Earthquake Engineering, Tenth World Conference © 1994 Balkema, Rotterdam. ISBN 90 5410 060 5

Modeling of nonlinear stress strain relations of sands for dynamic response analysis

Hiroshi Hayashi, Mitsuhiko Honda & Takemine Yamada
Kajima Technical Research Institute, Tokyo, Japan

Fumio Tatsuoka
Institute of Industrial Science, University of Tokyo, Japan

ABSTRACT: This paper presents a new flexible and versatile model, which is capable of accurately fitting to a given experimentally-obtained shear stress versus shear strain relation of sand over a wide range of strain level, say from around 0.0001% up to near failure-corresponding strain. The model is validated by using the data from a systematic test series of simple shear tests. This model has a generalized form of the well-known hyperbolic model, modified by introducing three new parameters n_L, n_U, and α. The values of n_L and n_U are independent of soil type (0.3 and 1.0, respectively) whereas the parameter α is closely related to the grain size distribution characteristics (D_{max} and U_C). This model is applied to cyclic loading conditions by using the Masing rule. It is shown that in dynamic response analyses, more accurate and reliable results could be obtained by using the new model, compared to the cases using other existing models.

1 INTRODUCTION

For earthquake response analyses of level grounds, slopes and geotechnical structures including fill-type dams, accurately modeled non-linear stress-strain relations of soils are required. In most previous studies, stress-strain relations during cyclic loading are formulated by using a so-called skeleton curve and some rules to generate hysteresis loops from it. Many researchers including Ramberg and Osgood (1964), Hardin and Drnevich (1972), Iwan (1967) and Prevost et al (1989) assumed that the stress-strain relation obtained from a monotonic loading test is the skeleton curve.

Based on some recent experimental data obtained by the authors from a systematic test series of cyclic simple shear tests, this paper shows that most of the above-mentioned previous models have some specific limitations and are not versatile when applying to (1) both monotonic and cyclic loading conditions, (2) a wide variety of geotechnical materials including clays, sands and gravels, and (3) a wide range of strain, say from 0.0001% to that at the peak of the order of 10%. In this paper, a new model is proposed, which is much more flexible and accurate than the above-mentioned models in fitting to the data obtained under largely varying conditions and in generating stress-strain relations under various cyclic loading conditions.

2 OUTLINE OF TESTING PROGRAM

As shown in Fig. 1, simple shear testing apparatus used i this research is of Kjellman-type (1951). In order t accurately evaluate the shear modulus, G, and dampin ratio, h, over a wide range of shear strain, γ, from 10^{-6} u to that at near-failure, several considerations for designin the apparatus are given (Hara and Kiyota, 1977) as fo lows: The damping ratio of flat springs supporting th shaking table is less than 0.1%; The vertical rollers be tween vertical loading frame and rigid frame is 1/1000 i its friction coefficient, thereby keeping applied overburde pressure, σ_v, constant throughout the testing; Confinin rings for producing Ko condition in soil specimen a

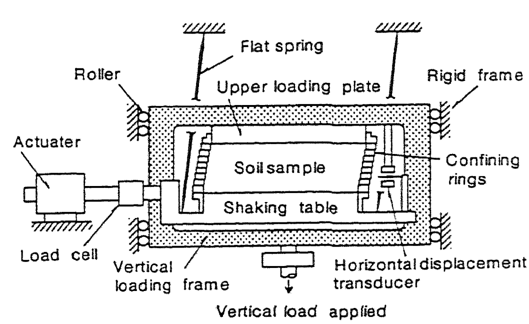


Figure 1. Schematic view of simple shear testing apparatus

coated with molybdenum disulfide such that they are minimal in friction while being rigid enough to confine the soil horizontally.

Testing Scheme is shown in Fig. 2: During very small strain levels of less than 10^{-5}, 10 cycles of shear strain are applied to soil samples; Otherwise, two cycles of sinusoidal shear strain at a frequency of 1 Hz are applied to soil specimen of 300mm in diameter and 100mm in thickness to evaluate the values of induced shear stress, τ, G and h at each strain level.

Main factors controlling the shear stress-shear strain (τ~γ) relation of soils are: 1) overburden pressure, σ_v, or mean principal pressure, σ_m; 2) void ratio, e; 3) grain size distribution; 4) the stress history in terms of overconsolidation ratio or preshearing; and so on. In order to study into the influence of the above-mentioned first three factors on the τ~γ relations of soils, a series of cyclic simple shear tests have been conducted on sands and gravels with a wide variety of grain size distribution as shown in Fig. 3 and Table 1.

