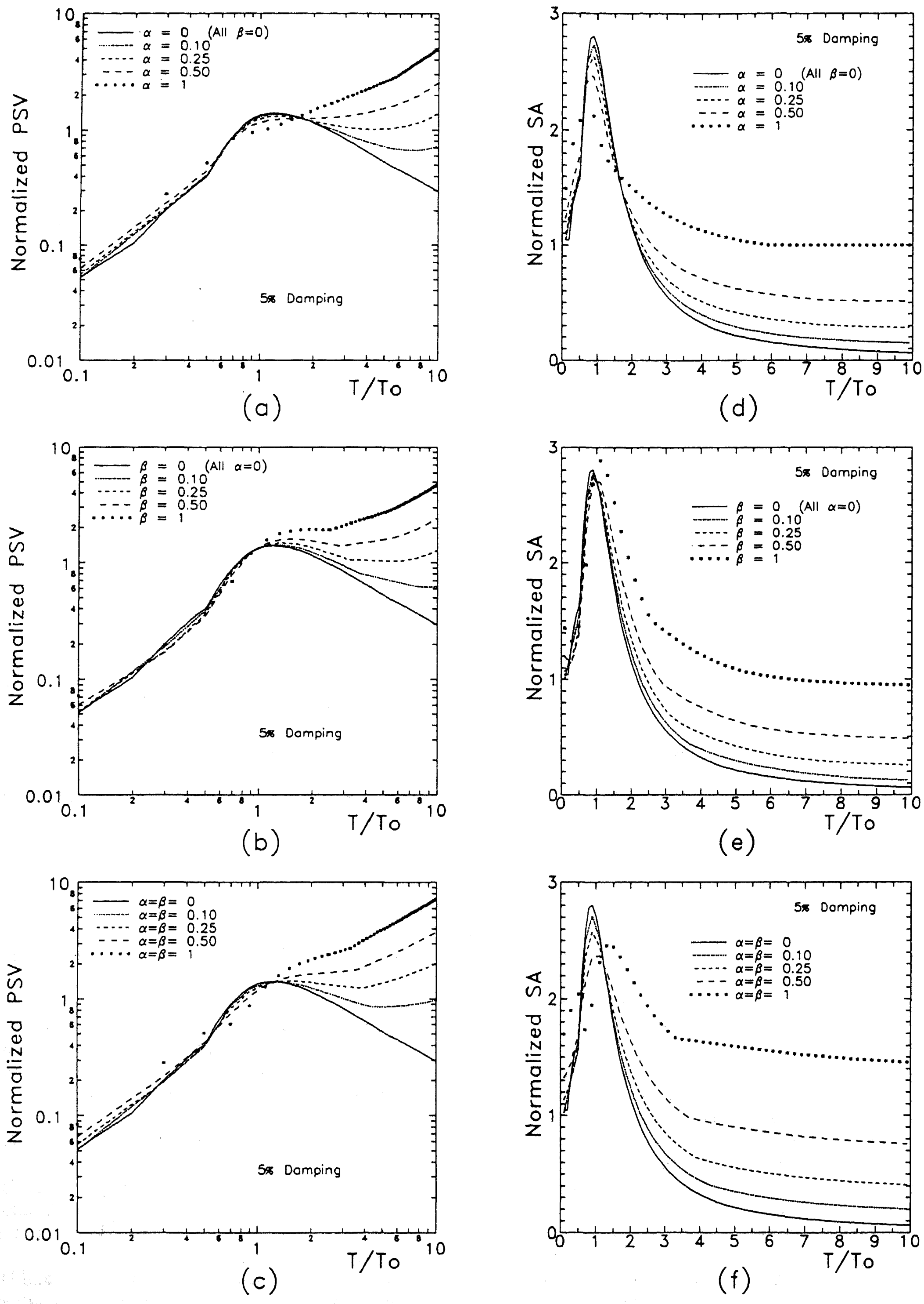

Figure 1. Influence of initial displacement (α) and initial velocity (β) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic systems subjected to sinusoidal base excitation.

3 RESPONSE OF ELASTIC SYSTEMS

The state of motion of an elastic SDOF system just before the occurrence of a ground excitation can be characterized in terms of the initial displacement, U_o, and initial velocity, $\dot{U}_o$. The response of the system to these initial conditions depends on its natural frequency and damping. Systems subjected to the same initial displacement and velocity, but with different natural frequency and damping, will respond differently.

