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Factors affecting response spectra in the long period range

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ABSTRACT: The seismic design of long period systems (e.g., base isolated structures) is based on ground velocities and/or ground displacements, rather than on ground accelerations. As a result, a number of factors affecting the recording and processing stages of strong motion accelerograms become very significant and need to be reanalyzed. These factors sometimes are out of control, since they emanate from geophysical or geologic phenomena, and generally lead to spurious noise in the records. Besides, their relative influence on the reliability of response spectrum ordinates depends very closely on the type of accelerogram (analog or digital), the trigger characteristics of the instrument and the procedures used to retrieve the original data. In this paper, the salient features of the earthquake ground motions in the long period range are outlined. The limitations imposed by the spectral analysis on the velocity and pseudo-velocity response spectra are discussed and the sensitivity of the spectrum ordinates to various tectonic, geotechnical and processing factors is investigated.

1 THEORETICAL BACKGROUND

Response spectra plots have become a standard design tool to characterize the frequency content of an earthquake and its effects on structures. They are obtained by maximizing the solution, u(t), of the equation of motion of the one-degree-of-freedom system (Fig. 1):

$$\ddot{u} + 2\zeta p\dot{u} + p^2u = -\ddot{y}(t) \qquad (1)$$

as well as the related magnitudes, $\dot{u}(t)$ and $\ddot{x}(t) = \ddot{u}(t) + \ddot{y}(t)$, in the following manner:

$SD = \lvert u(t)\rvert_{max}$	relative displacement response spectrum
$SV = \lvert \dot{u}(t)\rvert_{max}$	relative velocity response spectrum
$SA = \lvert \ddot{x}(t)\rvert_{max}$	absolute acceleration response spectrum
$PSV = p.SD = \frac{2\pi}{T}.SD$	pseudo-relative velocity response spectrum
$PSA = p^2.SD = \left(\frac{2\pi}{T}\right)^2.SD$	pseudo-relative acceleration response spectrum

In equation (1), $p = \frac{2\pi}{T} = \sqrt{k/m}$ is the circular frequency of the oscillator whose spectral ordinate is being computed for the damping ratio $\zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{k.m}}$ and the ground acceleration $\ddot{y}(t)$. The main advantage of this type of analysis in the frequency domain is the possibility of showing in only one diagram several response calculations in the time domain, each one being a function of the characteristics of the excitation, $\ddot{y}(t)$, and

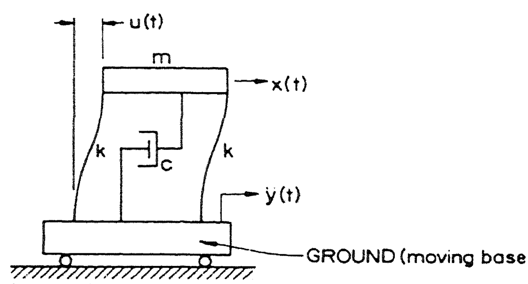


Figure 1. Dynamics of single-degree-of-freedom system.

the properties of the system (p, ζ). Furthermore, for most buildings and civil engineering structures, the damping ratio is low and a quasi-linear behavior of the ascillator can be assumed, thereby yielding the following relations (Blázquez, 1984):

$$SV \approx PSV \ ; \ SA \approx PSA \qquad (\zeta \to 0) \qquad (2)$$

The accuracy of equations (2) decreases as the damping ratio increases, whereas for small ζ values digitization errors are more critical than the above approximations.

Some advantages of using pseudo-response spectra instead of true response spectra are: a) significant savings in the cost and complexity of the calculations; b) the possibility of showing the three spectra (SD, PSV, PSA) together on a trilogarithmic graph, and "linearize" them for design purposes. From such a graph (Fig. 2), the spectral velocity, acceleration and displacement can be determined simultaneously along with some

