Earthquake Engineering, Tenth World Conference © *1994 Balkema, Rotterdam. ISBN 90 5410 060 5*

Simple physical models for foundation vibration
Towards a strength-of-materials approach

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ABSTRACT: As an alternative to rigorous boundary-element solutions, simple physical models can be used to determine e.g. the interaction force-displacement relationship (dynamic stiffness) of foundations. Translational and rotational cones and their corresponding lumped-parameter models together with simple one-dimensional wave patterns in the horizontal plane allow surface, embedded and pile foundations even for a layered site to be analysed and thus form a major step towards developing a strength-of-materials approach to foundation-vibration analysis. The analysis can mostly be performed directly in the time domain. The physical models provide physical insight which is often obscured by the mathematical complexity of rigorous solutions, offer simplicity in application as well as in the physics and in the rigorous mathematical solution of the physical model, are sufficiently general to enable reasonably complicated practical cases to be solved, exhibit adequate accuracy, allow physical features to be demonstrated and offer the potential for generalizations.

1 INTRODUCTION

A key aspect of any soil-structure-interaction or foundation-vibration analysis is the calculation of the interaction force-displacement relationship (dynamic stiffness) on the foundation-soil interface. An example follows. To determine this P_0-u_0 equation in the vertical direction for a rigid massless disk of radius r_0 on the surface of a homogeneous layer of depth d with shear modulus G, Poisson's ratio υ and shear-wave velocity c_s on rigid rock (Fig. 1a), *rigorous methods* exist: either the region below the disk is modelled with axisymmetric finite elements and sophisticated *consistent transmitting boundaries* are introduced to represent wave propagation towards infinity (Kausel et al 1975) or the *boundary-element method* is applied whereby the free and fixed surfaces of the layer must be discretized when the fundamental solution of the full space is used (Beskos 1987). In these rigorous methods a formidable theoretical background is required, and a considerable amount of expertise in idealising the actual dynamic system is necessary. A sophisticated computer program must be available. The computational expense involving the numerical evaluation of integrals and the solution of a large system of equations for just one run is large, making it difficult from an economical point of view to perform the necessary parametric studies. The mathematical complexity of rigorous methods often obscures the physical insight. The rigorous methods belong more to the discipline of applied mechanics than to civil engineering. These methods should only be used for large projects of critical facilities. For all other projects, *simple physical models* to represent the unbounded soil should be applied. For instance, the soil below the disk is modelled as a truncated rod (bar) with its area varying as in a *cone* (Fig. 1b). The vertical force applied to the disk produces dilatational waves propagating downwards from the disk. At the interface of the layer and the rigid rock the incident wave is reflected propagating back through the layer along the indicated cone in the opposite upward direction. The latter will reflect back at the free surface and then propagate downwards along its cone; upon reaching the interface of the layer and the rock, a reflection again takes place. The waves in the layer thus decrease in amplitude and spread resulting in radiation of energy towards infinity in the layer in the horizontal direction. Through the choice of the cone as a physical model, the complicated three-dimensional wave pattern with body and surface waves and three different velocities is replaced by the simple one-dimensional wave propagation governed by the one constant dilatational wave velocity of the conical rod, whereby *plane sections remain plane* (theory of strength of materials). Only one unknown needs to be introduced. For instance, the dynamic-stiffness coefficient equals (Meek and Wolf 1991)

$$S(\omega) = \frac{\left(1 + i\omega\frac{T}{\kappa}\right)K}{1 + 2\sum_{j=1}^{\infty}\frac{(-1)^j}{1 + j\kappa}e^{-ij\omega T}} \tag{1}$$

with the static-stiffness coefficient of the halfspace $K = 4Gr_0/(1-\upsilon)$, $T = 2d/c_p$ (c_p = dilatational wave velocity) and $\kappa = 2d/z_0$ (z_0 = apex height of cone, determining the opening angle, which follows from equating the static stiffness of the cone to that of the halfspace). After normalizing $S(\omega)$ by its static value, the real part

