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Instrument correction for a coupled transducer-galvanometer system

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ABSTRACT: Instrument correction of the records obtained by a coupled transducer-galvanometer system is necessary to eliminate the amplitude and phase distortions. A method for correction of the output from a seismograph or accelerometer with galvanometric registration is described. The procedure involves operations in the time domain only, and can be applied to any digitized record. Example tests are presented showing that a ground motion signal can be adequately reconstructed (for both phase and amplitude) in the frequency band which is wider than the nominal range of typical recordings.

INTRODUCTION

Seismological and strong motion measurements require use of a variety of devices with electrodynamic registration. The first systematic description of these devices was presented by Galitzin in 1912. Depending on the application, the response of a coupled "transducer-galvanometer" system may be required to reproduce displacement, velocity or acceleration of the moving point. By changing the constants of both devices, one can obtain a transfer function of the system that represents almost ideal displacement, velocity or acceleration meter in the defined frequency band. "Almost ideal" means that the device has the ability to reproduce the amplitude of the motion of interest. The direct instrument output is distorted in its phase response for all frequencies (Novikova and Trifunac, 1991).

A coupled transducer-galvanometer device is very popular in seismology and in earthquake engineering, and a great number of records is being produced by such devices in different countries. For example, structural vibrations are recorded in Soviet Union with the help of multi-channel systems based on such transducers as VEGIK (Vibrograph, Electrodynamic, Geophysical Institute, Kirnos), SPM-16 (Seismo-transducer, Mechanical), VBP (Vibrograph for Large Displacements), and galvanometers of GB type (Medvedev, 1962). Many seismologists are using transducers VEGIK, SGK and SKM with galvanometers of GB type. Variety of techniques are used to control the response of these systems (Khalturin, 1991; Medvedev, 1962). The strong-motion instruments often used in China are RDZ type devices with galvanometers (Lee and Wang, 1983).

There are certain advantages in using the coupled systems, as compared with the single degree of freedom devices: a) ability to get a broad range of amplifications, b) ability to separate recording from measuring locations and c) ability to gather and to write on the same medium (film, paper, magnetic tape) the response of several transducers, attached to different places of the object studied (this simplifies time-matching considerations). Thus, it is useful to process the records obtained by such devices to be as representative of ground (or structural) motion and, in as broad frequency band as possible. This can be accomplished by careful digitization of these records and the application of data processing and correction procedures (Trifunac, 1971 1972; Lee and Trifunac, 1979a,b, 1984, 1990).

Almost any study of the earthquake source, of wave propagation, or of vibration of structures requires information, supplied by the broad frequency range of the spectrum of the recorded motions. In consideration of the response of long structures to earthquake excitation, and in surface-wave propagation, the long-period end of the Fourier spectrum is of considerable importance. High frequency waves play a big role in any study of complex or stiff structures. Thus, the need for the information contained in the high and in the low frequency ends of the spectrum motivates us to attempt to broaden the frequency band available for the analyses, as much as possible. Majority of methods, used to test mathematical models of earthquake source, of wave propagation, or of structural vibration, require also accurate information about the phase of the motion throughout the frequency band under consideration. The instrument correction, then, gives the opportunity to increase the quality of the large number of records, obtained from coupled devices and to use them in various analyses.

Lee and Wang (1983) developed an instrument correction procedure, for the same type of device as discussed in this paper, on the basis of the approximation available for the Chinese RDZ-12-66 instrument. This paper presents a method (Novikova and Trifunac, 1991) that can be used to correct the direct response of almost any coupled transducer-galvanometer system. This method consists of the direct solution of the system of equations of motion for the coupled device in the time domain. Three numerical differentiations and one integration are used to obtain ground acceleration from the direct output of the system. If velocity or displacement are the quantities of interest, an additional integration(s) is (are) required. Because of the large number of filtering procedures involved, the accuracy of the filters becomes important in this method.