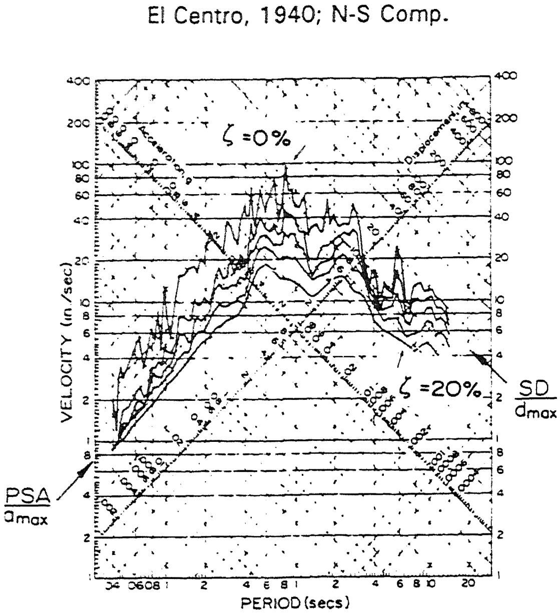


Figure 2. Example of use of tri- partite log-log plots of response spectra (Hudson, 1979).

estimates of its asymptotic values (e.g., the peak ground acceleration for zero period).

The latter point was developed further by Newmark (1967), who suggested factors to construct smooth response spectra from the peak motions of the ground, as a function of the structural damping. While this tripartite representation is adequate for low and intermediate-period systems, caution must be exerted in other cases, as is discussed next.

2 LIMITATIONS OF PSEUDO-RESPONSE SPECTRA FOR LONG PERIOD SYSTEMS

Since we are only interested in the zone of high periods of the response spectra, the so-called "shock spectra equations" for a short-duration sinusoidal wave are employed. Fig. 3 shows the normalized undamped spectra (with respect to the maximum ground motions) for a single sine wave pulse with period T_0 = 1 sec and amplitude 1 cm/sec^2, assuming that the oscillator starts from "at rest conditions". It can be seen that, at $\zeta = 0$, PSA = SA for all periods, but PSV ≠ SV. The reason for this discrepancy is that the expression for PSV involves a sine integral:

$$(PSV)_{\zeta=0} = p.(SD)_{\zeta=0} = \left| -\int_0^t \ddot{y}(\tau) \sin[p(t-\tau)]\, d\tau \right|_{max} \qquad (3)$$

whereas the SV formula involves a cosine integral which peaks differently with time:

$$(SV)_{\zeta=0} = \left| -\int_0^t \ddot{y}(\tau) \cos[p(t-\tau)]\, d\tau \right|_{max} \qquad (4)$$

Furthermore, beyond a certain critical period, T_{crit}, the SV curve departs clearly from the PSV curve and approaches its asymptotic value, the maximum velocity of the ground, $|\dot{y}(t)|_{max}$.

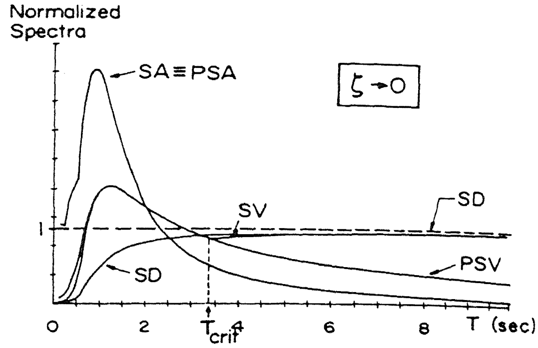


Figure 3. True and pseudo-response spectra for sine acceleration pulse (Blázquez and Kelly, 1988).

For the same conditions however, PSV = $\frac{2\pi}{T}$.SD → 0, since SD approaches its limiting value, $|y(t)|_{max}$, while T increases indefinitely (Hudson, 1979).

This simulation proves that, for a long period system, velocity and pseudo-velocity response spectra are not exchangeable, even in the case of very low damping, when the system works approximately as an undamped elastic spring.

3 IMPORTANCE OF THE LOW FREQUENCY COMPONENTS OF SEISMIC STRONG GROUND MOTION

In recent years, seismologists and engineers have been paying considerable attention to the topic of the characterization of earthquake motions in the long period range.

From the seismological perspective, the interest is focused in better knowing the strong ground motion at the epicentral zone, which would provide a better understanding of the rupture mechanism at the source of the quake. In addition, if the ground displacement could be restored from recorded accelerograms and the oscillatory and non-oscillatory motions differentiated, the residual part of the displacement would give us the actual slippage near the fault. Although such a possibility has been claimed by Graizer (1979), it is actually quite doubtful, since - as will be seen in section 4.3 - it is not feasible to separate permanent displacement (zero frequency component of the spectrum) from noise if standard filtering is applied to the accelerogram.