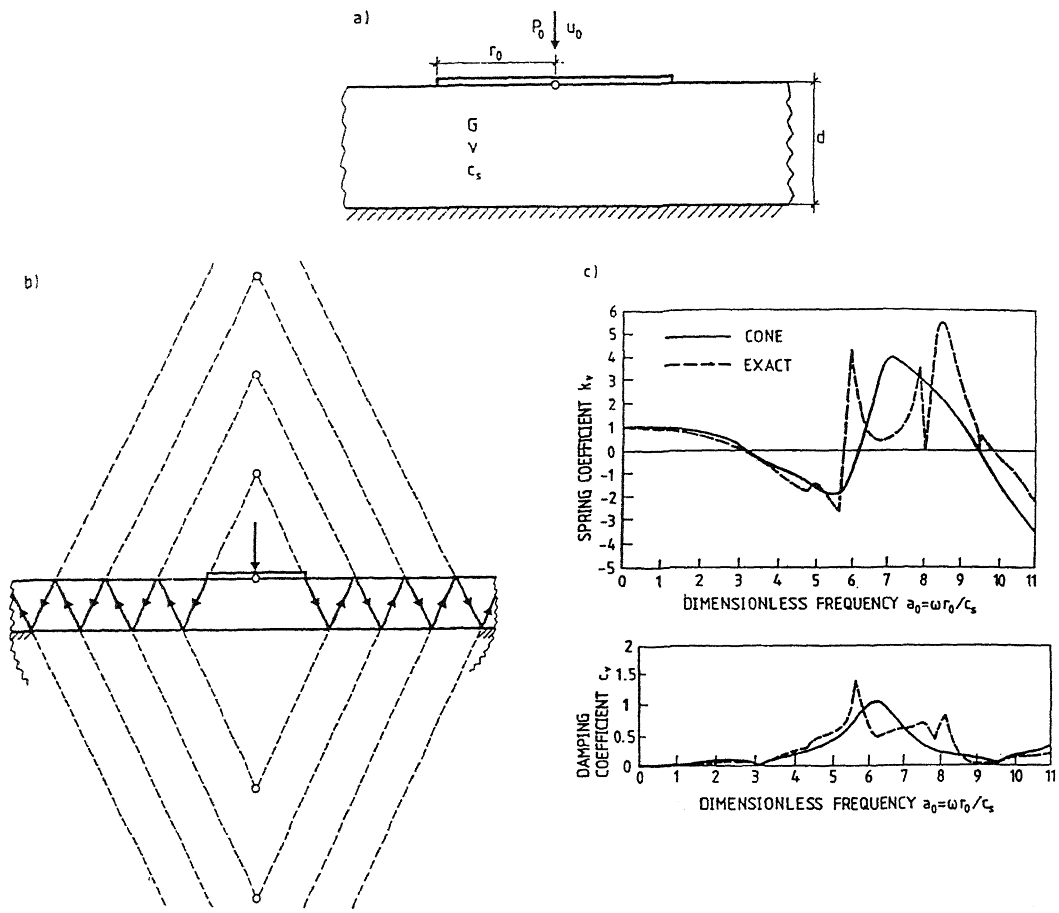

Figure 1. Vertical motion of surface foundation on soil layer on rigid rock.

a) Disk on layer (d/r_0=1, ν=1/3).

b) Cone model with reflections at fixed and free surfaces of layer.

c) Vertical dynamic-stiffness coefficient.

$k_v(a_0)$ and the imaginary part divided by a_0, $c_v(a_0)$, shown in Fig. 1c (Meek and Wolf 1992b), yield a smooth approximation in the sense of a good fit to the exact solution (Kausel 1990) which becomes increasingly irregular ($a_0 = \omega r_0/c_s$). Analyses with a *hand calculator* are possible using the cone model. The analysis can also be performed directly in the familiar time domain, which also allows nonlinear systems to be calculated. Applying these simple physical models leads to some loss of precision, which is more than compensated by their many advantages. As the simple models cannot cover all cases, they do not supplant the much more generally applicable rigorous boundary-element method, but rather supplement it.

In this paper, simple physical models to determine the interaction force-displacement relationship (dynamic stiffness) are discussed. For other tasks of foundation-vibration analysis, such as determining the foundation-input motion and the free-field response based on the same physical models and on others, the reader should consult the references. Emphasis is placed on describing the types of foundations which can be analysed with the physical models, stressing the physical insight provided, the simplicity in use and the adequate accuracy. The derivation and why the physical models work as well as they do are not addressed. This paper summarizes the practical results of a three-year research effort performed together with Dr. J.W. Meek, who developed almost 20 years ago a rotational cone with the corresponding lumped-parameter model (Meek and Veletsos 1974) which provided the starting point for much of this and other developments (Gazetas 1984). The work on physical models for foundation-vibration analysis is also summarized in a forthcoming book (Wolf 1994).

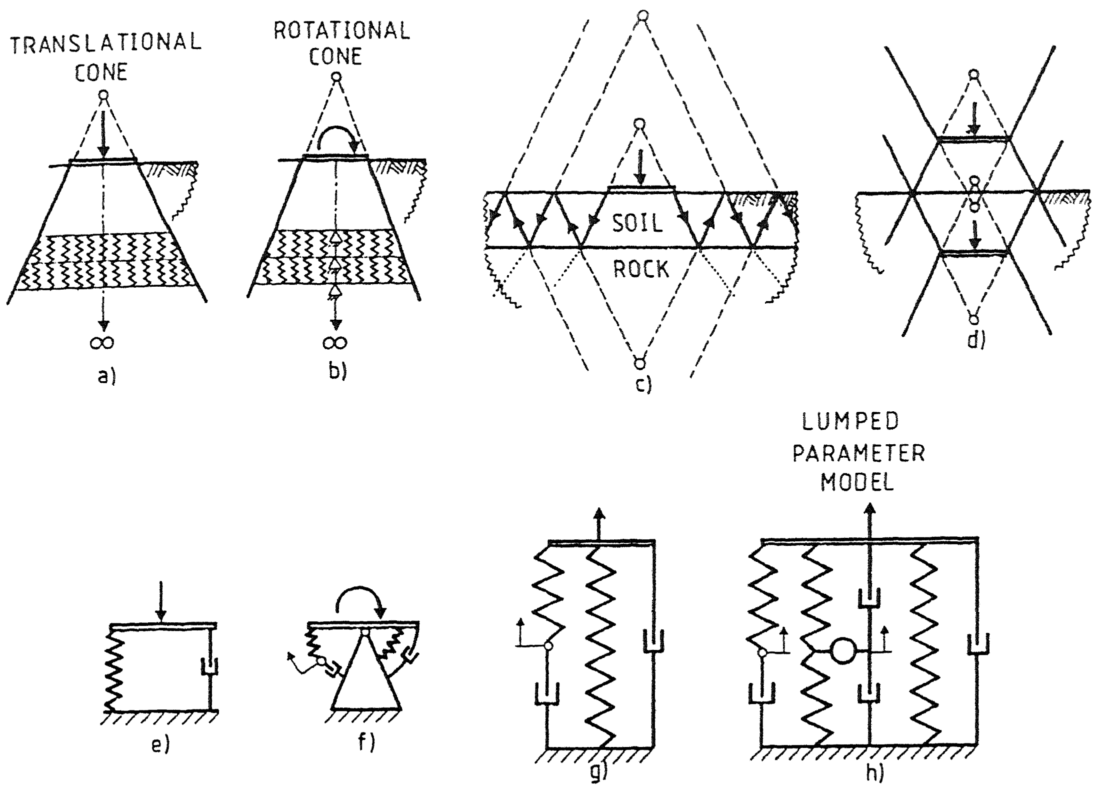

Figure 2. Cones and lumped-parameter models.

a) Disk on surface of halfspace with truncated semi-infinite translational cone.
b) Disk on surface of halfspace with truncated semi-infinite rotational cone.
c) Disk on surface of soil layer resting on flexible rock halfspace with corresponding cones.
d) Symmetry condition with respect to free surface for disk and mirror-image disk with corresponding double cones to calculate Green's functions.
e) Discrete-element model for translational cone.
f) Discrete-element model for rotational cone.
g) Lumped-parameter model consisting of springs and dashpots with one internal degree of freedom corresponding to *f*.
h) Lumped-parameter model consisting of springs, dashpots and a mass with two internal degrees of freedom.