RESPONSE OF A COUPLED TRANSDUCER-GALVANOMETER SYSTEM

The motion of the coupled system can be represented by the equations:

$$\ddot{\theta} + 2\omega_1\xi_1\dot{\theta} + \omega_1^2\theta = \frac{-1}{l_0}\ddot{x} + 2\omega_1\xi_1\sigma_1\dot{\varphi} \qquad (1a)$$

$$\ddot{\varphi} + 2\omega_2\xi_2\dot{\varphi} + \omega_2^2\varphi = 2\omega_2\xi_2\sigma_2\dot{\theta}, \qquad (1b)$$

where: θ and φ are the relative rotational responses of the transducer and of the galvanometer, ω_1 and ξ_1 are the natural frequency and the damping ratio of the transducer, and ω_2 and ξ_2 are the natural frequency and the damping ratio of the galvanometer. σ_1 and σ_2 are dimensionless coefficients describing the additional excitation of the transducer due to feed-back from the galvanometer and the transfer factor for the electrodynamic registration, respectively. l_0 is the generalized length of the pendulum of the transducer, and $\ddot{x}$ is the ground acceleration. All the coefficients involved can be obtained from the physical constants of the transducer and the galvanometer (Borisevich, 1981).

To get the transfer function of the system, one can assume harmonic excitation with unit amplitude

$$x = e^{i\omega t},$$

which will result in the harmonic output:

$$\varphi = B(\omega)e^{i\omega t} = |B(\omega)| \cdot e^{i(\omega t + \beta(\omega))}$$

where $|B(\omega)|$ and $\beta(\omega)$ are amplitude and phase responses:

$$|B(\omega)| = \frac{\sigma_2\eta_1^2}{l_0}\frac{2\eta_2\xi_2}{\{a^2+b^2\}^{1/2}}, \qquad \tan\beta(\omega) = \frac{a}{b}. \quad (2)$$

Here

$$a = (1-\eta_1^2)(1-\eta_2^2) + 4\xi_1\eta_1\xi_2\eta_2(\sigma_1\sigma_2 - 1),$$
$$b = 2[\eta_2\xi_2(1-\eta_1^2) + \eta_1\xi_1(1-\eta_2^2)],$$

and $\eta_1 = \omega/\omega_1$ and $\eta_2 = \omega/\omega_2$ are dimensionless frequencies. Simple analysis (Novikova and Trifunac, 1991) shows that for any combination of the parameters ξ_1, ξ_2, σ_1 and σ_2 the phase response of the system is not constant even in the flat portion of the amplitude response. An example of functions (2) is shown on Fig. 1. It is seen that the best phase response is achieved for complete coupling $\sigma_1\sigma_2 = 1$, which is usually not used in practice.

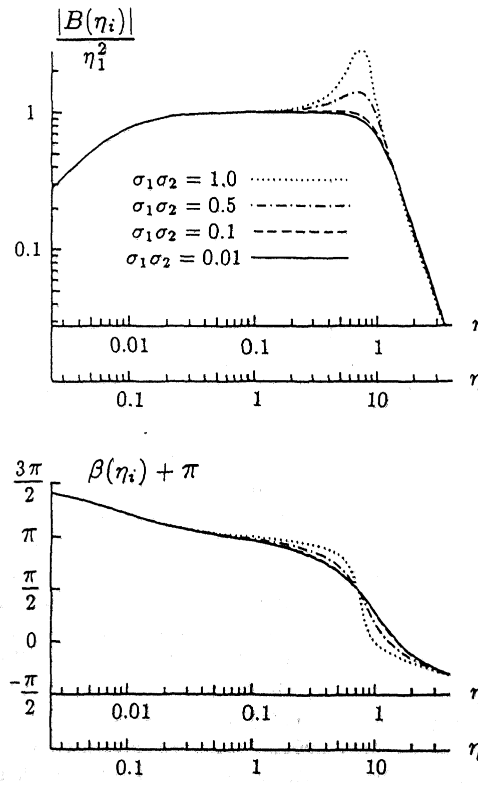

Fig. 1. Sample response of coupled transducer galvanometer system. $\sigma_2 = 1$, $\xi_1 = 0.6$, $\xi_2 = 6.0$, $\eta_2/\eta_1 = 10$.

The following conclusion can be made: the correction of the output of a coupled transducer-galvanometer system is necessary for any degree of coupling even in the frequency range where the amplitude response is stable, because of the phase distortion. Obviously, correction is also needed if we wish to increase the useful frequency range.