As for the engineering perspective, the trend to design more and more structures with high natural periods (suspension bridges, base-isolated structures, flexible high-rise buildings and like) has pushed the demand to better knowing the actual displacements imposed by the earthquake, as well as the response of the structure. In dealing with these cases, it must be kept in mind that the design of long period structures varies with respect to the conventional seismic design in two fundamental aspects: a) the uncertainties inherent in the recording and correcting processes of accelerograms are greater in the region of high periods; b) strictly speaking, the definition of the design earthquake for long period systems must be based on spectral displacements and velocities rather than on spectral accelerations (e.g., code specifications for base isolated structures).

4 FACTORS GOVERNING RESPONSE SPECTRUM ANALYSIS AT LOW FREQUENCIES

Response spectrum techniques are applicable to long period systems, provided that linearity is preserved and the calculations are carried out over a sufficient time interval, since the maximum response of the system is sometimes attained after the ground motion ceases. Thus, a quick assessment of the seismic performance of the system can be easily made. Nevertheless, such a performance is strongly affected by a number of factors whose relative influence on the response spectra must be carefully evaluated in each particular case. These factors are reviewed in the following sections from the engineering standpoint.

4.1 Source-and-path-dependent factors

Four governing factors are considered here: magnitude, epicentral distance, type of earthquake and directivity of the fault.

The magnitude and the epicentral distance affect both the attenuation curves and the response spectra of the earthquake. The effect is dual. First, the spectral ordinates for relatively long periods depend on the magnitude, both for soil and rock sites. Second, for a given magnitude and local geology the spectral ordinates depend also on the distance to the source.

Fig. 4 exemplifies these points with regard to the pseudo-velocity spectrum. It is clear from the figure that, especially for near-field records (for which epicentral distance is not adequate), scaling a fixed spectral shape by a peak motion parameter is fundamentally incorrect and leads to serious error in the long period region of the spectrum (Joyner and Boore, 1982).

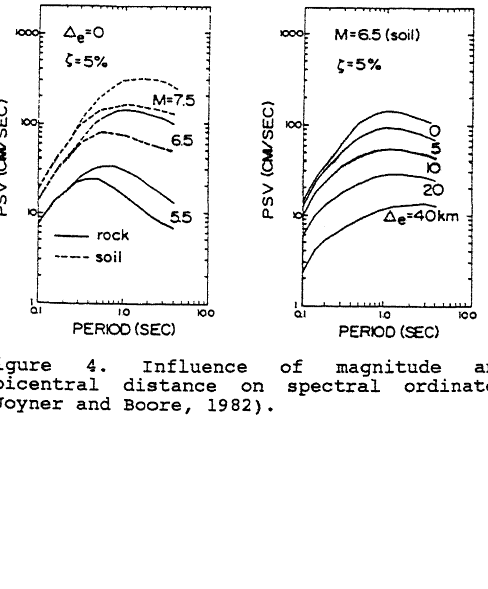


Figure 4. Influence of magnitude and epicentral distance on spectral ordinates (Joyner and Boore, 1982).

The type of earthquake also significantly affects the spectral curves. As expected, deep-focus earthquakes exhibit a poorer content on surface waves, which results in lower spectral ordinates for high periods, than shallow earthquakes (Fig. 5). Therefore, mean spectra and attenuation laws for subduction-zone earthquakes shall be derived independently, including the focal depth as an additional parameter (Kawashima et al., 1984).

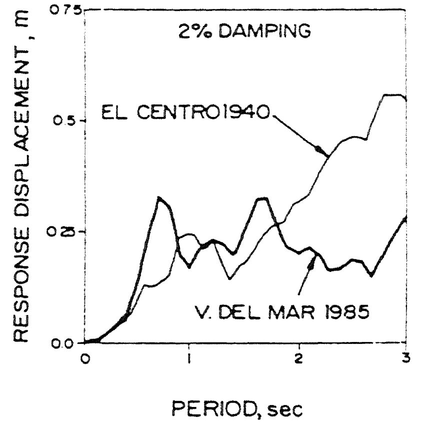


Figure 5. Displacement response spectra of typical deep-focus (Viña del Mar, 1985; Chile) and shallow (El Centro, 1940; California) earthquakes (Bariola and Fernández, 1991).