2 CONCEPTS, CLASSIFICATION AND EXAMPLES

An overview of the physical models is given next. The model shown in Fig. 1b is an extension of the well-known *translational* truncated semi-infinite *cone* to represent a disk on the surface of a *halfspace* (Fig. 2a), which, when working with shear distortions, can also be used to analyse the horizontal motion. For the rocking and torsional motions, *rotational cones* (Fig. 2b) can be identified using the same concepts (Meek and Wolf 1992 a).

Fig. 2c shows the extension of the cone model to calculate a disk on the surface of a *soil layer resting on flexible rock halfspace*. The concept is the same as explained in connection with rigid rock (Fig. 1b), but at the interface of the layer and the rock, a refracted wave propagating in the rock in the same direction as the incident wave along its own cone (dotted lines) is created in addition to the reflected wave. The only modification in the equation of the dynamic-stiffness coefficient (equation (1)) consists of replacing -1 by minus the reflection coefficient which is calculated based on the rod theory of cones (Wolf and Meek 1993b). High accuracy is again achieved. For instance, the ratios of the vertical static-stiffness coefficient of a disk on a layer resting on flexible rock with $d/r_0 = 1$, $\upsilon = 1/3$ calculated using cones to the exact value equal 0.962, 1.002 and 1.016 varying the ratios of the constrained modulus of the layer to that of the rock as 0 (rigid rock), 0.2 and 0.4, respectively. The other motions can also be processed.

The concepts of cone models can be expanded to the analysis of *embedded cylindrical foundations*. Again, the vertical degree of freedom is addressed in Fig. 2d, but the following argumentation is just as valid for the horizontal, rocking or torsional ones. To represent a disk within an elastic fullspace, a double-cone model is introduced (Meek and Wolf 1993a, Wolf and Meek 1993a). Their displacement fields define approximate Green's functions for use in an uncomplicated (one-dimensional) version of the boundary-element method. To enforce the stress-free condition at the free surface of the halfspace (Fig. 2d), two identical double cones placed symmetrically (with respect to the free surface) and excited simultaneously by the same forces are considered. Indeed, any halfspace problem amenable to solution via cone models may also be solved in the full space. It is only necessary to augment the actual foundation in the lower halfspace by its mirror image in the upper halfspace. By exploiting principles of symmetry and superposition, the soil's flexibility matrix defined at the disks located within the embedded part of the foundation can be set up. The rest of the analysis follows via conventional matrix methods. The methodology points towards a general *strength-of-materials approach to foundation dynamics* using the approximate *Green's functions of the double-cone models*. As an example, the vertical motion of a rigid cylinder embedded with the depth e in a layer on rigid rock (Fig. 3a) is discussed (Wolf, Meek and Song 1992). In the embedded part of the foundation (Fig. 3b) 8 disks with double cones are selected (two are shown in the figure, one with a solid and one with a dotted line). To enforce approximately the stress-free condition at the free surface and the fixed boundary condition at the base of