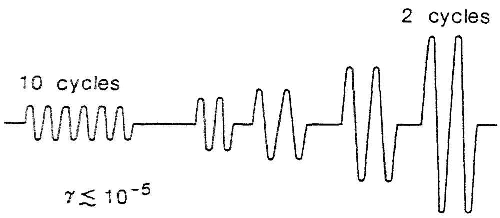


Figure 2. Testing scheme.

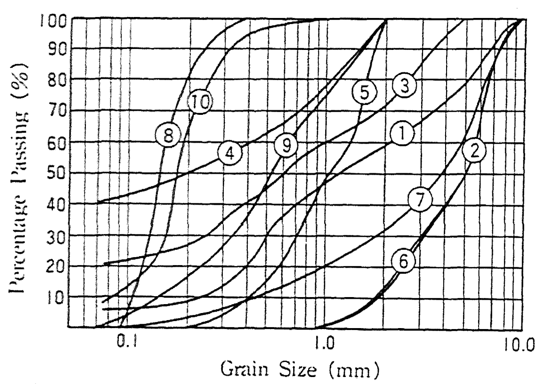


Figure 3. Grain size distribution curves of samples.

Table 1. Physical properties of specimen.

Sample	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
	Tokyo gravel					Crushed sand	Rock	Standard sand	Sand-1	Sand-2
Maximum grain size Dmax(mm)	9.52	9.52	4.76	2.0	2.0	9.52	9.52	0.42	2.0	2.0
Average grain size D_{50}(mm)	1.2	5.0	0.6	0.2	1.04	4.9	3.8	0.14	0.52	0.17
Uniformity coefficient Uc	14.6	2.8	—	—	3.0	3.0	10.4	1.5	4.26	2.12
Coefficient of curvatine Uc'	0.85	1.0	—	—	1.16	0.93	1.45	1.07	0.88	1.01
Percentage of 74μm finer (%)	5.8	0.0	20.0	40.0	0.0	0.0	0.0	0.0	0.0	6.5

3 INFLUENCE OF σ_v AND e ON THE τ~γ RELATION

3.1 *Determination of shear strength*

The cyclic simple shear testing apparatus used in this study was not capable of determining directly very accurate values of the shear strength, τ_f, of soil. Based on the general fact that τ_f estimated by assuming the τ~γ relation in relatively large strain level as hyperbolic is nearly the same as measured τ_f in any type of shearing test, τ_f was equated with the reciprocal of slope on the graph of γ/τ ~ γ plotting.

3.2 *Influence of σ_v on τ~γ relation*

It is convenient to convert the τ~γ relation of soil to a general form which is rather independent of various factors. This is achieved by normalizing the stress as $y=\tau/\tau_f$ and the strain as $x=\gamma/\gamma_r$ where γ_r is the so-called reference strain, defined as τ_f/G_o in which G_o is G at infinitesimally small strains. Fig. 4 shows the normalized τ~γ relation, i.e. y~x relation of Tokyo gravel under different vertical stress, σ_v. It may be seen from this figure that these y~x relations are rather independent of σ_v.

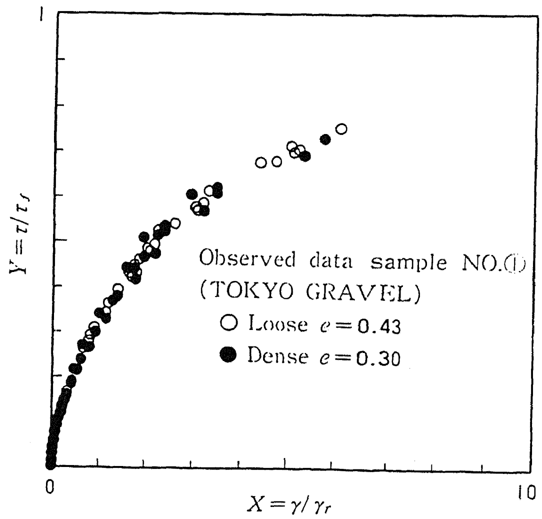


Figure 4. Y-X relations for different confining pressures, Tokyo gravel.