The directivity of the fault is important in the near-field zone and is similar to a Doppler effect caused by the moving seismic sources. Recent observations of this phenomenon have shown the appearance of low-frequency pulses in the velocity record (fling) to be more damaging than the acceleration pulses. The overall result is a spatial variation of the response spectra along the transmission path of the earthquake, as a function of the relative positions of the epicenter and the recording station. The spectral displacements are amplified in the direction of the rupture propagation; the same effect is observed on the spectral accelerations in the opposite direction (Fig. 6).

4.2 Site-dependent factors

The main factors related to the specific conditions at the site are topography, local geology, liquefaction and interaction phenomena.

Topographical effects are important for earthquakes recorded far away from the source on soft soil layers with sloping bedrock. In that case, pseudo-resonance phenomena take place between the predominant period of the input and the natural period of the layer:

$$T_s = \frac{4H}{V_s} \qquad (5)$$

H = thickness of layer
v_s = shear wave velocity

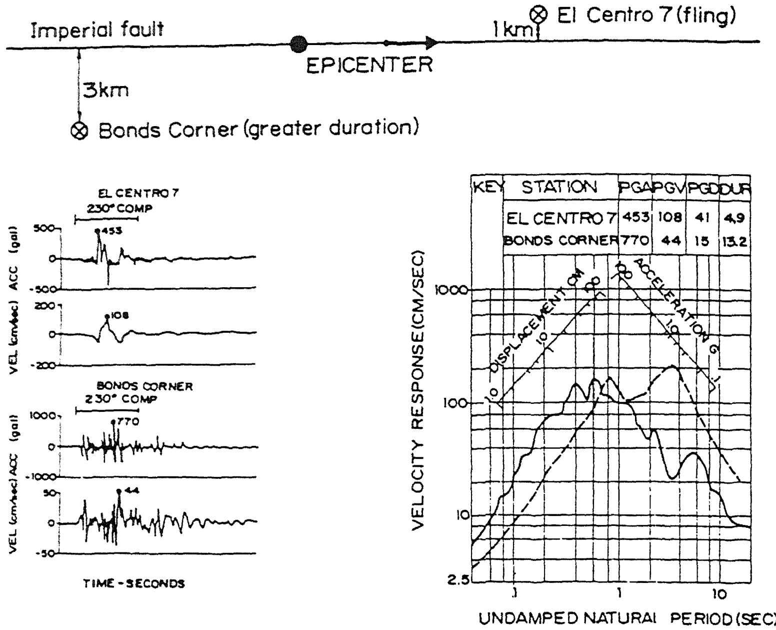


Figure 6. Directivity effects in near-field strong ground motion (Singh, 1981).

as has been repeatedly shown in the catastrophic Michoacán earthquake (Fig. 7) in 1985. Characteristically, the spectral shapes also vary with the strength of the foundation soil, and - according to eq. 5 - the softer the soil, the higher the response spectrum ordinates in the long period range. Recent observations of the in situ seismic performance of soil deposits in the epicentral zone (Loma Prieta earthquake, 1989) corroborate this well known local geology effect.

Sometimes liquefaction phenomena are responsible for the shift of the acceleration spectrum towards higher periods, since the dynamic porewater pressures developed in the liquefiable soil lead to a further softening of the layer and a subsequent increase in its natural period. This effect has been demonstrated by Finn (1981) comparing a total and an effective stress analysis of the seismic soil response. Indeed this could also be the explanation for the anomalous patterns of soil/rock amplification ratios which have