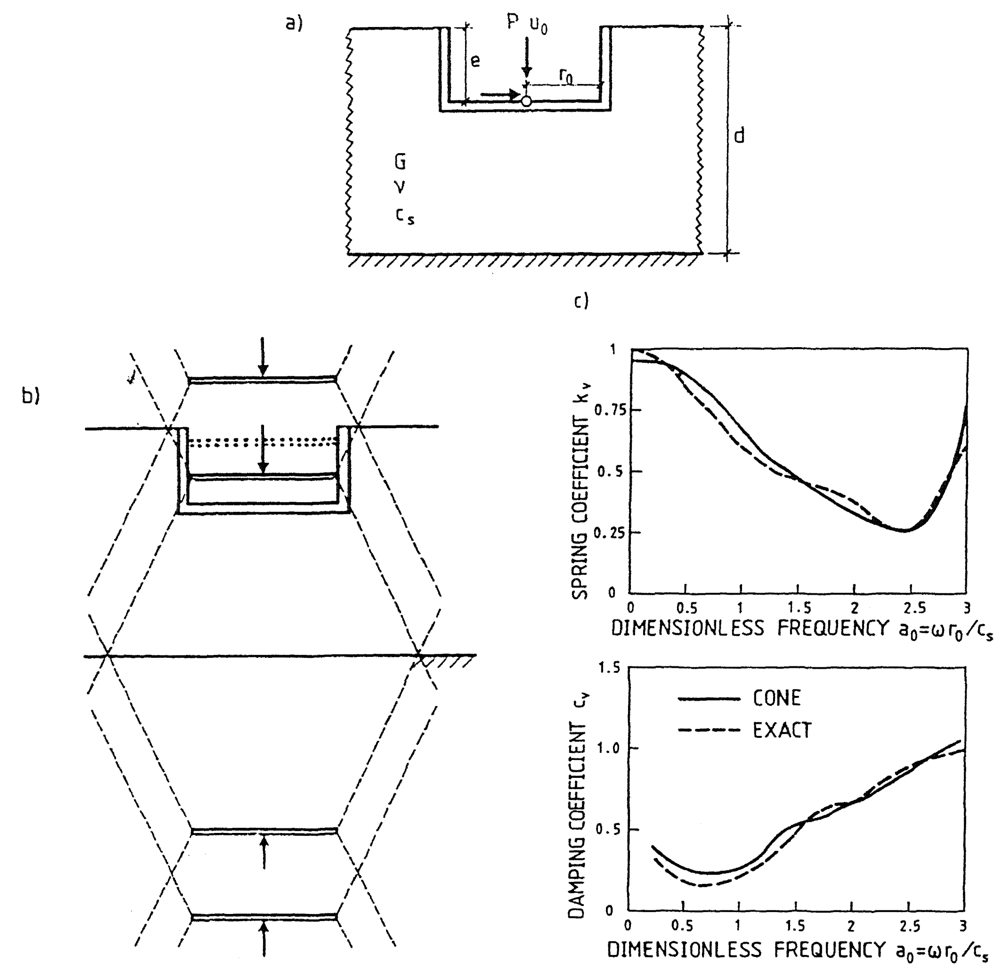

Figure 3. Vertical motion of foundation embedded in soil layer on rigid rock.

a) Cylinder embedded in layer (e/r_0=1, d/r_0=3, ν=1/3, 5% material damping).
b) Array of disks with double-cone models and mirror image disks.
c) Vertical dynamic-stiffness coefficient.

the layer, mirror images of the disk with the loads acting in the indicated directions with the corresponding double cones (dashed lines) are introduced. The dynamic-stiffness coefficient (Fig. 3c) is surprisingly accurate, as can be seen from a comparison with the rigorous solution determined with a very fine mesh of boundary elements.

Returning to the translational and rotational cones of Figs. 2a and b, it should be emphasized that in an actual soil-structure-interaction analysis the cones are not represented physically by finite elements of a tapered rod. For practical applications it is not necessary to compute explicitly the displacement field propagating along the cone. The translational cone's dynamic stiffness can rigorously be represented by the *discrete-element model* shown in Fig. 2e. It consists of a spring with the static-stiffness coefficient (of the halfspace) in parallel with a dashpot with the coefficient determined as the product of the density, the dilatational-wave velocity (for the vertical motion) and the area of the disk (doubly-asymptotic approximation). One rigorous representation of the rotational cone's dynamic stiffness (Fig. 2f) consists of a spring with the static-stiffness coefficient in parallel with a dashpot with as coefficient the exact high-frequency limit of the dynamic stiffness (density times dilatational-wave velocity times disk's moment of intertia). An additional degree of freedom is also introduced, connected by a spring (with a coefficient equal to minus a third of the static stiffness)

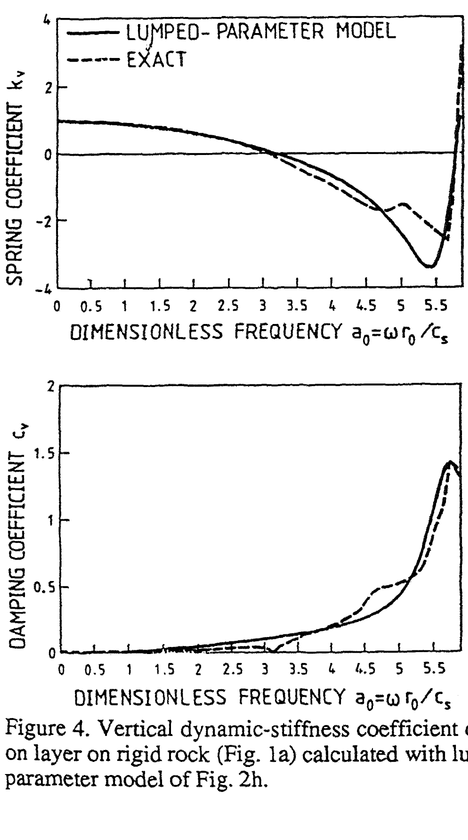

Figure 4. Vertical dynamic-stiffness coefficient of disk on layer on rigid rock (Fig. 1a) calculated with lumped-parameter model of Fig. 2h.

to the footing and by a dashpot (with a coefficient equal to minus the high-frequency limit) to the rigid support.

The model of Fig. 2f shown in Fig. 2g for the translational motion forms the starting point to develop systematically a family of consistent *lumped-parameter models*. The direct spring is chosen to represent the static stiffness. The coefficients of the other spring and the two dashpots are selected so as to achieve an *optimum fit* between the dynamic stiffness of the lumped-parameter model and the corresponding exact value (originally determined by a rigorous procedure such as the boundary-element method). If the direct dashpot is used to represent the high-frequency limit of the dynamic stiffness, the number of coefficients available for the optimum fit is reduced to two. To increase the number of coefficients and thus the accuracy, several systems of Fig. 2g can be placed in parallel. Fig. 2h shows the lumped-parameter model for three such systems, whereby two of them are combined to form a new system consisting of two springs, one independent dashpot (the two dashpots in series have the same coefficient) and a mass. A total of six coefficients keeping the doubly-asymptotic approximation thus results. It can be shown that these six frequency-independent coefficients, which can be determined using curve fitting applied to the dynamic stiffnesses (involving the solution of a linear system of equations only) will be real (but not necessarily positive). The *eight springs, dashpots and mass* will represent a *stable lumped-parameter model with only two additional internal degrees of freedom*. Use is thus implicitly made of the results obtained with the state-of-the-art formulation which leads to the rigorous dynamic stiffnesses used in the optimum fit. For illustration, the lumped-parameter model of Fig. 2h is used to calculate the vertical dynamic-stiffness coefficient shown in Fig. 4 of a disk on a layer on rigid rock (Fig. 1a). The lumped-parameter model for the *coupled horizontal and rocking motions* of a cylinder embedded in a layer on rigid rock (Fig. 3a) is shown in Fig. 5a (Wolf and Paronesso 1992). The coupling term is represented by placing the lumped-parameter model of Fig. 2h at an eccentricity *e*. The agreement for the horizontal dynamic-stiffness coefficient in Fig. 5b is good.