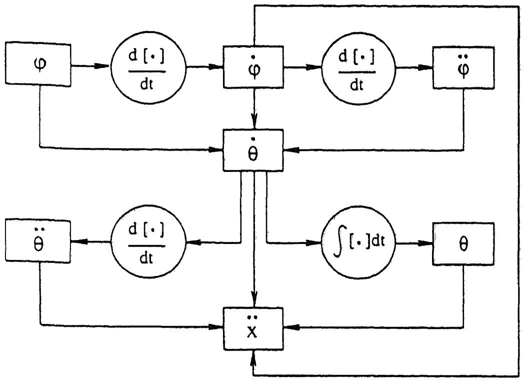

Fig. 2. General scheme of the instrument correction procedure for the coupled $2DOF$ system, represented by the equations (1). Having the response of the system φ one can get $\dot{\varphi}$ and $\ddot{\varphi}$ by two numerical differentiations. Knowing all constants involved in the equation (1b) the first derivative of the transducer response $\dot{\theta}$ can be obtained. Further, numerical differentiation and integration gives $\ddot{\theta}$ and θ. The last step is to compute $\ddot{x}$ from equation (1a) having $\theta, \dot{\theta}, \ddot{\theta}$ and $\dot{\varphi}$.

CORRECTION FOR THE INSTRUMENT RESPONSE

The general scheme for the proposed method is based on the Eq. (1) and can be presented by the flow-chart in Fig 2.

Before applying the numerical procedures, shown in Fig. 2, the frequency band of interest should be identified. This can be done by comparison of the Fourier spectrum of the direct output from the system with the average noise spectrum, typical for the type of the record used. By "noise" here we mean mostly digitization noise (Amini et al., 1987; Lee et al., 1982; Trifunac et al., 1973), and we consider the other distortions of the signal to be part of the signal itself. Having Fourier spectra both for the noise, and for the signal with noise, one can determine the upper (f_1) and the lower (f_2) limits of the frequency band for any desired signal to noise ratio. It is customary to assume this ratio to be equal to one, so that no information is lost.

Each differentiation and integration from the flow-chart in Fig. 2 should be accompanied by low- or high-pass filtering with a low cut-off frequency f_2 and a high cut-off frequency f_1. This is necessary because differentiation emphasizes high frequencies (higher than f_1) which do not have any useful information, and integration does the same with low frequencies (lower than f_2). So, low-pass, high-pass, differentiation and integration filters are all required.

We would like to emphasize, that only "ideal phase" filters are needed in the procedure (zero shift for high- and low-pass, $\pi/2$ for differentiation and $-\pi/2$ for integration). This is necessary if the reconstruction of the original phase of the signal, recorded by the device, is one of the tasks to be performed and, moreover, if it guaranties exact phase reconstruction.

We next discuss the restoration of the amplitude of motion. The accuracy of the procedure does not depend on the specific values of the damping ratios ξ_1 and ξ_2 and the coupling constants σ_1 and σ_2. The natural frequencies of the transducer and the galvanometer are the key parameters defining the corner frequencies of the amplitude responses of the coupled device. The frequency band, where adequate reconstruction of the original signal is possible, is somewhat wider than the flat portion of the amplitude response of the device that recorded the signal.

The computer program (ICR2 – the Instrument Correction for a 2-Degree of Freedom System) was designed to perform the correction of records for the instrument response. One way to check the quality of the algorithm is to get it's transfer function. Given the (generally) coupled device with known characteristics, its transfer function $B(\omega)$ is known Eq. (2). The relationship between the input (displacement of the moving point $x(\omega)$) and the output (rotational response of the galvanometer $\varphi(\omega)$), in the frequency domain, can be expressed as

$$B(\omega) \cdot x(\omega) = \varphi(\omega).$$

Therefore, acceleration of the moving point $\ddot{x}(\omega)$ can be obtained as

$$\ddot{x}(\omega) = -\omega^2[B(\omega)]^{-1} \cdot \varphi(\omega). \qquad (3)$$

The ideal instrument correction procedure should be able to reconstruct $\ddot{x}(\omega)$ inside a prescribed frequency band $f_2 < f < f_1$. Designating the transfer function of ICR2 as $R(\omega)$, we have

$$\ddot{x}(\omega) = R(\omega) \cdot \varphi(\omega). \qquad (4)$$

The accuracy of the instrument correction, then, can be measured by the discrepancy between the theoretical, Eq. (3), and the actual, Eq. (4), transfer function. The relative error is given by

$$\varepsilon(\omega) = \frac{-\omega^2[B(\omega)]^{-1} - R(\omega)}{-\omega^2[B(\omega)]^{-1}}. \tag{5}$$

However, direct implementation of (5) is impossible as the analytical expression for $R(\omega)$ is not known.