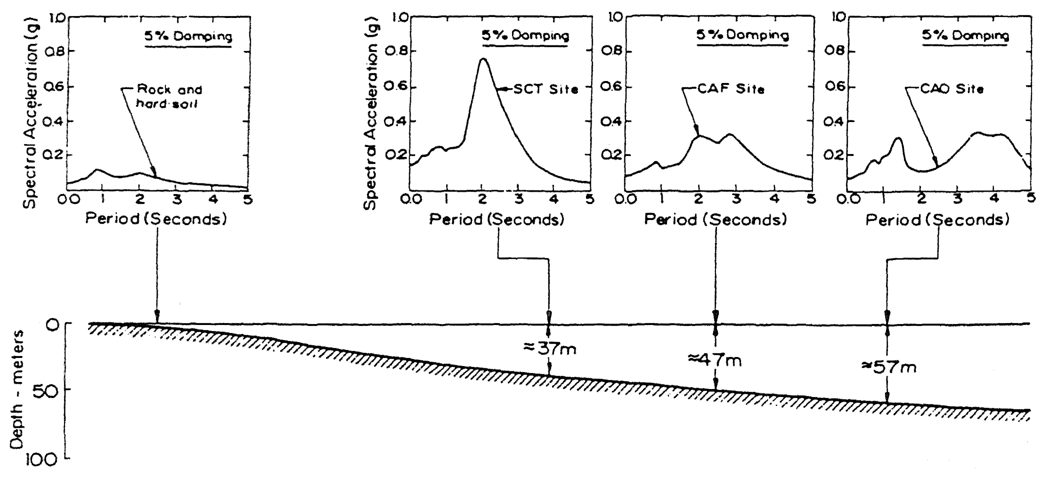


Figure 7. Site effects along propagation path of seismic waves in Mexico City (Michoacán earthquake; Sept. 19, 1985).

been found by Figueras et al. (1992) in ground motion from SMART-1 records at soft sites consisting of loose saturated silty sands.

Finally, the variation in shape of the spectrum with soil conditions may cause interaction phenomena, such as the "progressive collapse" of yielding structures founded on soft soils. It can be seen in Fig. 8 that in these cases, when the excitation goes on, the natural period of a cracked structure becomes progressively coupled with the predominant period of the input. As a result, the spectral amplification of the acceleration increases with the duration of shaking (normally high in these structures), and so does the damage in the structure.

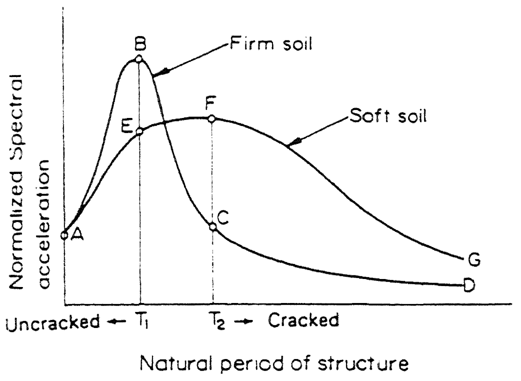


Figure 8. Progressive collapse phenomenon (Ohsaki, 1969).

4.3 Processing-dependent factors

These factors affect the long period response spectra differently, according to the type of record. In principle, the computational domain and the integration algorithm must be regarded. In addition, for non-digital records, the digitization process and the initial state of motion are critical factors for the accuracy of the analysis.

Due to the processing of analog accelerograms, digitization procedures are the main source of long period errors. These errors, as well as the methods to correct them for semi-automatic digitizers, have been described rigorously by Hudson (1979) as follows. The human error of digitization comes in the form of a high-frequency random error (operator-dependent) superimposed onto the curved trace resulting from averaging the baselines of the digitization samples. Precisely, this false baseline (which should be a straight line) constitutes a low-frequency **systematic error (Fig. 9), introduced by the digitizing machine itself.** Furthermore, the noise associated with this error is magnified by the repeated integration of the accelerogram in order to obtain velocities and displacements of the ground. This can be easily demonstrated, assuming that the false baseline of the accelerogram is a low-frequency sinusoidal curve, $a = A.\sin(\omega t)$, in the following manner:

$$d = \iint a\,dt = -\frac{A}{\omega^2}\sin\omega t = D\sin\omega t \qquad (6\text{-}1)$$

$$D = -\frac{A}{\omega^2}, \quad \omega \to 0 \Rightarrow D \to \infty \qquad (6\text{-}2)$$

To correct the digitization error, a high-pass filter (usually a Butterworth or Ormsby filter) is applied to the signal in order to secure the accuracy of the information above certain cut-off frequency, f_c. Although various agencies still use quite different fixed values of f_c for data processing (e.g., Caltech, f_c = 0.05-0.07 hz; USC, f_c = 0.7-1.0 hz), it is now widely accepted that the f_c parameter is accelerogram-dependent and varies with the level of noise associated with the digitization system, which in turn increases with the period.