The models shown in Fig. 2 prescribe a displacement pattern varying with depth along the axis of the cone. To extend the application, *displacement patterns in the horizontal plane* other than those corresponding to the strength-of-materials assumption of plane cross-sections remain plane can be introduced. One-dimensional wave propagation is again prescribed. Using nonmathematical physical reasoning and calibration with rigorous solutions, the vertical displacement on the free surface of a halfspace for a loaded source subdisk (Fig. 6a) can be derived (Meek and Wolf 1993b). This approximate *Green's function* exhibits a different dynamic behaviour in the *near and far fields*. *Arbitrary shaped surface foundations* can be treated as an assemblage of subdisks. The dynamic-stiffness coefficients of the vertical and rocking motions of a rigid square foundation of length 2a on the surface of a halfspace are calculated (Fig. 7), whereby one

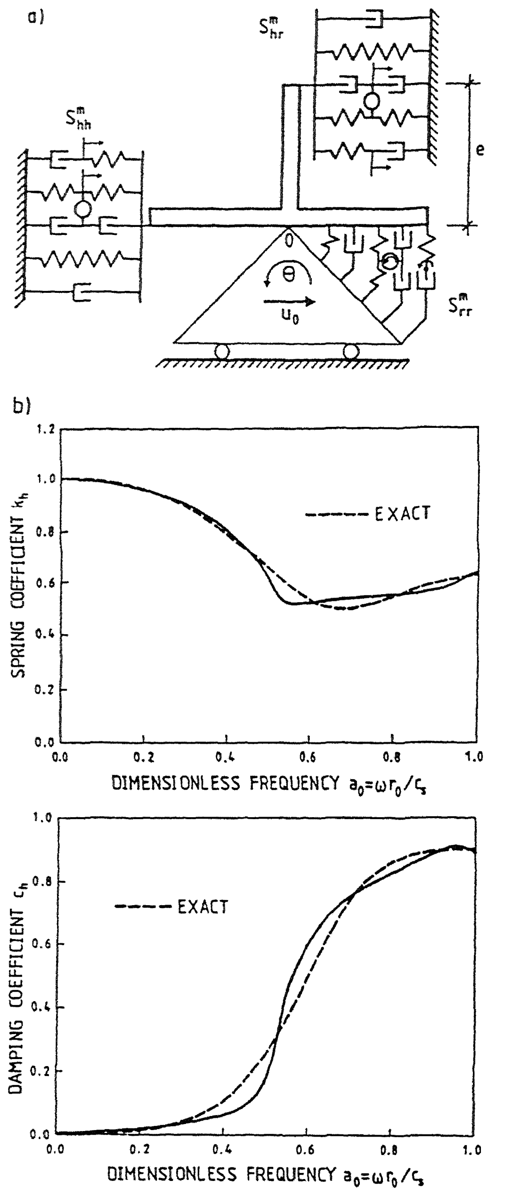

Figure 5. Coupled horizontal and rocking motions of cylinder embedded in soil layer on rigid rock (Fig. 3a).
a) Lumped-parameter model.
b) Horizontal dynamic-stiffness coefficient.

quadrant is discretized with 7x7 subdisks. As another displacement pattern, *cylindrical waves* can be assumed to propagate with the propagation velocites indicated in Fig. 6b to determine *dynamic interaction factors*, describing the effect of a source pile on a receiver pile (Dobry and Gazetas 1988). This allows *pile groups* taking *pile-soil-pile interaction* into consideration to be analysed. As an example the 3x3 pile group shown in Fig. 8a is analysed (E = Young's modulus of elasticity,

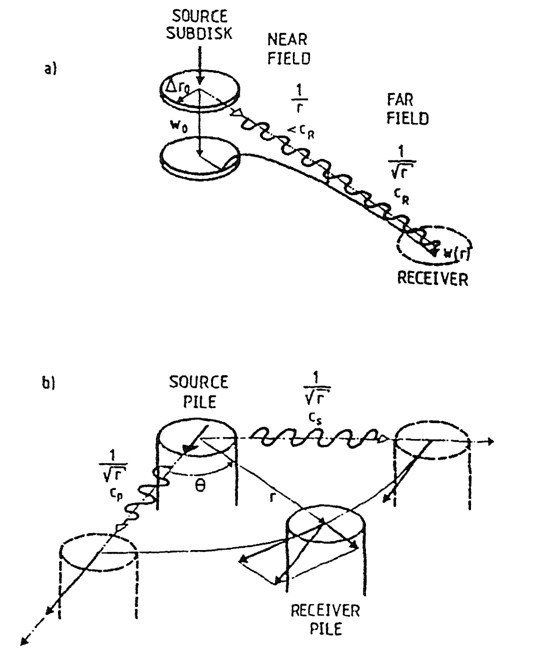

Figure 6. Displacement patterns in horizontal plane.
a) Vertical displacement on free surface from loaded subdisk.
b) Horizontal displacement from loaded pile.

ρ = density, s = distance between axes of two neighbouring piles, ℓ = length of pile, $2r_0$ = diameter of pile). 25 disks with the corresponding double cones (Fig. 2d) are used to model the single pile (Wolf, Meek and Song 1992). Dynamic-interaction factors are determined based on the sound physical approximation of Fig. 6b, modified approximately for the vertical motion. The vertical dynamic-stiffness coefficient of the pile group calculated with cones is astonishingly accurate (Fig. 8b). Even details of the strong dependency on frequency are well represented.