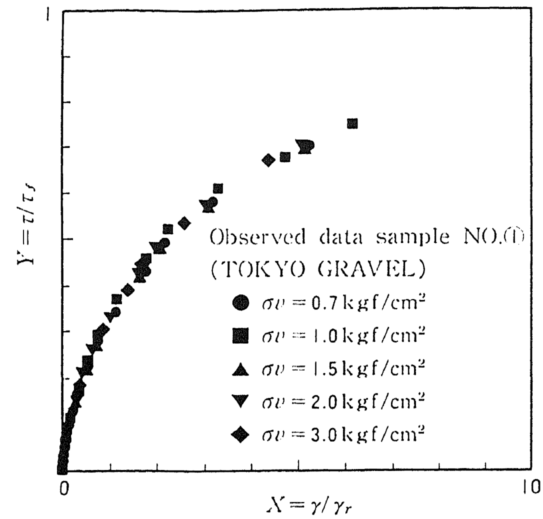


Figure 5. Y-X relations for different void ratios, Tokyo gravel.

3.3 *Influence of e on τ~γ relation*

In the same way as above, y~x relations of Tokyo gravel with two different void ratio, e, are plotted on Fig. 5. It is seen from this figure that these y~x relations are also almost the same. Therefore, these results suggest that the normalized τ~γ relation, i.e. y~x relation, of any soil could be virtually independent of σ_v and e. Inversely, the normalizing parameters, i.e. τ_f and $\gamma_r=\tau_f/G_o$, of the soil are dependent mainly on σ_v and e.

4 PROPOSED MODEL

In order to simulate accurately a given actual y~x relation of soil as shown in Figs. 4 and 5, modeling of y~x relation must satisfy the following requirements:

$$y\,(x=0) = 0 \qquad (1)$$

$$y'\,(x=0) = 1 \qquad (2)$$

$$y' > 0 \text{ and } y'' < 0 \text{ for } 0 \leqq x < x_f \qquad (3)$$

$$y\,(x=x_f) = 1 \qquad (4)$$

$$y'\,(x_f) = 0 \qquad (5)$$

where x_f is the value of x at failure (y=1). Eqs. (4) and (5) are difficult to satisfy unless some complicated formulation is introducted (Prevost et al., 1989). When two of the requirements are relaxed; e.g., when

$$y\,(x=\infty) = 1 \qquad (6)$$

$$y'\,(x=\infty) = 0 \qquad (7)$$

are used, a very simple model, the hyperbolic y~x relation (Hardin and Drnevich, 1972); i.e. y = x / (1+x), satisfies the other requirements. It is readily seen that this original hyperbolic (0H) model is too rigid in fitting varing relations of a wide range of soils. In order to alleviate this point, one new parameter n was introduced (Hayashi and Sugahara, 1990) as;

$$y = \frac{\left(\frac{2}{n}x+1\right)^n - 1}{\left(\frac{2}{n}x+1\right)^n + 1} = f(x,n)\,;\, n>0 \qquad (8)$$

Eq. (8) is reduced to the original hyperbolic form for n = 1. Fig. 6 compares Eq. (8) for different n's with typical data. It is seen from this figure that with any value of n, Eq. (8) cannot fit well the actual data over an entire range of γ. It was also found that two different values of parameter n, n_L and n_U, are needed so that the equation fits the observed stress-strain relation at small value of x less than about 1.0 and large values of x up to 10, respectively. Then, in order to increase the versatility of the model, the two equations $y=f(x,n_L)$ and $y=f(x,n_U)$ were combined in a quasi-linear way by using another parameter α;

$$y = e^{-\alpha x} \bullet f(x,n_L) + (1-e^{-\alpha x}) \bullet f(x,n_U) \qquad (9)$$

where α is the parameter controlling how y transfers from $f(x,n_L)$ to $f(x,n_U)$ as x increases. In the special case where $n_L = n_U = n$, Eq. (9) is reduced to Eq. (8).

Fig. 7 compares the proposed model, Eq. (9) using relevant parameter values, and other existing models with typical data. From this figure, it is seen that the proposed model is much superior to these other models in modeling the actual data. Fig. 8 compares the observed h~γ relation with the relation derived from this proposed model by using the Masing rule (1926). The agreement of the proposed model with the observed data is fairly good.