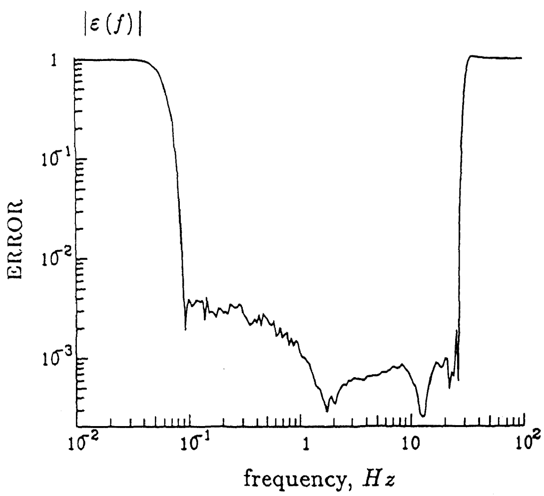

Fig. 3. Relative error of the ICR2 procedure in the case of a harmonic input.

To evaluate $|\varepsilon(\omega)|$ the following test was performed. Given harmonic input $\varphi_\omega(t) = \sin\omega t$ both actual $\ddot{x}_\omega(t) = r_\omega[\sin(\omega t + \beta(\omega))]$ and theoretical $\ddot{x}^0_\omega(t) = b_\omega[\sin(\omega t + \beta^0(\omega))]$ responses of the instrument correction algorithm can be obtained: first - by just running ICR2 program, and second - analytically. As all filters involved are symmetric (or antisymmetric), one can assume that the instrument correction procedure reconstructs the phase perfectly $(\beta(\omega) = \beta^0(\omega))$. This allows one to obtain an estimation of r_ω/b_ω as the discrepancy between $\ddot{x}_\omega(t)$ and $\ddot{x}^0_\omega(t)$ at their (say) maxima. Carrying out calculations for a wide range of frequencies, the estimation of the relative error of the instrument correction procedure can be obtained:

$$|\varepsilon(\omega)| = \frac{b_\omega - r_\omega}{b_\omega} = 1 - \frac{r_\omega}{b_\omega}. \tag{6}$$

Fig. 3 presents the graph of Eq. (6) for the case when the frequency range requested for reconstruction is bounded by $f_2 = 0.05$ Hz and $f_1 = 30$ Hz. The coupled device used in the "recording" consists of a transducer with $f_{tr} = 5$ Hz and $\xi_{tr} = 5$ (velocity type), and of a galvanometer with $f_{\text{galv}} = 10$ Hz and $\xi_{\text{galv}} = 0.6$ (acceleration type). The coupling constants are $\sigma_1 = 0.01$ and $\sigma_2 = 1$. Fig. 4 shows the transfer function of the coupled transducer-galvanometer with these parameters. As one can see, the device is only a hypothetical one and has "poor" characteristics. A device with these constants was chosen in order to make the effect of the instrument correction more pronounced. It is seen from Fig. 3 that ICR2 performs well with accuracy of 3×10^{-3} in the range 0.09 Hz $< f <$ 27 Hz and with accuracy 10^{-1} in the ranges 0.065 Hz $< f <$ 0.09 Hz and 27 Hz $< f <$ 29 Hz. Comparison of these values with the graph in Fig. 4 shows that the algorithm does reconstruct the amplitude characteristics of the motion far beyond the "flat" portion of the system response.