The peak value of the ground motion is a parameter commonly used for describing the level of ground excitation and for comparing it with the maximum response of the system. It would also be practical to relate the initial conditions of the system to this parameter. To this end, the peak ground acceleration, $\ddot{Y}$, the initial conditions and the natural frequency of the system can be interrelated by the dimensionless coefficients:

$$\alpha = -p^2 U_o/\ddot{Y} \quad \text{and} \quad \beta = -p\dot{U}_o/\ddot{Y} \qquad (3)$$

The α and β coefficients represent the ratio of equivalent instantaneous accelerations of magnitude p^2U_o and $p\dot{U}_o$, respectively, to the peak base acceleration, $\ddot{Y}$. Alternatively, α can be described as the ratio of a base shear (kU_o) produced by a static displacement U_o to the equivalent static force ($m\ddot{Y}$) produced by the peak ground acceleration. Similarly, β can be described as the ratio of a base shear ($mp\dot{U}_o$) produced by an impulse $m\dot{U}_o$ to the equivalent static force $m\ddot{Y}$ produced by the peak ground acceleration.

The sensitivity of the response to various combinations of α and β can be evaluated for SDOF systems subjected to a simple, one-cycle sinusoidal base excitation of the form $\ddot{y}(t)=\ddot{Y}\sin(2\pi t/T_o)$, in which T_o is the duration of one full cycle of excitation.

It has been shown by Veletsos and Newmark, 1964, that for base excitations of the complexity of strong motions earthquake records, the response spectra are similar to those for simple excitations, and that their salient features can be reasonably identified from the spectra for simple pulses, provided the gross characteristics of the ground acceleration, velocity and displacement are known. To this end, the pseudo-velocity, $PSV=pU$, and absolute acceleration, SA, response spectra of systems with 5% damping were computed for various combinations of α and β. The results are shown in Fig. 1 where the spectra are plotted as a function of the ratio T/T_o. The PSV spectra are drawn in log-log scale with the PSV values normalized with respect to the peak base velocity. The SA spectra are drawn in linear-linear scale and the SA values are normalized with respect to the peak base acceleration.

Figure 1 shows that nonzero initial conditions, when expressed in terms of α and β, have a significant effect on the response of very flexible systems for which $T>>T_o$. The effect is negligible for rigid systems that are more sensitive to the high frequency components of the input motion than to the initial state of motion.

Ventura and Blázquez, 1990, have presented a more detailed study on the influence of α and β on the response of elastic systems subjected to sinusoidal excitations.

For more complex excitations the same trends on the variation of the response spectra can be expected. The response spectra for the 1940 El Centro earthquake (north-south component) are shown in Fig. 2, and the spectra for the 1985 Mexico earthquake recorded at the SCT station (east-west component) are shown in Fig. 3. The El Centro earthquake was selected for this study because it produces significant responses over a wide band of system periods, while the Mexico earthquake was selected because it produces significant responses for a narrow range of periods, mostly periods around 2 to 3 sec. Figures 2(a) and 3(a) show the 5% damped PSV elastic spectra and Figs. 2(d) and 3(d) show the SA elastic spectra for different values of α (with $\beta=0$). These spectra represent the peak values of pseudo-velocity and absolute acceleration during the duration of the excitation only.

As in the case for sinusoidal base motion, the long period regions of the PSV and SA spectra are more sensitive to the effects of the initial conditions than the other regions of the spectra. The SA spectra also show that the base shear for a long-period system with nonzero initial conditions is larger than that for a system initially at rest. This may lead to unconservative designs of long period structures if the initial conditions effects are not properly accounted for.

4 RESPONSE OF INELASTIC SYSTEMS

The parameters that usually characterize the response of inelastic systems are the yield displacement, U_y, and the associated force level, R_y, that produces this displacement. To obtain the dynamic response of an inelastic system for which its force-deformation characteristics are known, Eq. 1 can be solved directly. However, it is more desirable to express this equation in a normalized form such that the specific parameters that influence the response can be more readily identified, as in the case for elastic systems (Mahin and Lin, 1983). The normalized version of Eq. 1 can be written as:

$$\ddot{\mu}(t) + 2\xi p\dot{\mu}(t) + p^2\rho(t) = -A\ddot{y}(t) \qquad (4)$$

in which $\mu(t)=u(t)/U_y$ is defined as the displacement ductility and its peak value is referred to as the Ductility Factor (Clough and Penzien, 1977); and $\rho(t)=R(t)/R_y$ and $A=p^2/\eta\ddot{Y}$. The dimensionless parameter $\eta=R_y/m\ddot{Y}$ represents the system's yield strength relative to the maximum inertia force of an infinitely rigid system.