Hence, in terms of spectra, the calibration data of the digitization system imposes a basic upper limit, $T_c = 1/f_c$, for determining the range of reliable long periods of the record (Fig. 10). Additionally, since in this region signal and noise amplitudes have similar levels, one must be careful to avoid excessive filtering that may eliminate valid information, uncontaminated by the digitization process. On the other hand, beyond the cross-point of the signal and noise spectra, the ground accelerations and their integrals are not distinguishable from the noise of the system, let alone the fault displacements (for an infinite period) recoverable from the record.

The influence of the initial state of motion of the system on the computation of response spectra has been recently investigated by Blázquez and Kelly (1988), and Ventura and Blázquez (1990, 1992). To state the problem, the solution of the differential equation of motion of the oscillator (eq. 1) is obtained as follows:

$$u(t) = u_t + u_f = u_t + U_0\,g(t) + \dot{U}_0\,h(t) \qquad (7)$$

where u_t and u_f stand, respectively, for the forced and free vibration parts of the motion, g(t) and h(t) are unit response functions, and U_0 and $\dot{U}_0$ are the system initial conditions. The physical existence of such conditions in optical accelerographs can be explained by analyzing the way these instruments function. As for levels of signal which fall below a prefixed threshold level, the instrument is not triggered. Unavoidably, that portion of the accelerogram remains unrecorded, originating some unknown values of the ground and the mass motion at time t = 0 (information gap). These values depend on the characteristics of the excitation and the mechanical properties of the system.

Using a sine-wave excitation of period T_0, Blázquez and Kelly (1988) have shown that, for long period systems ($T \gg T_0$), the free vibration motion can be neglected and "at rest" initial conditions apply:

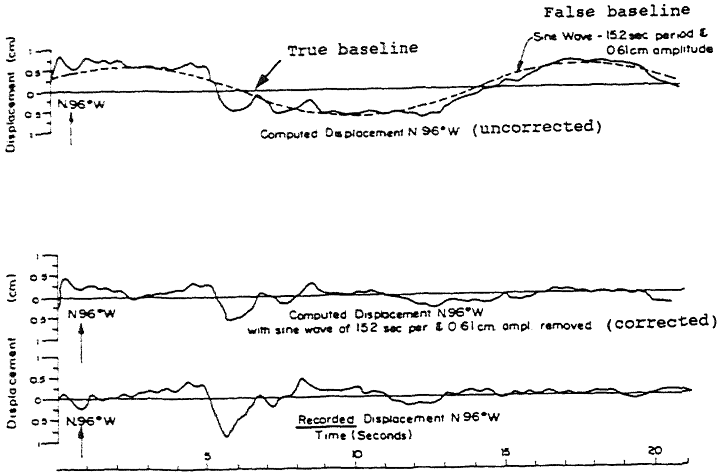
$$U_0 = u(0) = 0 \quad ; \quad \dot{U}_0 = \dot{u}(0) = 0 \qquad (8\text{-}1)$$

whereas, for short period systems ($T \ll T_0$), the following asymptotic relations:

$$U_0 = u(0) = -y_0 \quad ; \quad \dot{U}_0 = \dot{u}(0) = -\dot{y}_0 \qquad (8\text{-}2)$$

are satisfied, regardless of the damping level (Fig. 11).

Since the standard method for calculating response spectra assumes zero initial conditions, it is concluded that, strictly



Long Beach earthquake (Oct 2, 1933)

L.A.Subway Terminal St., Comp N96W

Figure 9. Spurious long-period noise in computed displacement records (adapted from Housner, 1947).

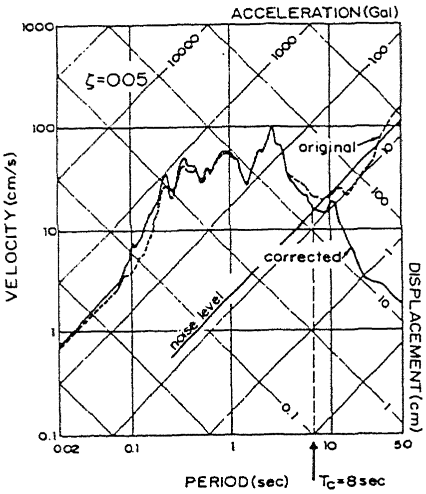


Figure 10. Determination of upper limit of reliable long periods of seismic record (Omote et al., 1987).