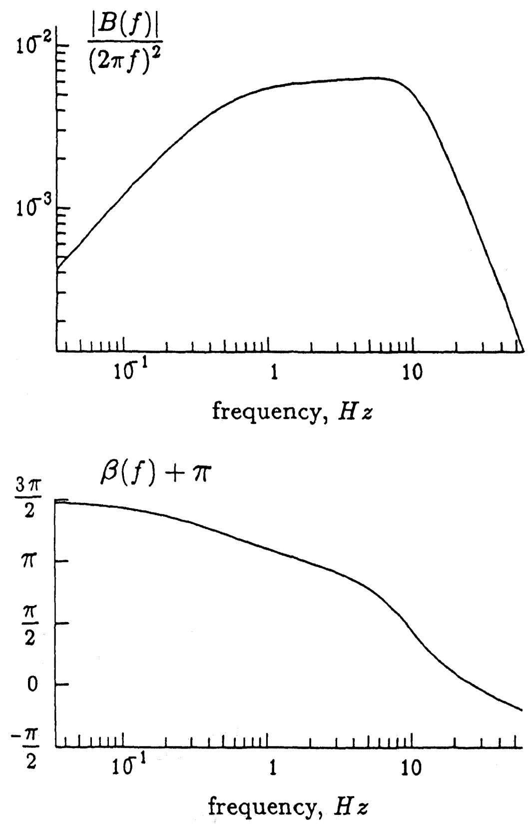

Fig. 4. The amplitude (top) and phase (bottom) transfer function for the device with parameters: transducer natural frequency 5 Hz and ratio of critical damping 5, galvanometer natural frequency 10 Hz and ratio of critical damping 0.6, and coupling constants $\sigma_1 = 0.01$ and $\sigma_2 = 1$.

APPLICATION TO AN EARTHQUAKE RECORD

Another test performed consisted of a case study.

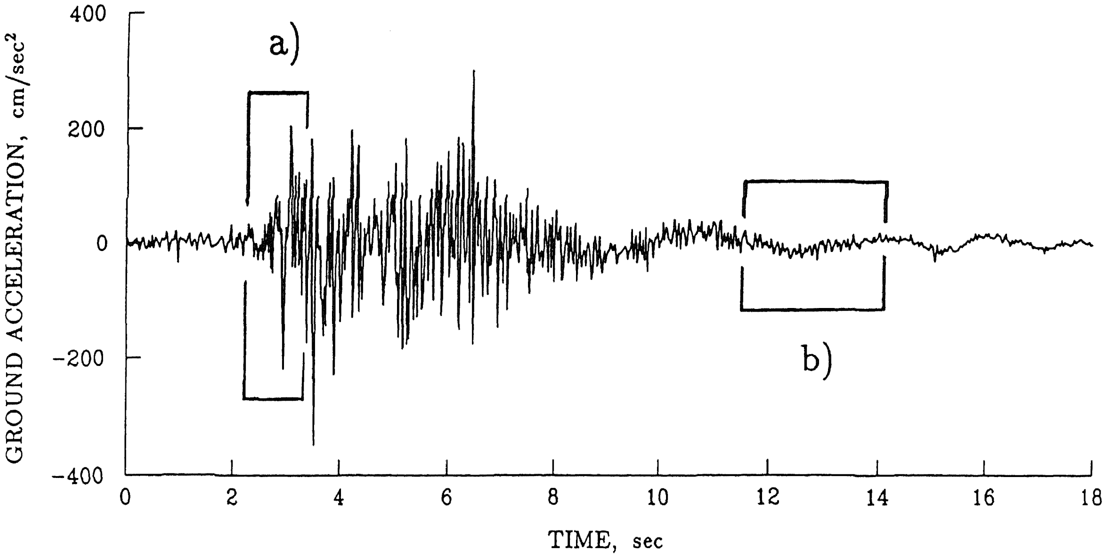

Fig. 5. Vertical component (total length 37.9 sec) of the Imperial Valley earthquake in California (Oct. 15, 1979), recorded at the epicentral distance 27 km. This record was adopted as the exact absolute ground acceleration. Boxes a) and b) refer to Fig. 6.