3. REQUIREMENTS

The following requirements are essential for successful physical models, which are suitable for everyday practical foundation-vibration analysis.

a) *Physical insight.* By simplifying the physics of the problem, *conceptual clarity* with *physical insight* results (see e.g. Fig. 1b).

b) *Simplicity.* Due to the simplification of the physical problem, the physical model can be solved mathematically rigorously in closed form even in the time domain, satisfying the fundamental principles of wave propagation and dynamics. The *practical application* is also simple, together with the *physics* and the *mathematical solution*..

c) *Generality.* To be able to provide engineering solutions to reasonably complicated practical cases, the physical models must reflect the following key aspects

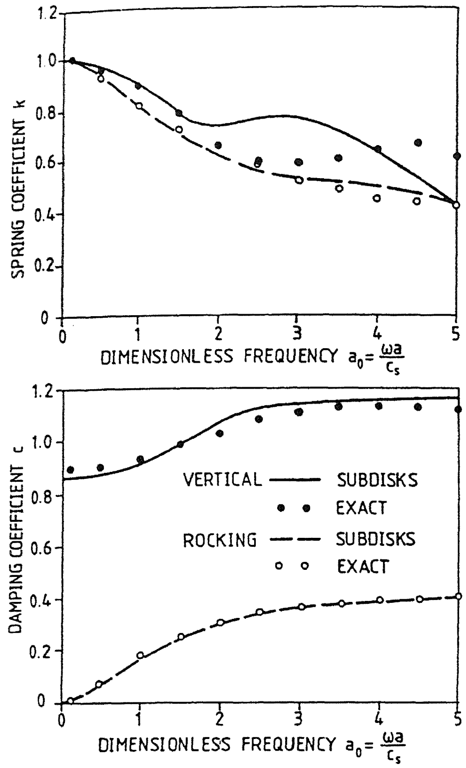

Figure 7. Dynamic-stiffness coefficients of square foundation on soil halfspace ($\nu = 1/3$).

for all translational and rotational motions in all frequency ranges: the *shape* of the foundation-soil interface (see e.g. Fig. 7), the *soil profile* (see e.g. Fig. 2c) and the amount of *embedment* (see e.g. Figs. 3a, 8a) must be able to vary significantly.

d) *Accuracy*. Due to the many uncertainties, the accuracy of any analysis will always be limited. A deviation of ± 20% of the results of the physical models from those of the rigorous solution for one set of input parameters is, in general, sufficient. This *engineering accuracy* criterion is, in general, satisfied.

e) *Demonstration of physical features*. The physical models are also well suited to demonstrate certain unexpected features. Placing a row of an infinite number of vertical point loads on the surface of a halfspace, the vertical dynamic stiffness of a rigid surface strip (two-dimensional) can be derived in closed form, whereby the static stiffness is zero, the spring coefficient increases abruptly to a more or less constant value, and the damping coefficient begins from infinity and then diminishes for increasing a_0 asymptotically. The same features exist in the rigorous solution.

To model a strip foundation on a halfspace, wedges can be used which are again based on rod theory, just as cones represent a disk on a halfspace (Meek and Wolf 1992c). For both the translational and rotational motions, the *damping ratios* $\zeta = b_0 c(b_0)/[2k(b_0)]$ of the *wedges* are significantly *larger* than those of the *cones* (Fig. 9). The multipliers are also specified. z_0 in the definition of b_0 shown in the figure is the apex height of the cone or the wedge.

For the frequency range below the cutoff frequency of the layer on rigid rock, the radiation damping and thus the damping coefficient of the dynamic stiffness vanishes, which is well simulated using cones and lumped-parameter models (see e.g. Figs. 1c and 4, where below the *cutoff frequency* for dilatational waves, $a_0 = \pi$, $c(a_0)$ is very small).

f) *Potential for generalization*. The *concepts* and certain *features* of the physical models can be *generalized* and the results applied in much more sophisticated calculations. The Green's functions of the physical models pave the way to understand the rigorous boundary-element method. A consistent lumped-parameter model for the dynamic-stiffness matrix of any general flexible foundation can be systematically constructed starting from the same basic lumped-parameter model (Fig. 2g).

Summarizing, the cone models with the prescribed deformation of rod (bar) theory, the lumped-parameter models based on them and the deformation patterns in the horizontal plane present a major step towards developing a *strength-of-materials approach to foundation dynamics*. The aim is the same as that used routinely in stress analysis of structural engineering, where e.g. for very complicated skew curved prestressed concrete bridges beam theory is applied successively and the general three-dimensional theory of elasticity is not needed. Concluding the dynamic analyst should always «Make things as simple as possible but no simpler» (H. Einstein). Or to state it differently: *«Simplicity that is based on rationality is the ultimate sophistication»* (A.S. Veletsos).

4. ENGINEERING APPLICATIONS

As a first example, a machine foundation on the surface of a soil halfspace excited by a three-cylinder compressor operating at 9 Hz with the cranks at 120° resulting in a moment (Fig. 10a) is investigated (Wolf 1994). The discrete-element models of the cones shown in Figs. 2-f and 2-e are used to represent the soil in the rocking and horizontal motions. Including the coupling between the horizontal and rocking motions increases the response (Fig. 10b).

In the second example, a rigid disk on the surface of a soil layer on rigid rock is loaded with a unit impulse force. The agreement of the vertical displacement as a function of time calculated with the (layered) cone model of Fig. 1b (or Fig. 2c) with the exact result is very satisfactory (Fig. 11). The jump discontinuities arising from the reflected waves are well represented (Meek and Wolf 1992b).