Equation 4 provides an efficient way to evaluate $\mu(t)$ for all systems having the same natural frequency, the same hysteretic characteristics and the same strength over inertia index (η) subjected to ground motions having the same shape.

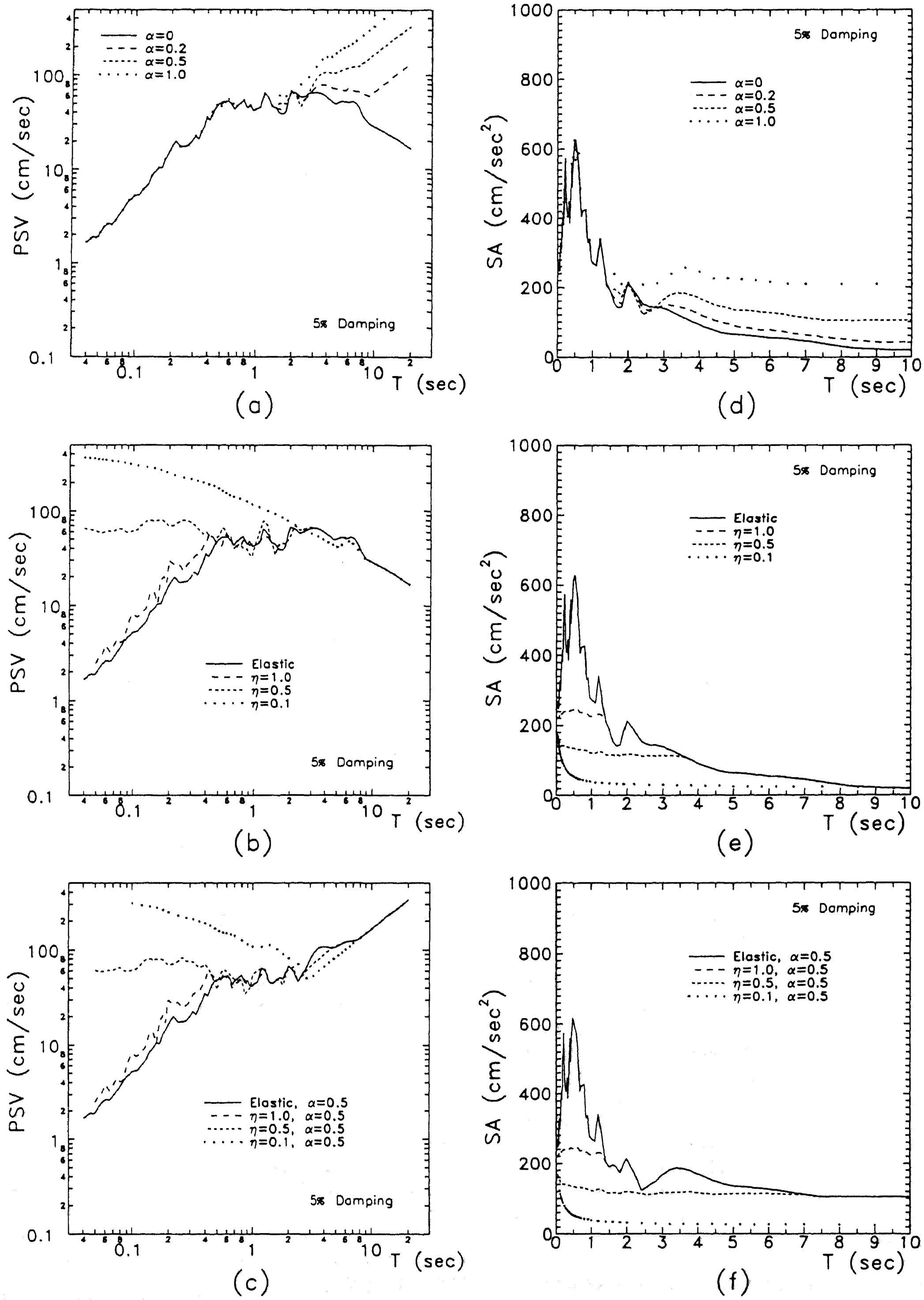

Figure 2. Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic and elasto-plastic systems subjected to El Centro 1940 earthquake.