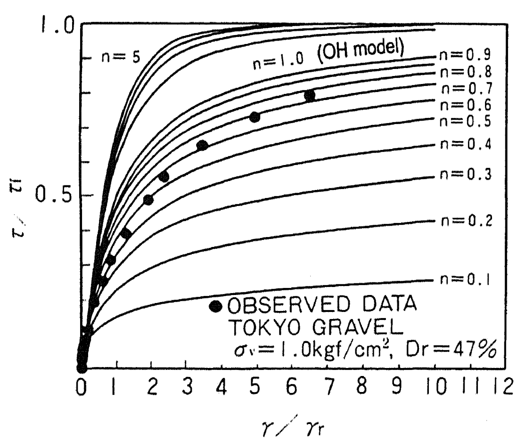


Figure 6. τ/τ_f~γ/γ_r relations with different n's.

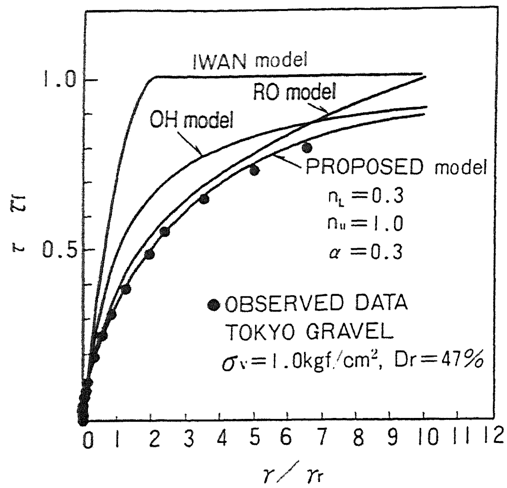


Figure 7. Comparison of proposed model with observed data ($\tau/\tau_f \sim \gamma/\gamma_r$ relation).

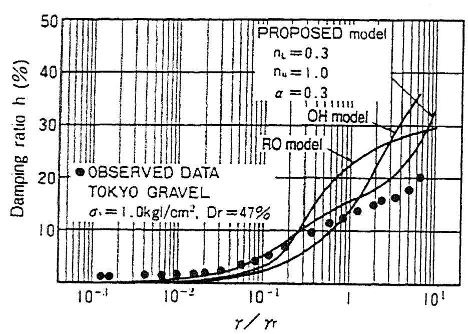


Figure 8. Comparison of proposed model with observed data ($h \sim \gamma/\gamma_r$ relation).

5 ANALYSIS ON PARAMETERS

The parameters used in the proposed modeling on y~x relation of soils are n_L, n_U and α. This section shows some relations between these parameters and physical properties of soil.

5.1 *Parameter* n_L

The values of n_L is to be determined so that $f(x,n_L)$ is best fit to the observed y at the small strains of less than 10^{-4} (0.0001). Table 2 shows the values of n_L for soils of different grain size distribution characteristics. It may be seen that parameter n_L is a rather constant value around 0.3 regardless of the different grain size distributions, particularly at smaller strain levels. This may be due to that only the grain-to-grain friction-type shearing resistance is effected at very small shear strains without shearing resistance due to interlocking being mobilized, which is a function of grain size distribution.

Table 2. Values of the parameters n_L determined by regression analysis.

Case No. \ Sample No.		①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	Mean	Standard Dimension
1	$\gamma \leq 5\times10^{-5}$	0.29	0.28	0.29	0.32	0.32	0.29	0.30	0.37	0.24	0.30	0.30	0.033
2	$\gamma \leq 10^{-4}$	0.29	0.29	0.29	0.31	0.31	0.28	0.31	0.39	0.23	0.32	0.30	0.04
3	$\gamma \leq 10^{-3}$	0.30	0.48	0.30	0.31	0.36	0.43	0.33	0.45	0.26	0.33	0.35	0.08

5.2 *Parameters* n_U *and* α

As described in sub-section 3.1, the $\tau \sim \gamma$ relation at large shear strains of 10^{-3} up to failure can be closely modeled by the hyperbolic model by using a value of G_0 smaller than the actual value. This means that the y-x relation of soil at large values of x is upperbounded by $y=f(x, n_U)$ with $n_U=1$. Therefore, assuming that parameters n_L and n_U are constants, 0.3 and 1.0 respectively, the parameter α for each case is to be determined so that the difference between observed and predicted relations be minimal. Table 3 shows the determined values of α for various types of soils.