speaking, such a procedure applies only to high-frequency systems (e.g., rigid structures). For other types of systems (particularly flexible ones), the initial conditions have a significant effect on the long period regions of the true spectra and pseudo-spectra, leading to unconservative designs of these structures if the correct initial conditions are not properly accounted for. Similar results have been reported by Pecknold and Riddell (1978), who also found that, if standard spectral analysis of long period systems is performed, the pseudo-velocity response spectrum approaches asymptotically the initial ground velocity (instead of zero) for an infinite period. Therefore, it seems that at long periods the

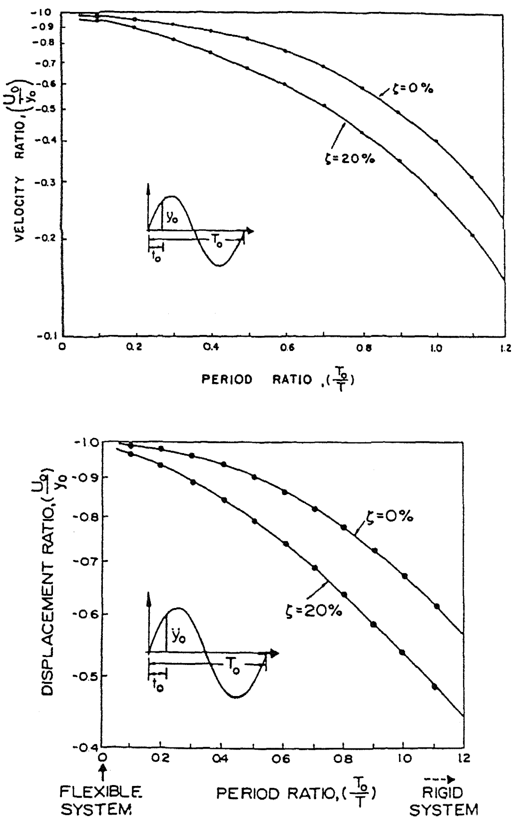


Figure 11. Ratio of initial motions of the oscillator to the initial motions of the ground for sinusoidal acceleration wave (Blázquez and Kelly, 1988).

initial ground velocity - rather than the initial ground displacement - dominates the response of the system.

Digitization procedures and system initial conditions have no influence on digital accelerographs, since these instruments incorporate a buffer memory which allows easy retrieval of the initial portion of the record. In these cases, only the numerical part of the processing scheme has to be reviewed.

The influence of the computational domain in the calculation of seismic response spectra has been investigated to some extent by Figueras (1991), although not isolated from the digitization, correction and integration steps of the processing scheme. The basic idea is to take advantage of the properties of the Fourier transform in the frequency domain and the FFT algorithm in order to make the calculations more effective and less time consuming. Fig. 12 shows an example of two corrected displacement records, one obtained in the time domain (Caltech method) and the other in the frequency domain, from the same uncorrected accelerogram. Although the similarity of both curves is encouraging, a rigorous study on the computation of the Duhamel integral involved in response spectra evaluations (u_t in eq. 7) is still lacking.

Restricting the analysis to the more frequently used time domain, the question of the integration algorithm comes into play due to: a) the assumption made on the variation of $\ddot{y}(t)$ between sampling points; b) the amplitude of the time integration step, $\Delta\tau$, used in Duhamel's formula (eqs. 3 and 4). The first issue has been discussed by Beaudet and Wolfson (1970) and Preumont (1982), by using the frequency domain analysis of time integration operators. It must be realized that, in the end, the integration scheme of acceleration values acts as a recursive low-pass digital filter that unevenly distorts the frequency content of the signal (which may result in misleading velocity, displacement or response spectrum curves).

The deleterious effect of the amplitude of time integration step on the accuracy of the spectral ordinates is portrayed in Fig. 13, which gives, in percentage (ϵ), the errors of the velocity and displacement spectrum ordinates (computed by different numerical integration schemes) relative to the exact analytical values. *It is clear from the figure* that the integration errors are larger for short-period systems and the Nigam-Jennings scheme is more accurate than the Runge-Kutta or Newmark algorithms. In all cases the ϵ-values increase with the length of the integration intervals, but, for high period systems, integration errors themselves are less critical than digitization or initial conditions errors.