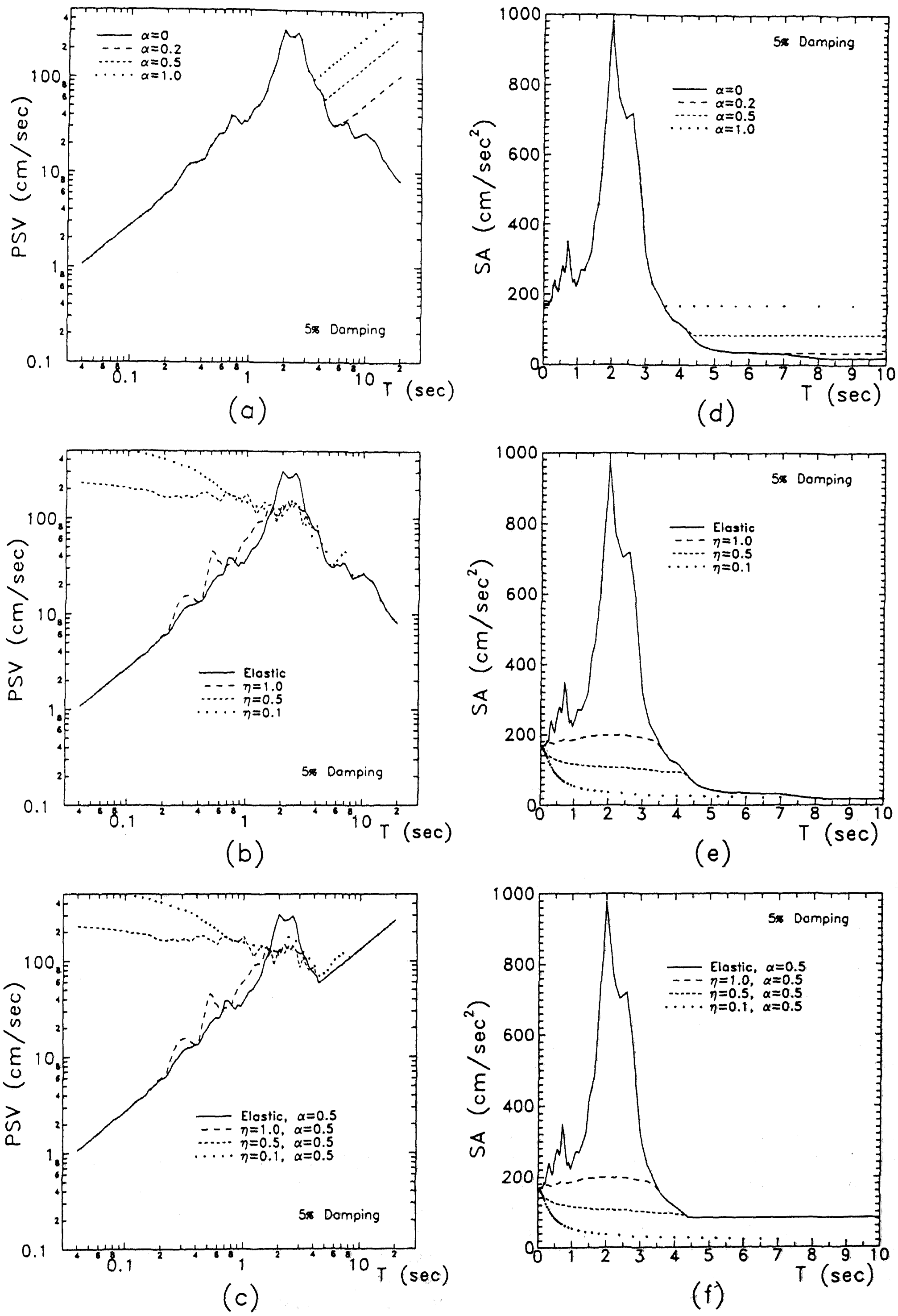

Figure 3. Influence of initial displacement (α) on pseudo-velocity (PSV) and absolute acceleration (SA) response spectra for elastic and elasto-plastic systems subjected to Mexico 1985 earthquake.