The third example addresses nonlinear soil-structure-interaction analysis (Fig. 12a). The vibration of a hammer foundation embedded in a soil layer on rigid rock (d/r_0=2) with an excentrically mounted anvil is

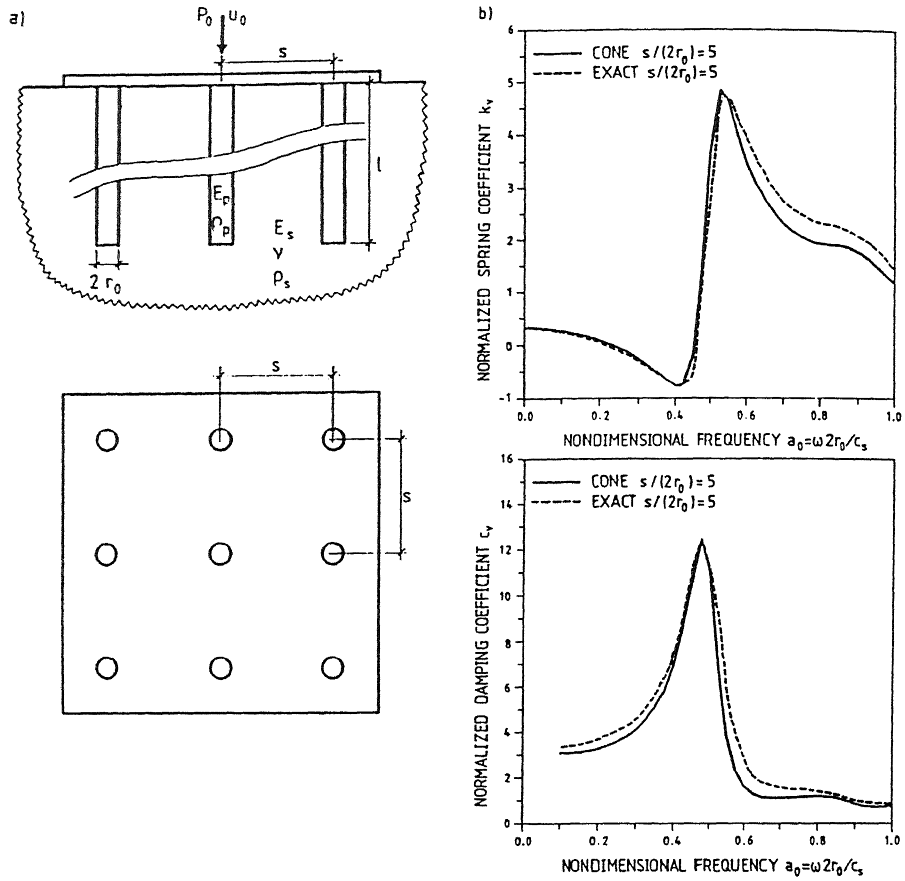

Figure 8. Vertical motion of floating pile group in soil halfspace.

a) Elevation and plan view of 3x3 pile group [$s/(2r_0)$ = 5, $l/(2r_0)$ = 15, ν = 0.4, E_p/E_s = 1000, ρ_p/ρ_s = 1.4, 5% material damping].

b) Dynamic-stiffness coefficient.

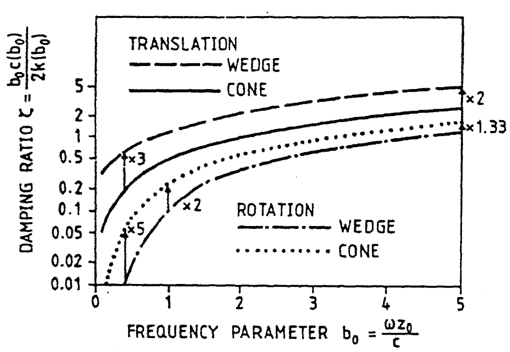

Figure 9. Comparison of three-dimensional modelling (cone) and two-dimensional modelling (wedge).

examined (Wolf and Paronesso 1992). The head impacts with a velocity 5 m/s against the anvil. As a tension-resistant connection for the pads of the anvil is not provided, the anvil will partially uplift from the block, when the dynamic stress in tension exceeds the static stress. The dynamic system with 14 degrees of freedom is constructed using the lumped-parameter models of Figs. 5a and 2h. As expected, the partial uplift of the anvil increases the motion significantly when compared with the result of a linear analysis (Fig. 12b). The response for a soil halfspace is also plotted.

As a final example, a rigid block with individual footings, which can uplift, resting on the surface of an soil layer with c_s = 750 m/s and d/a = 1 is discussed (Fig. 13a). An idealized horizontal earthquake acts during 2s (Wolf and Paronesso 1991). Only the vertical rocking motions of the block's bottom centre are considered in the calculation. The lumped-parameter model of Fig. 2-h is used to represent the soil neglecting through-soil coupling. As the fundamental frequency in rocking lies below the cutoff frequency of the layer, no radiation damping occurs during the free vibration phase after 2s. This leads to no decay