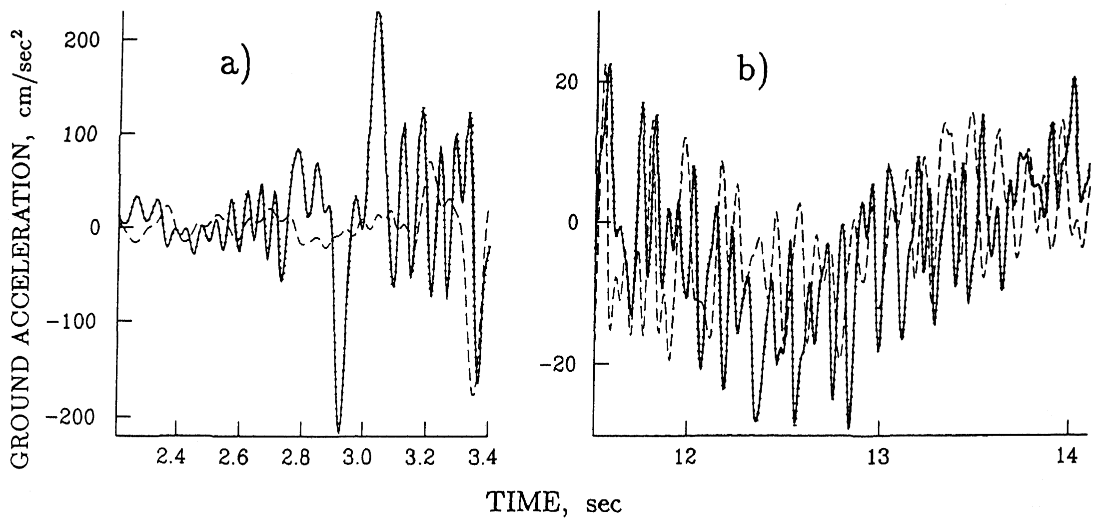

Fig. 6. Actual (solid line), "recorded" (dashed line), and corrected (dotted line) ground acceleration for two time intervals shown in the boxes in Fig. 5.

A typical strong motion accelerogram (Lee and Trifunac, 1987) was taken to represent the exact acceleration of the ground. This was the vertical component of the record obtained during the Imperial Valley earthquake in California, on October 15, 1979, at epicentral distance of 27 km (Fig. 5).

Eq. (1) was solved using the forth order Runge-Kutta method with small time steps. This procedure simulates the work of the recording device. The parameters chosen for the coupled system were the same as for the first test, and are summarized in Fig. 4. The "working range" (flat amplitude response) of the device adopted is narrower than the spectrum of the recorded acceleration. We will demonstrate here how the "lost" information from high and low frequencies can be reconstructed during the instrument correction procedure.

The output of the Runge-Kutta integration - the "recorded" motion - was corrected by ICR2, with frequency range $f_2 = 0.05$ Hz $< f < 25$ Hz $= f_1$ requested for reconstruction. Fig. 6 compares the actual ("recorded") and the reconstructed accelerations in two time intervals. These are the most critical intervals for high (left) and low (right) frequency range. As one can see, the original ground acceleration was adequately reconstructed throughout the whole required frequency range. The accuracy of the result was estimated to be $\approx 5 \times 10^{-2}$. The largest error appears in time intervals with mostly high frequency content.

CONCLUSIONS

The results of the work can be summarized as follows:

1. The need for accurate representation of both phase and amplitude of the original motion being measured requires the correction of the direct output from coupled transducer-galvanometer system for the instrument response. This procedure does not only significantly increase the frequency band beyond the "flat" portion of the system amplitude response, but it can also correct the phase, which always depends on the frequency.

2. The proposed instrument correction algorithm involves numerical differentiations and integrations in the time domain, and it can be applied to the output from any coupled system which can be described by Eqs. (1). It is not necessary to design instruments so that their output is proportional to displacement, velocity or acceleration of the moving point, if the direct output from the system is corrected by the proposed procedure. It is also not necessary to worry about small coupling between devices if it is more convenient to design an instrument with a large coupling coefficient.

3. The tests we presented show that the relative error of the procedure is about 5% inside the frequency band which was chosen to be corrected. In the case study considered, this frequency band was much broader than the "flat" portion of the "device" "recording" the motion (the "recording" was modeled by Runge-Kutta integration of the governing Eqs. (1)).

4. The program written on the basis of the proposed algorithm can be included into the package of common strong motion and seismological data processing programs (Trifunac and Lee, 1979a,b; Lee and Trifunac, 1990), and a great number of records obtained by coupled transducer-galvanometer systems can be corrected for the instrument response.

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