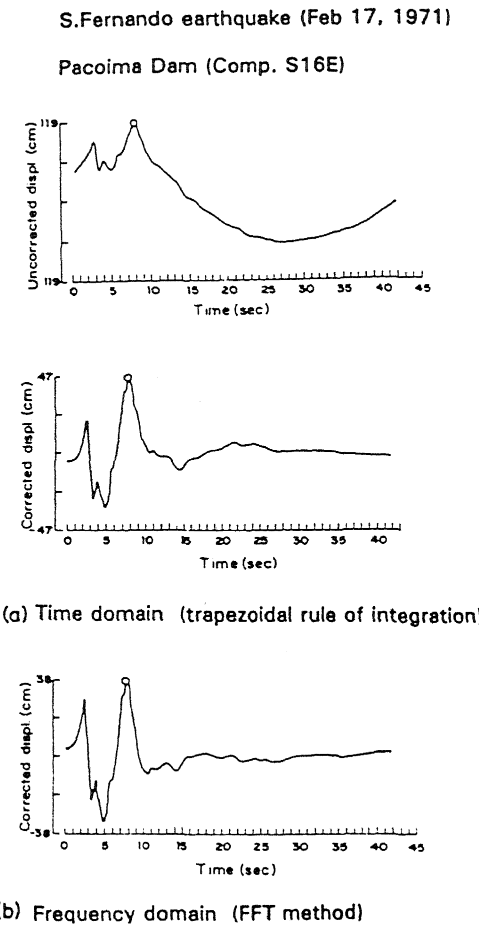


Figure 12. Effect of the computational domain on the values of the corrected displacement record (adapted from Figueras, 1991).

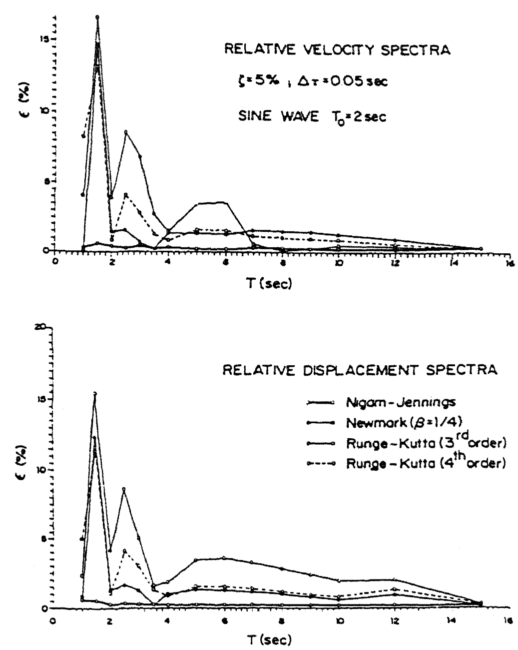


Figure 13. Effect of integration algorithm in the calculation of response spectra in the time domain.

5 CONCLUSIONS

A comprehensive review of the factors which govern the response spectrum values in the long period range has been presented. On the basis of the trends evidenced by this study, the following conclusions can be advanced:

a) velocity and pseudo-velocity response spectra are divergent for systems with long periods and/or high damping ratios, and are not exchangeable.

b) response spectrum values for high periods are very sensitive to source and site conditions; therefore, scaling a fixed spectral shape by a peak motion parameter is misleading in this region.

c) low-frequency errors due to the digitization process are enhanced by integration and pose a severe limitation on the accuracy of computed displacement records.

d) to eliminate the noise caused by baseline distortions, a high pass filter with cut-off frequency dependent on both the accelerogram and the digitization system must be used.

e) for long period systems, the assumption of zero-initial conditions is quite incorrect and introduces contaminating noise which magnifies the spectral velocity and the spectral displacements even more.

f) omitting initial conditions makes the pseudo-velocity spectrum deviate from zero at an infinite period, and asymptotically approach the peak ground velocity; therefore, the lower the damping, the greater the desviation is from the theoretical behavior.

g) all integration schemes act as low-pass filters which can introduce errors in the spectrum ordinates. Although those that deamplify long period noise perform advantageously (e.g., Nigam-Jennings algorithm), it can be concluded that the greater the integration interval, the greater the errors.

In summary: interpretation of response spectrum ordinates in the long period range requires some knowledge of the measurement and correction process of the data, especially for analog accelerograms. In order to avoid misinterpretation of the characteristics of the seismic records, potential users of this kind of information should exercise caution in differentiating signal values from spurious noise.

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