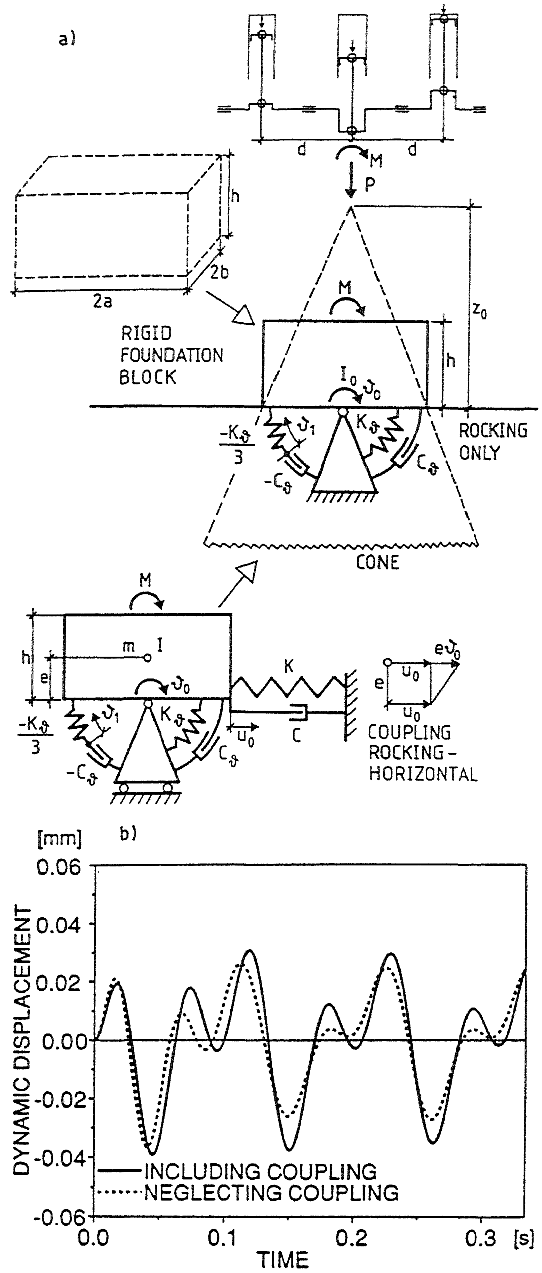

Figure 10. Machine foundation.
a) Three-cylinder machine with dynamic models.
b) Vertical displacement at edge of foundation block.

occurring in the gaps (Fig. 13b). If the soil is a halfspace, the gaps are much smaller and decay rapidly after 2s (Fig. 13c).

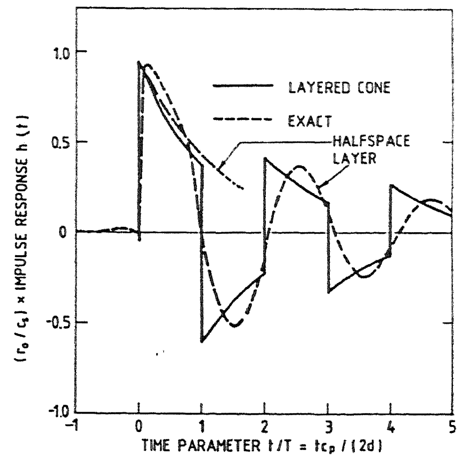

Figure 11. Vertical impulse response of soil layer on rigid rock (d/r_0=1, υ=1/3).

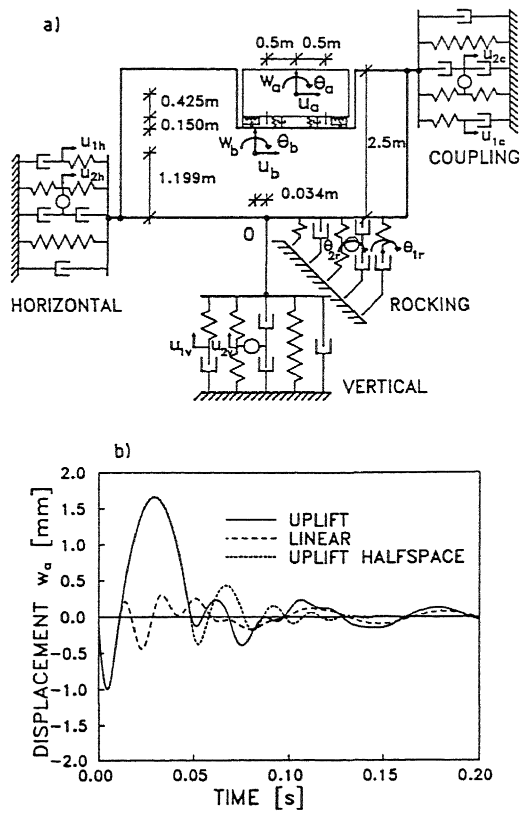

Figure 12. Hammer foundation with partial uplift of anvil.
a) General layout.
b) Vertical displacement at centre of anvil.

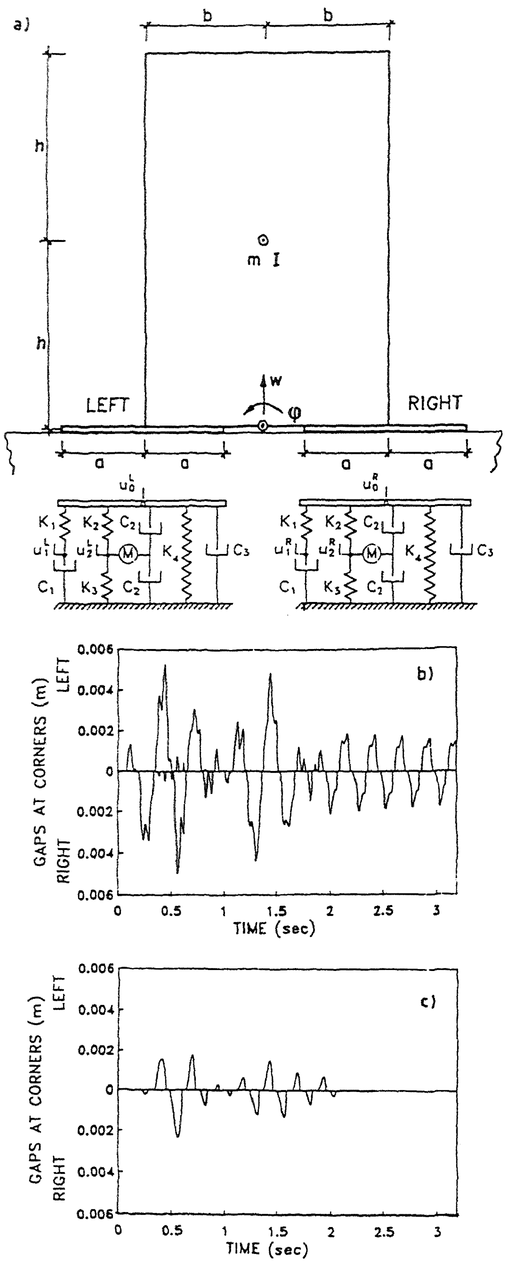

Figure 13. Rigid block on layer and halfspace with partial uplift.
a) Rigid block on disk with lumped-parameter models.
Vertical gaps between block and disk for:
b) Layer.
c) Halfspace.

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