For systems with hysteresis diagrams that exhibit an initial elastic behaviour, like the elasto-plastic systems, R_y and U_y can be related by $R_y=kU_y$ and the displacement ductility can be expressed directly in terms of η as

$$\mu(t) = p^2 u(t)/\eta \ddot{Y} \quad (5)$$

In a response spectrum analysis, for a given value of η, the peak value of u(t) can be obtained from the corresponding response spectrum and the Ductility Factor can be readily computed from Eq. 5.

To illustrate the sensitivity of the response of elasto-plastic systems to variations of the η index, the maximum responses due to El Centro and Mexico records were computed using a modified version of the computer program described in Mahin and Lin, 1983.

The maximum displacements for systems initially at rest ($\alpha=\beta=0$) are shown in Figs. 2(b) and 3(b) where they are presented as pseudo-velocity spectral values using the natural period of the elastic portion of the hysteresis diagram as the reference period. The corresponding absolute acceleration spectra are shown in Figs. 2(e) and 3(e). For reference, the associated elastic spectra are also included in the figures and are used as a basis of comparison between elastic and inelastic responses.

These figures show that for $\eta>0.5$, the responses of rigid and medium period systems up to 3 sec. are generally more sensitive to variations of η than those for very flexible systems. As the value of η increases, the response approaches that of the truly elastic systems, and the results are less sensitive of variations of η. For small values of η ($=0.1$) a wider range of systems are affected, since low values of η correspond to systems that have a low yield level resistance with respect to the maximum inertia force and therefore are expected to undergo large excursions beyond the yield level.

Introducing initial conditions to the computation of the PSV spectra for elasto-plastic systems has the effect shown in Figs. 2(c) and 3(c). The effect on the SA spectra is shown in Figs. 2(f) and 3(f). In these cases, including only an initial displacement ($\alpha=0.5$, $\beta=0$) in the computation of the inelastic response of systems to the El Centro and Mexico records completely alters the shape of the spectrum in the long period region when compared to that of the inelastic systems with zero initial conditions. Including the initial velocity effects, will further alter the shape of the spectrum. The middle and low period regions of the spectrum remain more sensitive to variations of the strength over inertia force index (η) than to variations of the initial conditions. For small values of η the SA values are insensitive to the effects of initial conditions.

5 CONCLUSIONS

The nature and effects of initial conditions on the computation of response spectra for elastic and inelastic systems have been identified. It has been shown that for elastic systems, the long period regions of response spectra for relative displacement and for absolute acceleration are very sensitive to variations on the initial conditions. The results also show that in a response spectrum dynamic analysis, the design shear computed from the SA spectrum for a long-period system with nonzero initial conditions may be significantly larger than that for a system initially at rest. The response spectra for inelastic systems is also influenced by the initial conditions. However, in this case, the inelastic spectra are also sensitive to the relative value of the system's yield strength with respect to the maximum inertia force of an infinitely rigid system (η). The responses of low period, rigid elastic and inelastic systems are, in general, not sensitive to variations on the initial conditions.

REFERENCES

Blázquez, R. and Kelly, J.M. 1988. Effect of initial conditions and computational algorithm on long period response spectra. *Proc. Ninth World Conference on Earthquake Engineering*: 1051-1056. Tokyo-Kyoto, Japan.

Clough, R.W. and Penzien, J. 1977. *Dynamics of structures*. McGraw-Hill, Inc.: 602-603. New York.

Mahin, S.A. and Lin, J. 1983. Construction of inelastic response spectra for single-degree of freedom systems - computer program and applications. *Report No. UCB/EERC-83/17, Earthquake Engineering Research Center*: University of California, Berkeley, California.

Pecknold, D.A. and Riddell, R. 1978. Effect of initial base motion on response spectra. *Journal of Engineering Mechanics Division, ASCE*: Vol. 104, No. EM2, 485-491. Discussions: February 1979, 204-206 and December 1979, 1057-1060.

Uang, C.-M. and Bertero, V.V. 1990. Evaluation of seismic energy in structures. *Journal of Earthquake Engineering and Structural Dynamics*: Vol. 19, No. 1, 77-90.

Veletsos, A.S. and Newmark, N.M. 1964. Design procedures for shock isolation systems of underground protective structures, Vol. III, response spectra of single-degree-of-freedom elastic and inelastic systems. *Report to Air Force Weapons Laboratory by Newmark, Hansen and Associates*: RTD-TDR-63-3096.

Ventura, C.E. and Blázquez, R. 1990. Sensitivity analysis of response spectra to system initial conditions. *Proc. Ninth European Conference on Earthquake Engineering*. Moscow, USSR.