Table 3. Values of the parameter α determined by regression analysis.

Sample No.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	Mean	Standard Deviation
n_L	0.30											
n_U	1.0											
α	0.20	0.66	0.20	0.19	0.19	0.42	0.25	0.26	0.20	0.16	0.27	0.15

Table 4. Regression analysis of α on several variables.

Step	Regression Coefficients						Correlation coefficient R
	Average grain size D_{50}	Maximum grain size Dmax	Uniformity coefficient Uc	Percentage of 74μm finer	Coefficient of curvature U'c	Constant Term	
1	0.063	—	—	—	—	0.153	0.80
2	—	0.037	-0.027	—	—	0.217	0.87
3	—	0.036	-0.024	-0.006	—	0.222	0.88
4	-0.141	0.113	-0.055	-0.032	—	0.290	0.89
5	-0.277	0.189	-0.088	-0.049	0.205	0.136	0.90

The values of α scatter from 0.16 to 0.66 depending on the soil type. In order to investigate the relation of α with the corresponding grain size distribution of soil, a step-wise regression analysis was performed on α in relation to several parameters characterizing grain size distribution. Based on the results shown in Table 4, step-2 through step-5 are not so different from one another in terms of the correlation coefficient R. Therefore, α may be estimated rather accurately from maximum grain size, D_{max}(mm), and uniformity coefficient, Uc, as follows:

$$\alpha = 0.037 \times (D_{max}) - 0.027 \times (Uc) + 0.217 \quad (10)$$

Fig. 9 compares the observed values of α with those evaluated on Eq. (10). Judging from this figure, Eq. (10) can be used as rough estimate of α as a function of D_{max} and U_c.

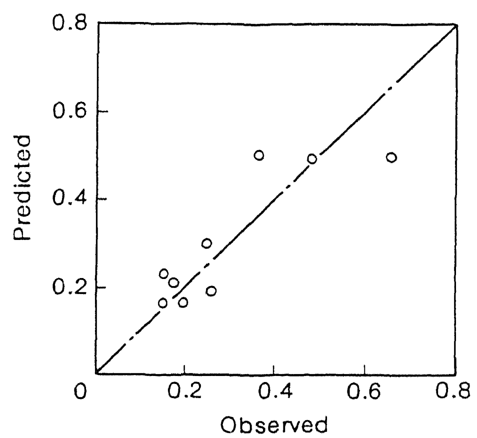


Figure 9. Comparison between observed and predicted values of α.

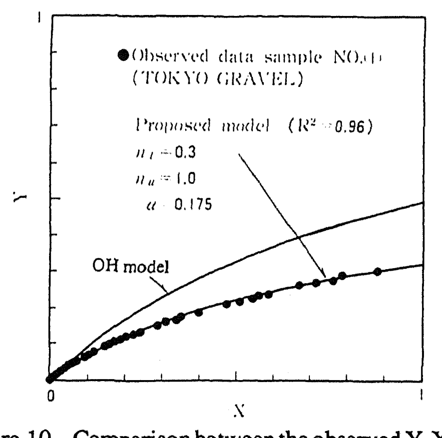


Figure 10. Comparison between the observed Y-X relations and those predicted by the OH model and the newly proposed model at small values of X, Tokyo gravel.

5.3 *Comparison of the proposed model with observed data*

Figs. 10 to 13 compare the observed y~x relation of sample No. 1 (Tokyo gravel) with the proposed modeling on various forms of plotting. Here, n_L and n_U are 0.3 and 1.0 respectively, and α is estimated to be 0.175 from Eq. (10). It is seen that the proposed model agrees very well with the observed data in all the figures, which covers ranges from very small to large values of x.

6 ILLUSTRATIVE EXAMPLES

In order to demonstrate the importance of the accurate modeling of the stress-strain relations of soils, a seismic stability analysis of a model rock-fill dam was performed.

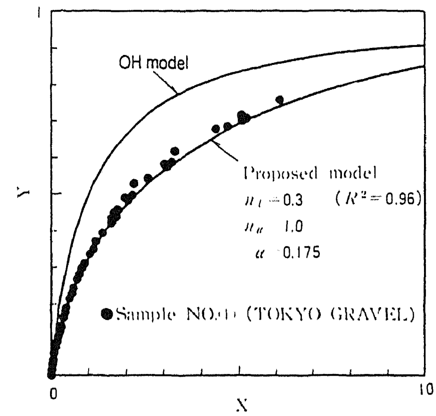


Figure 11. Comparison between the observed Y-X relations and those predicted by the OH model and the newly proposed model at large values of X, Tokyo gravel.

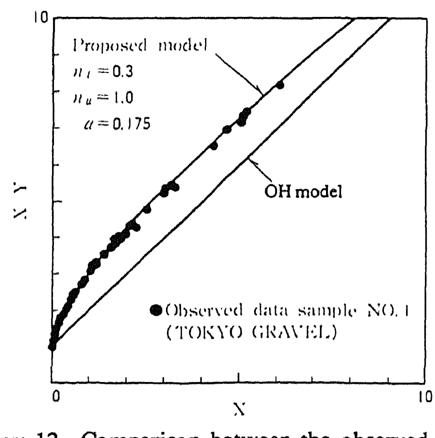


Figure 12. Comparison between the observed X/Y-X relations and those predicted by the OH model and the newly proposed model, Tokyo gravel.

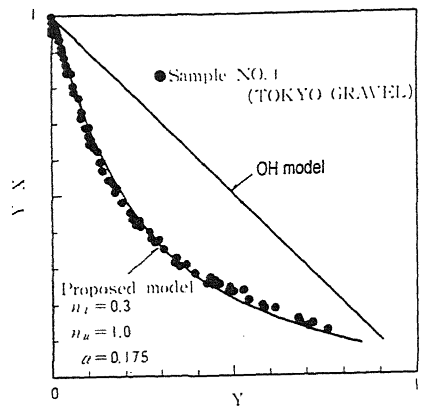


Figure 13. Comparison between the observed Y/X-X relations and those predicted by the OH model and the newly proposed model, Tokyo gravel.

This analysis is, in principle, based on Newmark's sliding concept (1965). The analytical procedure is as follows : (1) One-dimensional shear slice dynamic analysis are performed by using both the original hyperbolic model and the newly proposed model for nonlinear τ~γ relation of dam material. From the results, horizontal seismic coefficient, k (t, z), is evaluated as a function of time, t, and the height, z ; (2) Then, for a specified potential sliding mass, the average seismic coefficient, k (t), is obtained from k (t, z) weighted by mass distribution inside the sliding wedge (Watanabe et al., 1984) ; (3) Next, the acceleration of the sliding mass in the downhill direction along the failure surface, a (t), is evaluated as

$$a(t) = g\frac{\cos(\phi-\delta)}{\cos\phi}\left[k(t)-kc\right] \qquad (11)$$

where g is gravitational acceleration, ϕ is the friction angle of dam material, δ is the angle of the assumed failure surface relative to the horizontal and kc is the critical seismic coefficient, tan (ϕ-δ), under which the sliding is to initiate; (4) Finally, by integrating Eq. (11) twice with respect to time, ultimate post-earthquake sliding movement, D, can be obtained.

Fig. 14 shows the configuration of model dam analyzed and the assumed potential sliding mass, together with assumed model properties of dam material. Input motion is El-Centro 1940 NS with peak acceleration of 250 gal as shown in Fig. 15. Step-by-step integration with respect to time was carried out using Newmark's β method with β=1/4. Fig. 16 shows the comparison of the time histories of D for the two different modeling methods using the same value of G_0 and τ_f. The value of D when using the original hyperbolic model is larger by as much as 130% compared with that when using the proposed model. This result indicates that the accuracy of modeling of the τ~γ relation of soil has a marked effect on the evaluated sliding movement of dam subjected to earthquake loading.

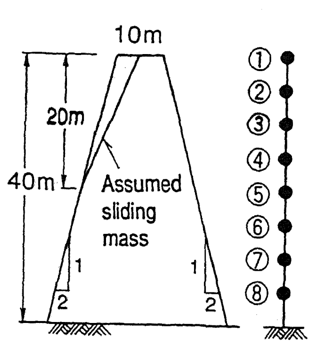
7 SUMMARY

Based on the results of a comprehensive series of cyclic simple shear tests on sands and gravels, a new model of the τ~γ relation of soils is proposed. The relation between the parameters used in the model and the physical properties of the soil was analyzed. From the results, the following conclusions are obtained :

i) The normalized τ~γ relation, i.e. the y~x relation ($y=\tau/\tau_f$, $x=\gamma/\gamma_r$, $\gamma_r=\tau_f/G_0$), for a given type of soil is independent of its overburden pressure, σ_v, and void ratio, e.

ii) The parameters n_L and n_U used in the proposed modeling on y~x relation have fixed values of 0.3 and 1.0 respectively, regardless of the grain size distribution of the soils.

iii) The remaining parameter α is closely related to the maximum grain size, D_{max}, and the uniformity coefficient,



Thickness of layer m	Unit weight tf/m³	Shear modulus G_0 kgf/cm²	Shear strength τ_f kgf/cm²	Parameters n_L	n_u	α
5	1.9	1030	0.37	0.3	1.0	0.3
5		1790	1.11			
5		2310	1.86			
5		2740	2.60			
5		3100	3.34			
5		3430	4.08			
5		3730	4.83			
5		4000	5.57			

Figure 14. Analytical model and soil properties.

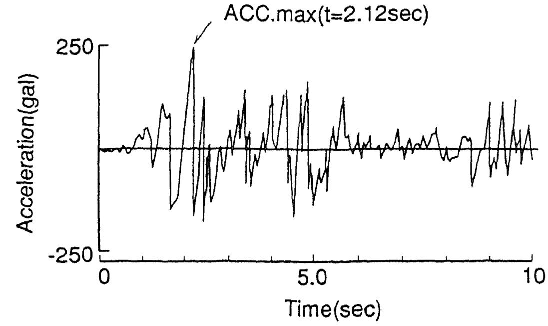


Figure 15. Input motion (EL Centro).

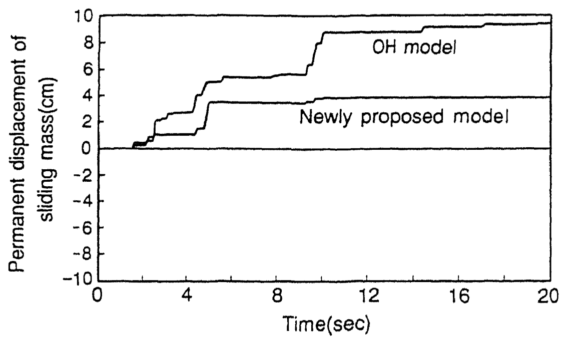


Figure 16. Comparison of permanent displacements D of sliding mass predicted based on the OH model and the newly proposed model.

Uc, and can be approximately estimated by Eq. (10).

iv) The proposed model can fit to a given observed y~x relation of soil over a wide range of shear strain much more accurately than the existing other models.

v) The accuracy of modeling on the τ~γ relation has a significant effect on the evaluated earthquake-induced displacement of dam.

REFERENCES

Hara, A. and Kiyota, Y. 1977. Dynamic shear tests of soils for seismic analysis. *9th ICSMFE (Tokyo) Vol. 2*: 247-250.

Hardin, B. O. and Drnevich, V. P. 1972. Shear modulus and damping in Soils; measurement and parametric effects. *Proc. ASCE 98 SM6* : 603-624.

Hayashi, H. and Sugahara, T. 1990. Modeling of the non-linear shear stress-strain behavior of soils. *8th Japan Symposium on Earthquake Engineering* : 777-782.

Iwan, W. P. 1967. On a class of models for the yielding behavior of continuous and composite systems. *Journal of Applied Mechanics 34 E3* : 612-617.

Jennings, P. C. 1964. Periodic response of a general yielding structure. *Proc. ASCE 90 EM2* : 131-166.

Kjellman, W. 1951. Testing the shear strength of clay in Sweden. *Geotechnique 2 No. 3*: 225-235.

Masing, G. 1926. Eigenspannungen und verfestigung beim messing. *Proc. 2nd International Congress of Applied Mechanics* : 332-335.

Newmark, N. M. 1965. Effects of earthquakes on dams and embankments. *Geotechnique 15 No. 2* : 139-160.

Prevost, J. H. and Keane, C. M. 1989. Shear stress-strain curve generation from simple material parameters. *Journal of GE ASCE 116 No. 8* : 1255-1263.

Watanabe, H. , Sato, S. and Murakami, K. 1984. Evaluation of earthquake-induced sliding in rockfill dams. *Soils and Foundation 24 No. 3* : 1-14.